Kelvin–Helmholtz instability of magnetohydrodynamic waves traveling along a coronal mass ejecta in the lower corona

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Introduction Coronal mass ejections (CMEs) are huge clouds of magnetized plasma that erupt from the solar corona into the interplanetary space. They propagate in the heliosphere with velocities ranging from 20 to 3200 $\mathrm{km\,s^{-1}}$ with an average speed of 489 $\mathrm{km\,s^{-1}}$, based on *SOHO*/LASCO coronagraph measurements between 1996 and 2003. CMEs are associated with enormous changes and disturbances in the coronal magnetic field and are the major contributor to severe space weather at the Earth. Flows and instabilities play a major role in the dynamics of magnetized plasmas including the solar corona. In 2011, Foullon et al. [1] have reported the first observations of the temporally and spatially resolved evolution of the magnetic Kelvin–Helmholtz (KH) instability, developing at the surface of a fast CME less than 150 Mm above the solar surface in the inner corona. Unprecedented high-resolution imaging observations of vortices developing at the surface of a fast coronal mass ejecta were taken with the Atmospheric Imaging Assembly (AIA) on board the *Solar Dynamics Observatory* (*SDO*), validating theories of the nonlinear dynamics involved. An updated and detailed study by Foullon et al. [2] of the dynamics and origin of the CME on 2010 November 3 by means of the Solar TErrestrial RElations Observatory Behind (*STEREO-B*) located eastward of *SDO* by 82° of heliolongitude, and used in conjunction with *SDO* give some indication of the magnetic field topology and flow pattern. At the time of the event, Extreme Ultraviolet Imager (EUVI) from *STEREO*'s Sun–Earth Connection Coronal and Heliospheric Investigation (SECCHI) instrument suite achieved the highest temporal resolution in the 195 Å bandpass: EUVI's images of the active region on the disk were taken every 5 minutes in this bandpass. The authors applied the Differential Emission Measure (DEM) techniques on the edge of the ejecta to determine the basic plasma parameters – they obtained electron temperature of 11.6 ± 3.8 MK and electron density $n = (7.1 \pm 1.6) \times 10^8$ $\mathrm{cm^{-3}}$, together with a layer width of $\Delta L = 4.1 \pm 0.7$ Mm. Density estimates of the ejecta environment (quiet corona) vary from (2 to 1)$\times 10^8$ $\mathrm{cm^{-3}}$ between 40 and 100 Mm, at heights where the authors started to see the KH waves developing. The final estimation based on a maximum height of 250 Mm and the highest DEM value on the northern flank of the ejecta yields electron density of $(7.1 \pm 0.8) \times 10^8$ $\mathrm{cm^{-3}}$. The adopted electron temperature in the ambient corona is $T = 4.5 \pm 1.5$ MK. The other important parameters derived by using the pressure balance equation assuming a benchmark value for the magnetic field B in the environment of 10 G are summarized in Table 2. The main features of the imaged KH instability presented on Table 3 include (in their notation) the speed of 131 Å CME leading edge, $V_{\rm LE} = 687$ km s^{-1}, flow shear on the 131 Å CME flank, $V_1 - V_2 = 680 \pm 92$ km s^{-1}, KH group velocity, $v_{\rm g} = 429 \pm 8$ km s^{-1}, KH wavelength, $\lambda = 18.5 \pm 0.5$ Mm, and exponential linear growth rate, $\gamma_{\rm KH} = 0.033 \pm 0.012$ s^{-1}.

The aim of this study is to model the imaged/registered KH instability via investigating the propagation of MHD waves along the ejecta. Our frame of reference for studying the wave propagation in the jet is attached to the surrounding magnetoplasma – thus $\mathbf{v}_0 = \mathbf{V}_{\rm i} - \mathbf{V}_{\rm e}$ is that velocity shear which can allow the instability developing. Recall that the magnetic KH instability occurs on an interface between two plasma regions in sheared flow, when the velocity shear become larger than a critical value. One important parameter in our modeling is the density contrast, $\eta = \rho_{\rm e}/\rho_{\rm i}$, where $\rho_{\rm i}$ and $\rho_{\rm e}$ are the homogeneous plasma densities inside and outside the ejecta. Our choice of that parameter is $\eta = 0.88$, which corresponds to electron densities $n_{\rm i} = 8.7 \times 10^8$ cm^{-3} and $n_{\rm e} = 7.67 \times 10^8$ cm^{-3}, respectively. As seen from Table 2 in Foullon et al. [2], the two plasma betas are correspondingly $\beta_{\rm i} = 1.5 \pm 1.01$ and $\beta_{\rm e} = 0.21 \pm 0.05$. It is clear, that in the first approximation, the jet's plasma can be considered as an incompressible medium while the environment might be treated as a cool plasma ($\beta_{\rm e} = 0$). If we steel keep $\beta_{\rm i} = 1.5$, and fix the Alfvén speed to be $v_{\rm Ae} \cong 787$ km s^{-1} (i.e., the value corresponding to $n_{\rm e} = 7.67 \times 10^8$ cm^{-3} and magnetic field of 10 G), the total pressure (sum of thermal and magnetic pressure) balance equation at $\eta = 0.88$ requires a sound speed inside the jet $c_{\rm si} \cong 523$ km s^{-1} and Alfvén speed $v_{\rm Ai} \cong 467$ km s^{-1} (more exactly, 467.44 km s^{-1}), which means that the magnetic field in the flux tube is 6.3 G.

