

Bayesian approach to Thomson Scattering data processing in RFX-mod and COMPASS

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1) Background

Thomson Scattering (TS hereafter) proved to be one of the most reliable diagnostics for electron temperature measurements on fusion plasma. Here we present a different method for data processing, based on Bayes Probability Theory (BPT) approach; this kind of analysis is not new in the TS data analysis literature [1]; on the other hand, here the flexibility of the method is exploited to relax the hypothesis on the electron distribution. TS profiles will be presented from both COMPASS [2] (tokamak discharges) and RFX-mod [3] (RFP discharges) discharges.

2) Procedure

Bayes formula (or theorem) is usually reported as follows:

$$P_o(M(k)|D) = \frac{L(D|M(k)) \cdot P_i(M(k))}{P_i(D)}$$

Where $P_i(M(k))$ describes the *a priori* probability for a given model M to describe the system via a k -set of parameters and $P_o(M(k)|D)$ the *a posteriori* probability distribution of a measure set D . L is the likelihood function and describes the probability for the given set of measures D conditioned the existence of conditions described by $M(k)$. In this perspective, data are not affected by errors in the sense that all the uncertainties of the measure operation is retained in the lack of knowledge of the parameters describing the system; hence the need to treat them as probability distributions and not single values or functions.

In our case, the problem is comparing the measured signals s_j on a set of expected signals which are functions $\Gamma_i(T_e, n_e)$ of plasma density and temperature. To be completely consistent, one has to assume that the temperature is not a uniquely defined function of electron velocity (the measured parameter), in particular because the electron distribution function can be non-Maxwellian (here, more generally, the distribution is assumed to be Lorentzian, while arbitrary electron distributions can be processed [4]). It is remarkable that the exact definition of P_i , which is unknown (and hence remains a free parameter), does not affect significantly

the results, though it affects the error amplitudes (the latter being directly related to the spread in parameters distributions)

Heuristically, one can consider $\Gamma_i(T_e)$ (expected signal for spectral channel j at temperature T_e), to be distributed as a function $P_i(\Gamma_j(T_e))$. The ratio k of Γ_j/s_j should be constant for a given temperature, so L can be expressed as $\prod_i P(\Gamma_j/s_j=k)$. The overall likelihood over temperatures becomes the product of the ratios over channel index j :

$$L(T_e, n_e | \{s_j\}) = \frac{\prod_j (\Gamma_j(T_e)/s_j) \cdot P_i(\Gamma_j(T_e))}{P_i(\{s_j\})}$$

This is implemented fading the Γ functions, scaling them and multiplying the resulting arrays, as shown in Fig 1. Temperature and scaling factors then can be interpreted as the value with maximum likelihood or the distribution mean value. Here the second quantity is chosen because it retains more information and does not require fitting on the distribution peak. Errors are obtained propagating the FWHM of peaks in the likelihood surfaces.

$P_i(\Gamma)$ optimization has to be done iteratively by maximizing the overall likelihood value: in particular, in the assumed Lorentzian distribution [5]:

$$f_L(p) = \left[\beta \left(\frac{5}{2}, \kappa - 3 \right) {}_2F_1 \left(\frac{5}{2}, -\frac{3}{2}; \kappa - \frac{1}{2}; \frac{4\alpha - \kappa}{4\alpha} \right) \right]^{-1} \frac{3\alpha^{3/2}}{4\pi(m_e c)^3 \kappa^{5/2}} \left(1 + \frac{2\alpha(\gamma - 1)}{\kappa} \right)^{-\kappa}$$

the parameter $\alpha = m_e c^2 / (2T_e)$ is related to the bulk electron temperature, γ is the Lorentz factor and κ a free parameter, upon which the analysis is iterated.

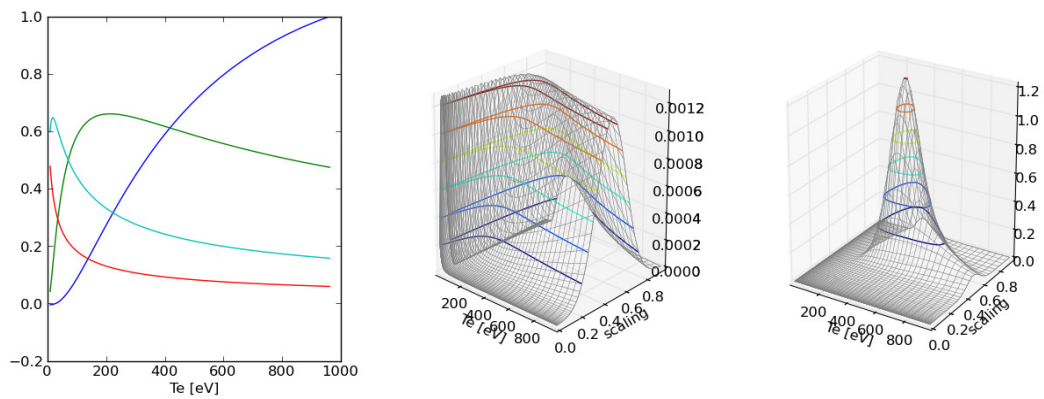


Fig 1: From left Γ_i functions, example of Γ distribution and likelihood function for a TS spatial point analysis; Γ distribution and shape are iteratively varied changing the distribution parameters and the a priori uncertainties on the scattered spectrum wavelength and intensity

3) Results In COMPASS

Fig.2 shows the comparison of temperature profiles obtained with bayesian and standard (frequentist) approach. In some cases BTP approach improves the description of the external gradient, however, the profiles show a marked discrepancy at intermediate radii, with BTP returning a lower temperature. Being a preliminary result, an instrumental effect cannot be excluded.

