Cosmic-ray acceleration at perpendicular shocks

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Diffusive shock acceleration (DSA) in supernova remnants is the dominant paradigm for the acceleration of cosmic rays in the Galaxy. However, the maximum possible energy to which parallel shocks can accelerate protons in these objects is only roughly 1 PeV [1], in tension with the observed cosmic ray spectrum. Perpendicular shocks present a possible alternative [4], and are expected to predominate if the supernova progenitor is a rotating, massive star with a powerful wind [2]. However, stochastic acceleration in this situation is complicated, because the underlying transport process is not necessarily diffusive, in which case the standard results concerning particle spectra and acceleration rates lose their validity [6]. This paper presents a new approximate, analytical approach to the test particle problem at perpendicular shocks, using an expansion in eigenfunctions of the pitch-angle and phase dependent scattering operator. An explicit expression for the spectral index of accelerated particles is found and compared with the results of Monte-Carlo simulations. Full details can be found in [7].

The goal is to solve for the stationary cosmic-ray distribution f at a perpendicular shock as a function of momentum $\vec{p}$ and distance from the shock, assuming that, in addition to their gyro-motion, particles are continually deflected by magnetic fluctuations whose effect can be modeled as isotropic diffusion in the direction of motion, i.e., as diffusion on the sphere of the end-point of the unit vector $\vec{p}/p$. In both the downstream and upstream plasmas, f then obeys a Fokker-Planck equation. When $\vec{x}$ and t are measured in the shock rest frame and $\vec{p}$ in the local fluid frame this takes the form:

$$\frac{\partial f}{\partial t}+(v_z-u)\frac{\partial f}{\partial z}=\omega_{\rm g}\left\{-\frac{\partial f}{\partial\phi}+\frac{1}{2\eta}\left[\frac{\partial}{\partial\mu}\left(1-\mu^2\right)\frac{\partial f}{\partial\mu}+\frac{1}{1-\mu^2}\frac{\partial^2 f}{\partial\phi^2}\right]\right\} \qquad (1)$$

Here, u (assumed $\ll 1$) is the shock speed and v the particle speed (both in units of c), $\omega_{\rm g}$ the gyro-frequency, μ the pitch-angle cosine and ϕ the gyro-phase. The uniform magnetic field lies along the y-axis and the shock is in the x–y plane, so that the component of the particle velocity along the shock normal is $v_z = v\sin\theta\sin\phi$. The ratio of $\omega_{\rm g}$ to the "collision frequency" is denoted by η. Solutions are found by separating the variables, leading to the eigenvalue problem

$$\Lambda_i (v_z - u) Q_i = \left\{ -\frac{\partial}{\partial \phi} + \frac{1}{2\eta} \left[\frac{\partial}{\partial \mu} (1-\mu^2) \frac{\partial}{\partial \mu} + \frac{1}{1-\mu^2} \frac{\partial^2}{\partial \phi^2} \right] \right\} Q_i \tag{2}$$

together with boundary conditions on $Q_i(\mu,\phi)$ that ensure regularity and single-valuedness on the sphere. Equation (2) has both positive and negative eigenvalues (labelled with $i > 0$ and $i < 0$ respectively), in addition to the eigenvalue $\Lambda_0 = 0$ with the isotropic eigenfunction $Q_0 =$ constant. These govern the spatial dependence of the solution: the isotropic part is independent of z, whereas the eigenfunctions with $i < 0$ decay exponentially upstream ($z > 0$), and grow exponentially downstream ($z < 0$) on a length scale that decreases as $|i|$ increases; eigenfunctions with $i > 0$ have the opposite behaviour. Following the procedure used for relativistic shocks [5], boundary conditions upstream are applied by limiting the expansion upstream to terms with $i < 0$. Matching conditions at the shock (essentially Liouville's theorem) introduce a dependence on p into the problem and enable one to compute the corresponding downstream distribution. This is then subjected to the boundary conditions far downstream by requiring it to be orthogonal to the eigenfunctions with $i < 0$. In the absence of a source term, f is a scale-free power law in momentum: $f \propto p^{-s}$ and s, as well as the spatial and angular dependence of f, is determined by the above procedure.

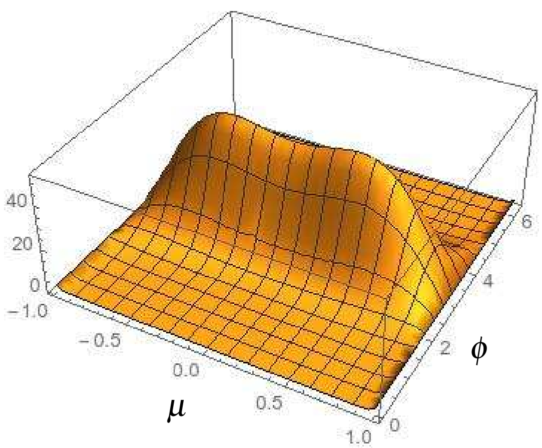

Figure 1: The leading eigenfunction (3) as a function of μ and ϕ, for $\eta u = 2$ and $v = 1$ (for which $\Lambda_{-1} = -5.2$).

The eigenfunction with $i = -1$ plays a special role here, since it corresponds to the only term in the expansion that survives at large distance upstream, and, therefore, may not change sign in the physically relevant range ($0 < \phi \leq 2\pi$, $-1 \leq \mu \leq 1$). Using this single term to represent the upstream distribution was found to give a reasonably good approximation in the one-dimensional case studied in [5]. This is the strategy followed here, motivated partly by the relatively large numerical effort involved in computing the higher-order functions in the current, two-dimensional problem, compared to the straightforward and efficient solution available via Monte-Carlo methods.

