Plasma surface currents in the presence of a resistive wall and their connection to Halo currents during disruptions in tokamaks

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1. Introduction. Major disruptions and VDEs result in halo and eddy currents flowing in the conducting structures surrounding plasma [1–4]. These currents can produce unacceptable forces on vacuum vessel and in-vessel components, and their careful evaluation is necessary. Recent analytical theory [5, 6] predicts that the ideal plasma surface current can contribute to the halo, especially when the plasma boundary almost coincides with the rational surface. Here we speculate that such a particular case must not be considered in the frame of the ideal MHD, because it leads to a singularity for the plasma displacement. Treating the cold post-disruption plasma edge as a resistive layer we derive a dispersion relation for the mode growth rate and frequency, expressions for the resistive plasma "surface" current and eddy currents in the wall. Our approach has some similarities with that in [7, 8].

2. Formulation of the problem. We consider a cylindrical plasma of radius a surrounded by a coaxial magnetically thin resistive wall with radius b, thickness d and uniform resistivity η_w. Assuming that the rational surface is sufficiently close, but still outside of the plasma boundary we treat the plasma edge as a thin resistive layer with given thickness W and resistivity η_{pl}, while the rest of the plasma is supposed to be ideal and inertialess. The plasma-wall gap and space behind the wall are treated as a vacuum. In this case, the equations governing the evolution of the system are:

$$\frac{\partial}{r\,\partial r}\left(r\frac{\partial\psi}{\partial r}\right)-\frac{m^2}{r^2}\psi-\frac{1}{(\mu-n/m)}\frac{\partial}{r\,\partial r}\left[\frac{\partial}{r\,\partial r}(r^2\mu)\right]\psi=0 \tag{1}$$

for the ideal plasma and vacuum regions;

$$\frac{\partial\psi}{\partial t}-i\Omega\psi=\frac{\eta_{pl}}{\mu_0}\Delta\psi \quad \text{with} \quad \Omega\equiv\frac{nV_z(a)}{R}-\frac{mV_\theta(a)}{r} \tag{2}$$

for the resistive plasma layer (at $\mu-n/m<<1$);

$$\frac{\mu_0}{\eta_w}\frac{\partial\psi}{\partial t}=\Delta\psi \tag{3}$$

for the resistive wall. These are derived in the cylindrical geometry (r,θ,z) for the (m,n) mode of the magnetic perturbation $\mathbf{b}=\nabla\psi\times\mathbf{e_z}$, assuming also $nb/(mR)<<1$, where R is the major radius, $\mu \equiv 1/q = RB_\theta / rB_z$, B_θ and B_z are equilibrium poloidal and toroidal magnetic fields, $V_\theta(a)$ and $V_z(a)$ are equilibrium poloidal and toroidal edge velocities, respectively. For the time dependence of the form $\exp(\gamma t + i\omega t)$ with the growth rate γ, and the mode rotational frequency ω, integration of Eqs. (2) and (3) across the resistive layer and the wall under the constant - ψ assumption yields the following boundary conditions

$$\tau_\eta \left[\gamma + i(\omega - \Omega)\right] = \left.\frac{r\psi'}{\psi}\right|_a - \frac{a}{r_-}\left.\frac{r\psi'}{\psi}\right|_{r-} \quad \text{and} \quad \tau_w(\gamma + i\omega) = \left.\frac{r\psi'}{\psi}\right|_{b-}^{b+}, \tag{4}$$

where $\tau_\eta \equiv \mu_0 aW / \eta_{pl}$, $\tau_w \equiv \mu_0 bd / \eta_w$, $r_- = a - W$ and prime means the radial derivative. For the parabolic distribution of the equilibrium current

$$j_z = j_0\left(1 - \alpha_j \frac{r^2}{a^2}\right), \tag{5}$$

the solution of Eq. (1) within the ideal plasma region can be found in terms of hypergeometric functions $\psi_{pl} \propto z^{m/2}(1-z)^{-1}F(A,B;A+B+1;z)$ [9], so that

$$\left.\frac{r\psi'}{\psi}\right|_{r-} = m + 2z(\ln F(A,B;A+B+1;z))'_z\Big|_{r-} \quad \text{with} \quad z = \frac{\alpha_j r^2}{2a^2(1 - nq_0/m)}, \tag{6}$$

where $A = (m - \sqrt{m^2+8})/2$, $B = (m + \sqrt{m^2+8})/2$ and q_0 is the safety factor at the magnetic axis. The solutions in the vacuum gap and external vacuum are $\psi_{gap} \propto Cr^m + Dr^{-m}$ and $\psi_{out} \propto r^{-m}$, where C and D are arbitrary constants. Applying the second of Eqs. (4) we have

$$\left.\frac{r\psi'}{\psi}\right|_a = -m\frac{2m + \tau_w(\gamma + i\omega)(1 + (a/b)^{2m})}{2m + \tau_w(\gamma + i\omega)(1 - (a/b)^{2m})}. \tag{7}$$

