

Numerical calculation of ion runaway distributions

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Background The phenomenon of particle runaway in a plasma is well known, arising because the friction force experienced by a fast charged particle decreases with particle energy, so a sufficiently strong electric field can allow the particle to be accelerated - or *run away* - to high energy. Ion runaway has long been of interest in the astrophysical community, where it is thought to contribute to the observed abundance of high energy ions in solar flares [1]. It has also been used to study the behavior of lightning channels [2] and was observed in magnetic confinement devices [3, 4]. The detailed mechanism of ion runaway differs from that of electron runaway. Friction with the electrons drifting in the electric field acts to cancel a portion of the accelerating field. This effect is captured by replacing the electric field with the effective electric field $E^* = (1 - Z_i/Z_{\text{eff}})E$, with the consequence that highly charged impurities may be accelerated anti-parallel to the electric field. The remaining friction against the bulk has a non-monotonic velocity dependence, depending on composition, with friction against electrons limiting the maximum energy attainable in moderate electric fields.

Earlier analytic treatments of runaway ions were focused on the initial stages of the process [3, 5], or limited to the high-electric field limit and unable to provide details of the runaway tail formation and its features [6]. Here we describe the formulation and implementation of an efficient, open-source [7], finite-difference-spectral-method tool, CODION (COllisional Distribution of IONs), that solves the two-dimensional momentum space ion kinetic equation in a homogeneous plasma. Using CODION we obtain illustrative two-dimensional ion velocity space distributions, which demonstrate the typical behaviour of runaway ions in a variety of physical scenarios.

Formulation We consider the problem of ion acceleration by induced electric fields parallel to the magnetic field in a plasma. We assume straight field line geometry and a homogeneous background plasma. The time evolution of the ion distribution is then given by the Fokker-Planck equation, $\partial_t f_i + \frac{Z_i e}{m_i} E^* (\xi \partial_v + (1 - \xi^2)/v \partial_\xi) f_i = \sum_s C_{is} \{f_i\}$, where $\xi = v_{\parallel}/v$ and C_{is} is the collision operator. We restrict the study to fully ionized plasmas. We consider the case where an electric field appears in what was previously an equilibrium state, and may linearize the distribution around a Maxwellian f_M if we only consider cases of small total runaway density,

or the initial phase of the process. The linearized collision operator for self-collisions is given by $C'_{ii}\{f_i\} = C_{ii}\{f_i, f_{Mi}\} + C_{ii}\{f_{Mi}, f_i\}$, where the first, the test-particle operator, describes collisions of the perturbed distribution with the bulk, while the second, the field-particle operator, describes the response of the bulk to the perturbation, for which we use the model described in Ref. [8]. Assuming the other ion species to remain near equilibrium, the collision operator is

$$\sum_s C_{is}\{f_i\} = \frac{1}{\tau_{ie}} \sum_s \frac{n_s Z_s^2}{n_e} \left\{ \frac{\phi(x_s) - G(x_s)}{2x_i^3} \frac{\partial}{\partial \xi} \left[(1 - \xi^2) \frac{\partial f_i}{\partial \xi} \right] + \frac{1}{x_i^2} \frac{\partial}{\partial x_i} \left[2 \frac{T_i}{T_s} x_i^2 G(x_s) f_i + x_i G(x_s) \frac{\partial f_i}{\partial x_i} \right] \right\} + \frac{1}{\tau_{ii}} [2v_s(v) x_i \xi u_{\parallel} + v_E(v) x_i^2 Q] f_{Mi},$$

where $\tau_{is}^{-1} = n_s e^4 Z_i^2 Z_s^2 \ln \Lambda / 4\pi \epsilon_0^2 m_i^2 v_{Ti}^3$, $x_s = v/v_{Ts} = \sqrt{m_s v^2 / 2T_s}$ and the usual error function $\phi(x) = (2/\sqrt{\pi}) \int_0^x dy e^{-y^2}$ and Chandrasekhar function $G(x) = [\phi(x) - x\phi'(x)] / 2x^2$ appear. The moments u_{\parallel} and Q of the distribution function appearing in the momentum and energy restoring terms of the self-collision operator are $u_{\parallel}\{f_i\} = \frac{3}{2} v_{Ti} (\int d^3 v v_s(v) v_{\parallel} f_i) / (\int d^3 v v_s(v) v^2 f_{Mi})$ and $Q\{f_i\} = v_{Ti}^2 (\int d^3 v v_E(v) v^2 f_i) / (\int d^3 v v_E(v) v^4 f_{Mi})$, with the scattering frequencies given by $v_s(v) = 4G(x_i)/x_i$, and $v_E(v) = 8G(x_i)/x_i - 2\phi(x_i)/x_i^3$.