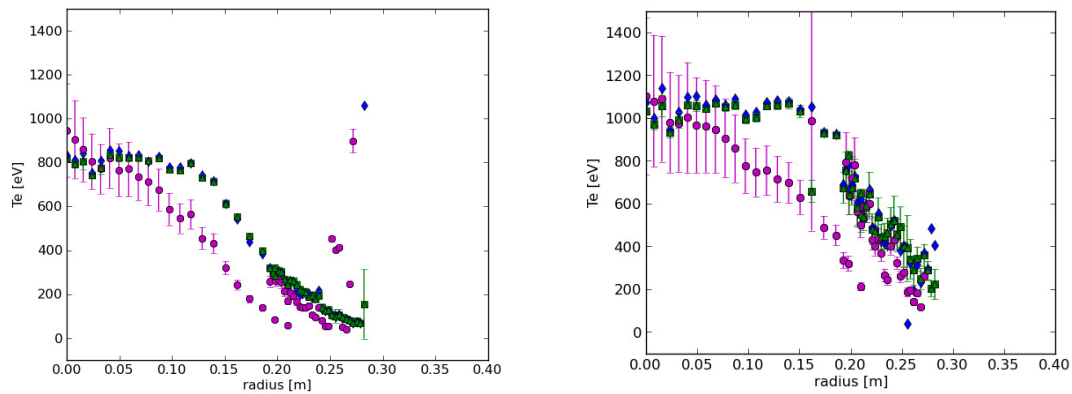


Fig 2: Example of bayesian (blue) and standard (green) Thomson Profiles (COMPASS shot 5943) Magenta dots refers to bayesian analysis with non Maxwellian electron distribution

4) Results in RFX-mod

Fig 3 shows an example of temperature profile obtained with standard analysis and bayesian technique in RFX-mod: BTP temperatures tend to be lower and the profile more scattered.

While the bayesian approach does not seem to improve the overall quality of the profile, it can be used to check if the maxwellian approximation in the standard analysis is correct or not. In the runs on standard discharges, the preferred value of the κ parameter remains for some point low (in the limit $\kappa \rightarrow \infty$ the distribution becomes maxwellian), indicating that the spectral shape is different from what expected. In high density shots, this effect is unlikely to be related to suprathermal tails, and it is still under investigation. It has to be remarked that lower values of the exponent occur where T_e profiles discrepancy is higher.

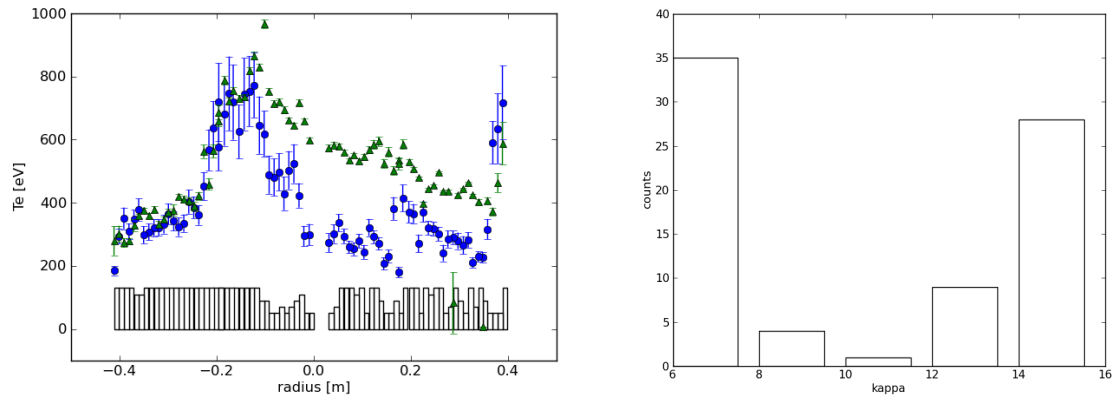


Fig 3: on the left, example of bayesian (blue) and standard (green) Thomson Profiles (RFX-mod shot 30011). BTP profile remains significantly lower. White bars describe in AU the preferred κ parameter in the distribution. On the right, distribution of κ parameter in this profile: non maxwellian (low κ) spectra are preferred in most cases

4) Conclusions and future work

BTP approach in Thomson Scattering data analysis is a robust tool for temperature and errors estimation, which often comes at the price of heavier numerical machinery. It is particularly suited for electron distribution analysis; in this perspective the procedure has been applied to COMPASS and RFX-mod plasmas. The COMPASS case shows a good agreement between frequentist and BTP data processing, as long as maxwellian spectra are maintained. RFX_mod plasmas are somehow more problematic. To clarify the mismatch, BTP method is foreseen to be applied to low density plasmas in RFX-mod, where the effect of suprathermal electron populations are expected to be higher. The flexibility of the algorithm and its capability to deal with scattered spectra generated by arbitrary electron distributions can be regarded as key features for application in high temperature, reactor like plasmas.

References:

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