Toroidally Symmetric Plasma Vortex at Tokamak Divertor Null Point

M.V. Umansky and D.D. Ryutov

Lawrence Livermore National Laboratory, Livermore, CA 94550, USA

Introduction

In next generation tokamak experiments and fusion reactors the divertor heat flux will challenge conventional means of peak heat flux reduction which makes it urgent to find innovative approaches to solving the divertor heat exhaust problem. One recent idea for tokamak divertor is using a higher order null (dubbed "snowflake") instead of the standard x-point [1]. Snowflake divertor configuration has a characteristic hexagonal separatrix structure, and it has a number of geometric properties that may affect edge plasma and may be helpful for alleviating the divertor heat flux problem: stronger fanning of the poloidal flux, stronger magnetic shear in the edge region, larger radiating volume, and larger connection length in the scrape-off layer [2].

Theoretical considerations [3, 4] and recent experimental observations from snowflake divertor experiments on several tokamaks indicate that presence of a near-second-order null of poloidal field may give rise to strong plasma mixing near the magnetic divertor null point, thereby providing sharing of heat and particle flux between multiple (three to four) strike points [5, 2].

Further, a semi-quantitative analytic model [6] was proposed to describe plasma dynamics driven by the pressure gradient and magnetic curvature near the null-point where the poloidal magnetic field is small. The analytic model predicts oscillatory twisting motion of plasma (dubbed "the churning mode") localized at the null point, and the size of the convective zone is much larger for snowflake than for a regular x-point configuration, for realistic tokamak parameters. Although the basic physics of the churning mode has been identified in the analytic model [6], the assumptions of the analytic model may be too restrictive, in particular, the motion was postulated to be in a particular form of differential rotation, the role of the resistive dissipation of the twisted poloidal field was not included, and there was no account for the heat conduction in the convectively mixed plasma. In the present study the formation of the convective cell is investigated by direct numerical simulation, solving plasma fluid equations describing time-evolution of plasma thermal energy and electric and magnetic fields.

Reduced MHD model

The standard ideal MHD equations include equations for mass continuity, momentum balance, adiabatic gas law, and magnetic induction [7]. The specific interest here is toroidally symmetric plasma motion in the vicinity of the null point (R_0, Z_0), where (R, θ, Z) are the usual cylindrical coordinates; so we'll further assume (i) toroidal symmetry, ∂_θ=0, (ii) large aspect ratio, $l_\perp/R \ll$1, (iii) strong toroidal guide field, $B_P/B_t \ll$1, and (iv) uniform density ρ=const. Instead of the cylindrical coordinates we'll be using more convenient here Cartesian coordinates (x,y) in the poloidal plane with the origin at the null-point, see Fig. (1), $x = R - R_0, y = Z - Z_0$

The poloidal magnetic field B_p is described by the poloidal flux function $\psi(x,y)$, $B_p = -(1/R_0)e_\theta \times \nabla\psi$, where $\nabla = (\partial_x, \partial_y)$, and the toroidal magnetic field is described by the reference value B_{t0} given at the reference point (R_0, Z_0), $B_t = e_\theta B_{t0}/(1 + x/R_0)$. The electric field is described by axisymmetric potential $\phi(x,y)$, and the fluid velocity V is poloidal and given by $V = -(c/B_{t0})(e_\theta \times \nabla\phi)$. From the magnetic induction equation one can obtain the evolution equation for the poloidal magnetic flux function, and expressing the poloidal current from the equation of motion and imposing a constraint, $\nabla \cdot J_p$=0 one finds an evolution equation for the vorticity which is defined as $\varpi = (c/B_{t0})\rho\nabla^2\phi$. For normalization of the equations, several reference parameters are chosen at the null point (subscripted 0) and at the outer mid-plane: R_0, B_{t0}, P_0, a_{mid}, B_{pmid}. These are combined into a reference speed and time unit: $C_{s0} = \sqrt{P_0/\rho}$, $t_0 = a_{mid}/C_{s0}$. Length units are normalized by a_{mid} and time units are normalized by t_0. Next, the pressure is normalized by P_0 so at the null-point the normalized pressure is 1.0. The poloidal flux is normalized by $\psi_0 = B_{pmid}R_0a_{mid}$ so the normalized flux is on the order of unity for normalized distance $\sqrt{x^2 + y^2} \sim$1. Normalization of the electric potential ϕ is such that $V_E = [e_\theta \times \nabla\phi]$, so $\varpi = \nabla^2\phi$. The dissipation coefficients are normalized by $a_{mid}C_{s0}$.

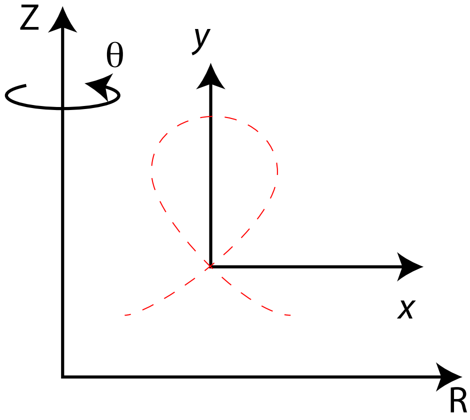

Figure 1: *Coordinates used for the model*

Finally, with dissipative terms added, the equations in normalized form are summarized as follows

$$\begin{cases} \dfrac{d}{dt}\varpi = 2\varepsilon\dfrac{dP}{dy} + \dfrac{2}{\beta_p}\{\psi, \nabla^2\psi\} + \mu\nabla^2\varpi \\ \dfrac{d}{dt}P = \chi\nabla^2 P \\ \dfrac{d}{dt}\psi = D_m\nabla^2\psi, \end{cases} \tag{1}$$

where $d/dt = \partial_t + V_E \cdot \nabla$, $V_E = [e_\theta \times \nabla\phi]$, and $\varpi = \nabla^2\phi$.