Substitution of (6) and (7) into the first of Eqs. (4) provides us with a dispersion relation.

Once one knows the mode growth rate and frequency the currents in the resistive layer and wall can be calculated by

$$\mu_0 i_\eta = -\psi'\Big|_{r-}^{a} = -\tau_\eta\left[\gamma + i(\omega - \Omega)\right]\psi(a) \quad \text{and} \quad \mu_0 i_w = -\psi'\Big|_{b-}^{b+} = -\tau_w(\gamma + i\omega)\psi(b). \tag{8}$$

3. Computation results and discussion. To provide a basis for the comparison Fig. 1 shows the results of [5, 6] where the whole plasma is treated as ideal. The region within which the rational surface is close to the plasma boundary is designated by the vertical lines. Predictions of our model are presented in Figs. 2-6 showing the dependences of the normalized growth rate on the edge safety factor for the $m/n = 2/1$ mode. In Fig. 2 the growth rate of the locked mode

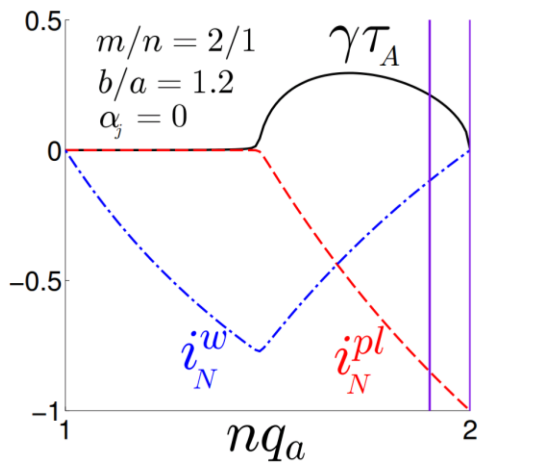

Fig. 1. Normalized growth rate, ideal plasma surface and resistive wall eddy currents vs. edge safety factor for flat current profile.

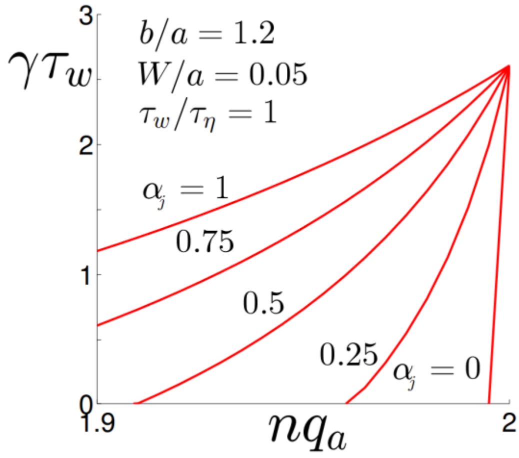

Fig. 2. Normalized growth rate of the locked mode vs. edge safety factor for different current profiles.

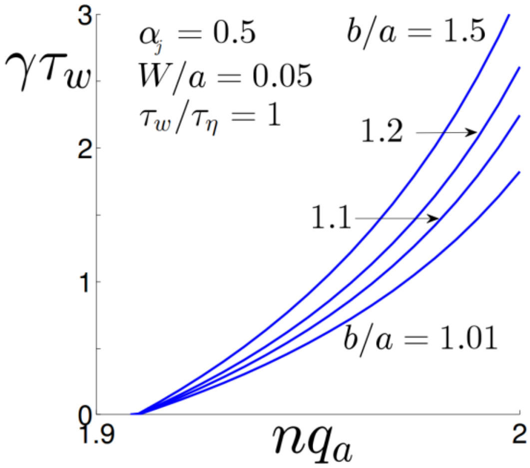

Fig. 3. Normalized growth rate of the locked mode vs. edge safety factor for different wall positions.