Geometry and MHD wave dispersion relation We consider a magnetic flux tube of radius $a = \Delta l/2$ embedded in a uniform field environment. The magnetic field inside the tube is helicoidal, with uniform twist, i.e., $\mathbf{B}_{\rm i} = (0, Ar, B_{\rm iz})$, where A and $B_{\rm iz}$ are constants. The magnetic field outside the tube is directed along the z-axis, $\mathbf{B}_{\rm e} = (0, 0, B_{\rm e})$. We consider the mass flow $\mathbf{v}_0 = (0, 0, v_0)$ to be along the z-axis. For studying the MHD wave propagation, we can use the dispersion relation of the normal modes propagating along a twisted magnetic tube of incompressible plasma with axial mass flow $\mathbf{v}_0$, surrounded by incompressible ionized medium [3], by adapting that equation to our case of cool environment, i.e., by changing the argument of Bessel function associated with that medium only. Accordingly, the modified dispersion equation is

$$\frac{\left[(\omega - \mathbf{k}\cdot\mathbf{v}_0)^2 - \omega_{\mathrm{Ai}}^2\right] F_m(\kappa_{\mathrm{i}}a) - 2mA\omega_{\mathrm{Ai}}/\sqrt{\mu\rho_{\mathrm{i}}}}{\left[(\omega - \mathbf{k}\cdot\mathbf{v}_0)^2 - \omega_{\mathrm{Ai}}^2\right]^2 - 4A^2\omega_{\mathrm{Ai}}^2/\mu\rho_{\mathrm{i}}} = \frac{P_m(\kappa_{\mathrm{e}}a)}{\frac{\rho_{\mathrm{e}}}{\rho_{\mathrm{i}}}\left(\omega^2 - \omega_{\mathrm{Ae}}^2\right) + A^2 P_m(\kappa_{\mathrm{e}}a)/\mu\rho_{\mathrm{i}}}, \tag{1}$$

where μ is the magnetic permeability, and

$$F_m(\kappa_{\mathrm{i}}a) = \frac{\kappa_{\mathrm{i}}a I_m'(\kappa_{\mathrm{i}}a)}{I_m(\kappa_{\mathrm{i}}a)} \quad \text{and} \quad P_m(\kappa_{\mathrm{e}}a) = \frac{\kappa_{\mathrm{e}}a K_m'(\kappa_{\mathrm{e}}a)}{K_m(\kappa_{\mathrm{e}}a)}.$$

Here,

$$\omega_{\mathrm{Ai}} = \frac{mA + k_z B_{\mathrm{iz}}}{\sqrt{\mu\rho_{\mathrm{i}}}} \quad \text{and} \quad \omega_{\mathrm{Ae}} = \frac{k_z B_{\mathrm{e}}}{\sqrt{\mu\rho_{\mathrm{e}}}}$$

are the local Alfvén frequencies, and

$$\kappa_{\mathrm{i}}^2 = k_z^2\left[1 - \frac{4A^2\omega_{\mathrm{Ai}}^2}{\mu\rho_{\mathrm{i}}\left[(\omega - \mathbf{k}\cdot\mathbf{v}_0)^2 - \omega_{\mathrm{Ai}}^2\right]^2}\right] \quad \text{and} \quad \kappa_{\mathrm{e}}^2 = k_z^2\left(1 - \omega^2/\omega_{\mathrm{Ae}}^2\right)$$

are squared wave attenuation coefficients of surface modes in both media. It is seen from Eq. (1) that the wave frequency in the moving media is Doppler shifted.