The non-dimensional parameters in the normalized equations are the aspect ratio, $\varepsilon = a_{mid}/R_0$, and the poloidal plasma beta, $\beta_p = 8\pi P_0/B^2_{pmid}$, μ is the kinematic viscosity, χ is the thermal diffusivity, D_m is the magnetic diffusivity. In the equations one can identify the driving term arising from the pressure gradient and curvature, the stabilizing forcing term from magnetic field deformation, and dissipative terms. Note that the toroidal magnetic field does not enter the normalized equations, and the toroidal geometry effects show up only in the curvature drive term.

Numerical simulations

For numerical solution, the equations are discretized by finite-difference on a computational grid and then time-integrated using standard numerical software. The simulations are carried out in a square domain $L \times L$ centered at the null point location. On the domain boundaries the values of all evolved variables are maintained constant, which is consistent with the expectation that the studied physical phenomena are localized near the null point.

The initial vorticity is ϖ=0, and the initial magnetic flux ψ_0 is chosen as either a first order null x-point configuration (XPT), $\psi_{(1)} = x^2 - y^2$, or as an exact second order null snowflake configuration (SNF), $\psi_{(2)} = y^3 - 3x^2y$, or as an inexact second order null configuration, snowflake-minus (SNF-) or snowflake-plus (SNF+), $\psi_{(2\mp)} = y^3 - 3x^2y \pm 3\sigma^2 y$, where for SNF- two first order nulls aligned horizontally and located at $(\pm\sigma, 0)$, and for SNF+ two first order nulls aligned vertically and located at $(0, \pm\sigma)$.

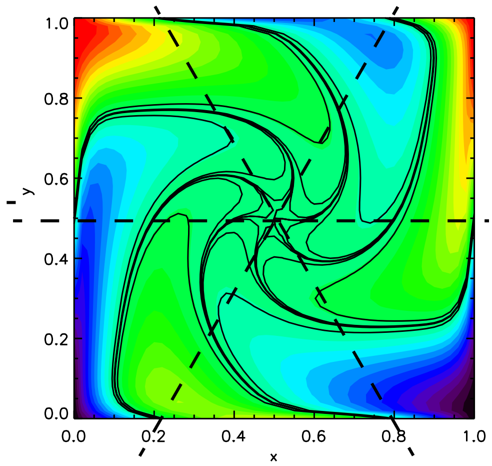

Figure 2: *Plasma vortex forming in SNF configuration*

Time-integration of Eqs. (1) shows formation of a vortex-like structure localized at the null point. Depending on the model parameters, there are various scenarios of evolution: (i) relaxation to a rotated quasi-steady state where the vertical pressure gradient dP/dy is near zero, or (ii) rotation of the central kernel with occasional relaxation events, or (iii) turbulent motion in the central zone. One can see in the simulations that for a SNF configuration there is always a convective zone at the null point, and the size of it increases with β_p. For the XPT configuration, there is a threshold value of β_p when convection starts. In general, the closer the configuration is to SNF the larger is the scale of convective motion. Furthermore, using parameters of a generic mid-size tokamak, one can conclude that in SNF configuration, during ELM the scale of convection should be significant for redistribution of thermal energy across the null-point. On the other hand, for same parameters, in XPT configuration there is virtually no perturbation of plasma pressure and magnetic field. This is consistent with the TCV experiments where redistribution of thermal energy between primary and secondary strike points was seen to be particularly large during ELM strikes in near snowflake configurations [5]. The churning mode can be a significant player in transport of thermal energy near the null point, in particular during ELMs when β_p is large at the null point, and in particular in near-snowflake configurations. Aside from the direct action of thermal convection, there may be other effects such as inducing magnetic stochasticity in the null point region due to perturbation of the poloidal magnetic field.

Note that the churning mode physics is very similar to free thermal convection of neutral fluid, in a vertical gravity field, driven by a horizontal temperature gradient, as described in, e.g., [8, 9, 10]. The hydrodynamics equations that are used for the the thermal convection problem can be cast in a form very similar to Eqs. (1), with the magnetic curvature playing the role of effective gravity. However one important difference is that the plasma equations have the restoring term due to magnetic field perturbations which plays the role of "elasticity" resisting plasma twisting.

Acknowledgments

This work was performed under the auspices of the US Department of Energy by Lawrence Livermore National Security, LLC, Lawrence Livermore National Laboratory, under Contract DE-AC52-07NA27344.

References

[1] D.D. Ryutov et al., Physics of Plasmas 14, 064502 (2007).
[2] V. A. Soukhanovskii et al., Journal of Nuclear Materials 438, S96-S101 (2013).
[3] D.D. Ryutov et al., Plasma Physics and Controlled Fusion 54 (2012).
[4] D.D. Ryutov et al., Contributions to Plasma Physics 52, 5-6, 539-543 (2012).
[5] W A J Vijvers et al., Nuclear Fusion 54, 2, 023009, (2014).
[6] D.D. Ryutov et al., Physica Scripta 89, 8, 088002 (2014).
[7] J. P. Freidberg, "Ideal Magnetohydrodynamics" (Plenum, New York, 1987).
[8] D. A. Bratsun et al., International Journal of Heat and Fluid Flow, 24, 6, 835 (2003).
[9] K. Buhler, Heat and Mass Transfer, 39, 631, (2003).
[10] G. Z. Gershuni and E. M. Zhukhovitskii, "Convective Stability of Incompressible Fluids" IPST (1976).