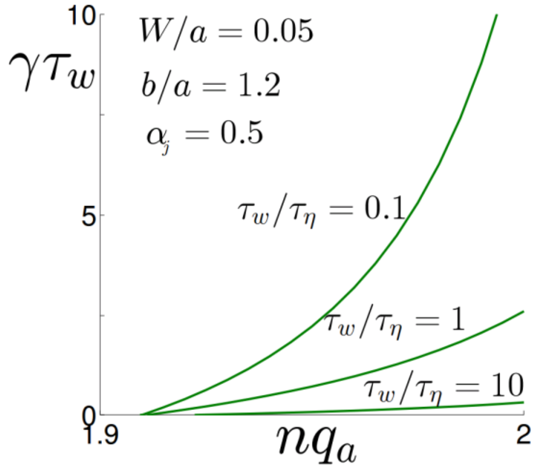

Fig. 4. Normalized growth rate of the locked mode vs. edge safety factor for different ratios of the wall and resistive layer times.

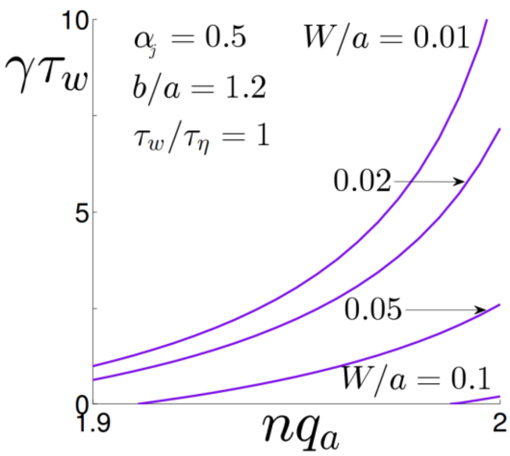

Fig. 5. Normalized growth rate of the locked mode vs. edge safety factor for the different thickness of the resistive layer.

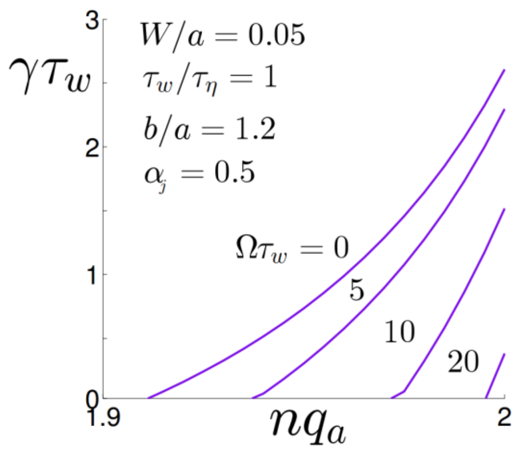

Fig. 6. Normalized growth rate of the rotating mode vs. edge safety factor for different plasma velocities.

is plotted for different current profiles from flat $\alpha_j = 0$ to the purely parabolic $\alpha_j = 1$. At the larger current gradient the mode becomes more unstable. Fig. 3 illustrates the stabilizing influence of the wall on the perturbation, which is slower when the wall is closer to the plasma. After the thermal quench the plasma is relatively cold, so the resistive effects becomes stronger as can be seen in Fig. 4 highlighting the dependence on the ratio τ_w / τ_η. The thickness of the plasma resistive layer W determines the plasma resistive time τ_η and also enters relation (6), Fig. 5 shows that for larger W the mode growth slows down. Adding of the plasma rotation to the model makes the dispersion relation complex that results in non-zero mode frequency, which in turn provides stabilizing effect on the perturbation. According to Fig.6 even moderate plasma rotation can completely suppress the mode.

Eqs. (8) allow to estimate edge plasma and wall eddy currents. For locked modes they are simply proportional to the mode growth rate and the amplitude of the perturbed poloidal magnetic flux at the plasma edge and the wall, respectively. In this case they are related as $i_\eta / i_w = \tau_\eta \psi(a) / \tau_w \psi(b)$.

4. Conclusion. In contrast with results in [5, 6] the model predicts increase of the perturbed plasma edge current for steeper profiles of the equilibrium current. For locked modes it becomes larger with a more distant wall, colder plasma and thinner resistive layer. Plasma rotation provides mode stabilization, to address its influence on the perturbed edge current dynamics further analysis is needed. Our consideration is free from the assumption that the mass density has a jump at the plasma boundary, which is the main reason leading to the existence of the surface currents within the ideal MHD.

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