Numerical solutions and results Before starting the numerical solving Eq. (1), one needs a normalization of all velocities and Alfvén frequencies with respect to the Alfvén speed $v_{\mathrm{Ai}} = B_{\mathrm{iz}}/\sqrt{\mu\rho_{\mathrm{i}}}$, and the wavelength $\lambda = 2\pi/k_z$ to the tube radius a, or equivalently introducing a dimensionless wavenumber $k_z a$. In particular, the normalized jet speed v_0/v_{Ai} defines the Alfvén Mach number M_{A}, while the normalization of ω_{Ae} requires the ratio $b \equiv B_{\mathrm{e}}/B_{\mathrm{iz}} = 1.58$ and already introduced density contrast $\eta = 0.88$. We note also that the normalization of ω_{Ai} needs a new input parameter, notably the magnetic field twist, $\varepsilon = B_{\mathrm{i}\phi}/B_{\mathrm{iz}}$. With these input parameters (M_{A} will be varied during computations) the solutions to the dispersion equation yield the dependence of the normalized wave phase velocity $\omega/k_z v_{\mathrm{Ai}}$ on $k_z a$. We have studied the wave propagation for three values of $\varepsilon = 0.025$, 0.1, and 0.2. It turns out that the threshold Alfvén Mach numbers (and respectively the critical jet' speeds) of the kink ($m = 1$) mode are rather high to provide the occurrence of KH instability. Fortunately, the picture dramatically changes for the $m = -2$ MHD mode. As seen from Fig. 1, one can observe the appearance of three instability windows on the $k_z a$-axis. The width of each instability window depends upon the value of the twist parameter ε; the narrowest window corresponds to $\varepsilon = 0.025$, and the widest one to $\varepsilon = 0.2$. It is worth noticing that the phase velocities of unstable $m = -2$ MHD waves are very close to the jet speeds (in the right panel of Fig. 1 one sees that the normalized wave phase velocity on given dispersion curve is approximately equal to its label

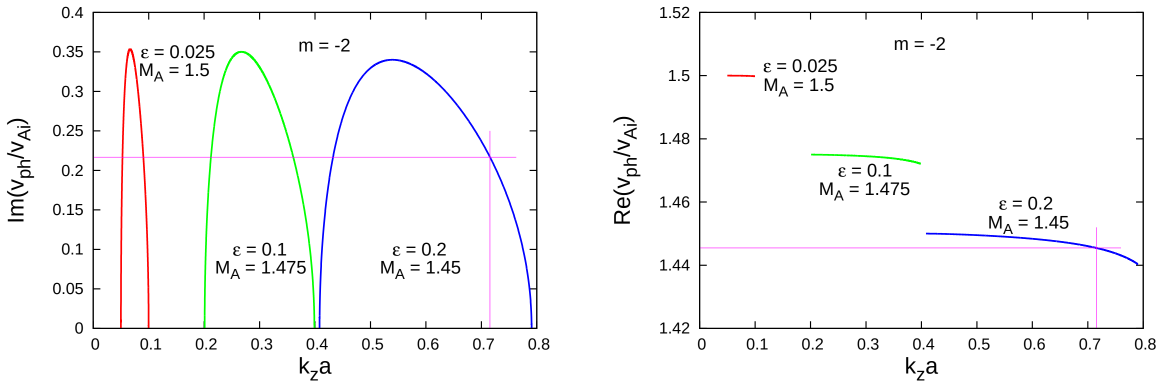

Figure 1: (*Left panel*) Growth rates of unstable $m = -2$ MHD mode in three instability windows. The best fit to the observational data one obtains at $k_z a = 0.7156$, for which value the normalized wave growth rate is equal to 0.2168. (*Right panel*) Dispersion curves of unstable $m = -2$ MHD waves for $\varepsilon = 0.025$, 0.1, and 0.2. The normalized phase velocity at $k_z a = 0.7156$ is equal to 1.4455.

M_A). All critical Alfvén Mach numbers yield acceptable threshold speeds of the ejecta which ensure the occurrence of KH instability – these speeds are equal to 701 $\mathrm{km\,s^{-1}}$, 689 $\mathrm{km\,s^{-1}}$, and 678 $\mathrm{km\,s^{-1}}$, respectively, in a very good agreement with the speed of 680 $\mathrm{km\,s^{-1}}$ found by Foullon et al. [2]. The best fit to the data listed in Table 3 in [2], however, yields $k_z a = 0.7156$ with $\mathrm{Im}(v_{ph}/v_{Ai}) = 0.2168$ in the third instability window (see the left panel of Fig. 1), which means that the computed wavelength is $\lambda = 18$ Mm, and wave growth rate $\gamma_{KH} = 0.035\ \mathrm{s^{-1}}$.

Conclusion Our study shows that the imaged by Foullon et al. [2] KH instability can be explained in terms of such instability arising during the propagation of the $m = -2$ MHD mode on a cylindrical jet contained in a twisted magnetic flux tube with twist parameter $\varepsilon = 0.2$ at a critical speed of 678 $\mathrm{km\,s^{-1}}$, as the wavelength of unstable mode is equal to 18 Mm and its growth rate to 0.035 $\mathrm{s^{-1}}$, in very good agreement with the data of Foullon et al. [2]. We note also, that the computed from Fig. 1 (right panel) wave phase velocity of 676 $\mathrm{km\,s^{-1}}$ is rather close to the speed of the 131 Å CME leading edge equal to 687 $\mathrm{km\,s^{-1}}$. The two “cross points” in Fig. 1 can be considered as a ‘computational portrait’ of the imaged in the 2010 November 3 coronal mass ejecta Kelvin–Helmholtz instability.

Acknowledgment This work was supported by Bulgarian Science Fund under Indo-Bulgarian bilateral project No. CSTC/INDIA 01/7, /Int/Bulgaria/P-2/12. We are indebted to Teimuraz Zaqarashvili for his valuable advice and helpful comments.

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