In the nonrelativistic limit $u \to 0$, this eigenfunction has a small ($\sim u$) anisotropy proportional to $\sqrt{1-\mu^2}\,(\eta \cos\phi - \sin\phi)$, and an eigenvalue $\Lambda_{-1} \approx -3u\eta$. Its anisotropic part gives the two-dimensional equivalent of the singular "diffusion" solution discussed by Fisch & Kulsrud [3], who were concerned with the incompleteness of the eigenfunctions in the case $u = 0$. In this limit, it is straightforward to show that the standard result of DSA follows: $s = 3r/(r-1)$, where the shock compression ratio $r = u_{\mathrm{s}}/u_{\mathrm{d}}$ is ratio of the shock speed in the upstream medium u_{s} to that in the downstream medium u_{d}. However, this result is valid only for $u_{\mathrm{s,d}} \sim \varepsilon \ll 1/\eta$. In supernova remnants, $u_{\mathrm{s,d}} \sim 10^{-1}$–$10^{-2}$, and may be comparable or larger than $1/\eta$. Therefore, an approximation scheme that sets $u \sim 1/\eta \sim \varepsilon \ll 1$ is more appropriate. The resulting eigenvalue is then $\sim \varepsilon^0$, and the eigenfunction, shown in Fig 1, is [7]

$$Q_{-1}(\mu,\phi) \approx \mathrm{e}^{\Lambda\nu\sqrt{1-\mu^2}\cos\phi}\mathrm{Ps}_0^0\left(\mu,-\Lambda^2/2\right) \tag{3}$$

where Ps_n^m is the angular, oblate, spheroidal wave function. This eigenfunction describes a strongly anisotropic fan-beam concentrated in the plane of the shock and drifting across the magnetic field in the same direction as the drift of unperturbed trajectories. For large ηu_{s}, the width of this beam in phase is approximately $(\eta u_{\mathrm{s}})^{-1/2}$, which is just the diffusive spread caused when the collision operator in (1) operates on a phase-collimated beam for a time $1/\left(\omega_{\mathrm{g}} u_{\mathrm{s}}\right)$ roughly equal to that needed for an unperturbed trajectory to traverse the shock.

In general, evaluation of the resulting power-law index requires a numerical integration. However, using a Taylor expansion of the eigenfunctions in the variable ηu_{s} one finds [7]

$$s = \frac{3r}{r-1} + \frac{9(r+1)}{20r(r-1)}\eta^2 {u_s}^2 + \mathrm{O}\left(\eta^4 {u_s}^4\right). \tag{4}$$

In Fig 2, this result is compared with those of fully relativistic Monte-Carlo simulations [7], for $r = 4$, ten values of η between 1 and 100, and thirty values of u_{s} between 0.01 and 0.2. Qualitatively, the agreement is good. In particular, the simulations confirm that, to a good approximation, s depends on η and mu_{s} only in ηu_{s}, since, for a fixed value of this parameter, Fig 2 shows only a weak variation of s with the colour coding (which denotes the η value). Also, the softening of the spectrum for $\eta u_{\mathrm{s}} > 1$ is confirmed. However, there is a quantitative discrepancy, which may result either from

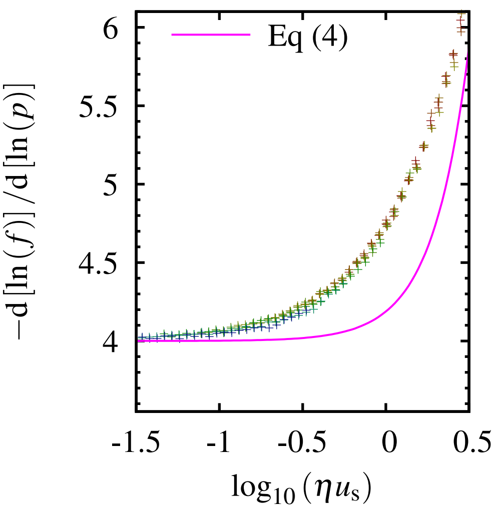

Figure 2: Comparison of the spectral index calculated using the leading eigenfunction, (Eq. (4)) with the results of fully relativistic Monte-Carlo simulations [7], for a matrix of parameters, colour-coded according to the value of η ranging from 1 (blue) to 100 (red).

the asymptotic expression used for the eigenfunction (which is valid only in the limit $u_{\mathrm{s}} \rightarrow 0$ and $\eta \rightarrow \infty$), or from the use of only the leading eigenfunction to represent the distribution, or, of course, from both of these approximations.

Nevertheless, we conclude that non-relativistic, perpendicular shocks are effective accelerators provided $\eta u_{\mathrm{s}} \leq 1$. Combining this with the conjecture advanced in [4] and confirmed in [7], that the acceleration timescale at these shocks decreases monotonically with increasing η, leads to the conclusion that the optimal configuration for acceleration at a nonrelativistic shock front is one where the shock is perpendicular and the turbulence is relatively weak, such that $\eta u_{\mathrm{s}} \approx 1$. In such a configuration, acceleration to energies significantly above 1 PeV appears possible for parameters thought appropriate for supernova that explode in the winds of massive stars [8].

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