Important features of the runaway behaviour can be demonstrated by reducing to the simpler case of an ion test particle, neglecting the effect of diffusion. The ion equation of motion takes the form $m_i dv_{\parallel}/dt = Z_i e E^* + F_i^{\text{test}}$ [2], where the collisional friction on a test particle is given by $F_i^{\text{test}}(v) = -Z_i^2 e E_D \sum_s \frac{n_s Z_s^2 T_e}{n_e T_s} (1 + m_s/m_i) G(x_s)$, and $E_D = n_e e^3 \ln \Lambda / 4\pi \epsilon_0^2 T_e$ is the Dreicer field. Thus ion acceleration can occur when $E/E_D > F_i^{\text{test}}/Z_i e E_D |1 - Z_i/Z_{\text{eff}}|$, where $Z_{\text{eff}} = \sum_i Z_i^2 n_i / n_e$ is the effective charge. The electric field required to accelerate a test ion with given velocity is illustrated in Fig. 1 for low and high Z_{eff} . The non-monotonic friction shows two maxima, near the ion and electron thermal velocities, and also shows a strong dependence on charge and composition. This qualitative behaviour is what allows a runaway ion population to form in the presence of a sufficiently strong electric field.

CODION The discretization scheme used in CODION is based on an expansion of the distribution function in Legendre polynomials, $f_i(t, p, \xi) = \sum_l f_l(t, p) P_l(\xi)$, as in CODE [9]. A description of the scheme, together with convergence tests and more, can be found in Refs. [10, 11].

Figure 2(a) shows a typical example of the evolution of the ion distribution for a case where the electric field is above the minimum required for runaway acceleration. The plasma parameters used are characteristic for tokamaks with a hot bulk deuterium plasma at 1 keV and fully ionized native carbon impurities. A contour plot of the distribution in velocity space when steady state is reached is shown in Fig. 2(b). The required fields of order 1 V/m are, however,

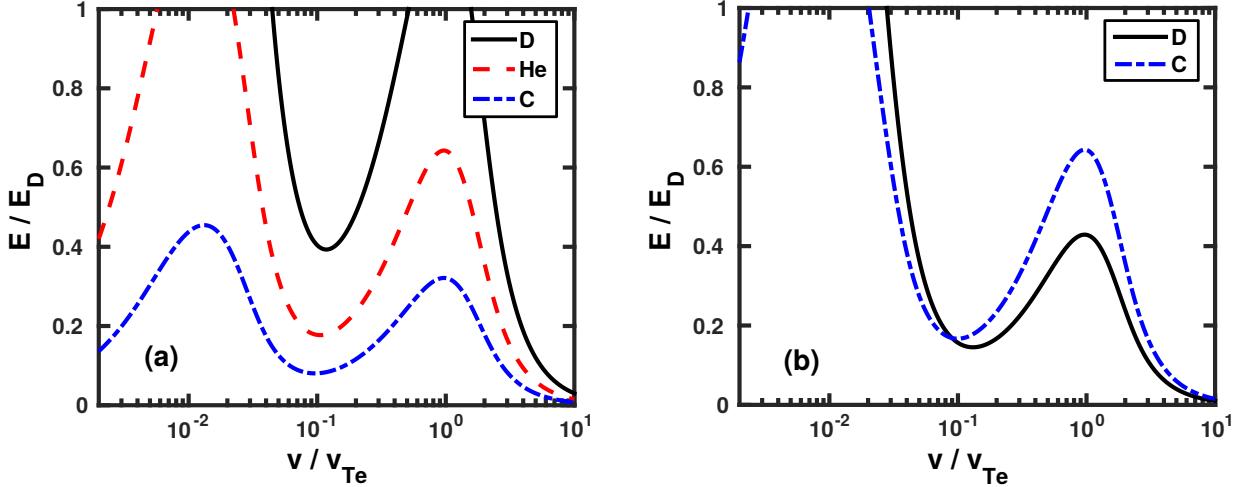


Figure 1: Electric field needed to accelerate fully ionized test particles of various ion species in impure deuterium plasmas. (a) $n_C/n_D = 0.4\%$, $n_{He}/n_D = 5\%$, $Z_{\text{eff}} = 1.2$ and (b) $n_C/n_D = 4\%$, $n_{He}/n_D = 5\%$, $Z_{\text{eff}} = 2$. All particle species are taken to be at the same temperature.

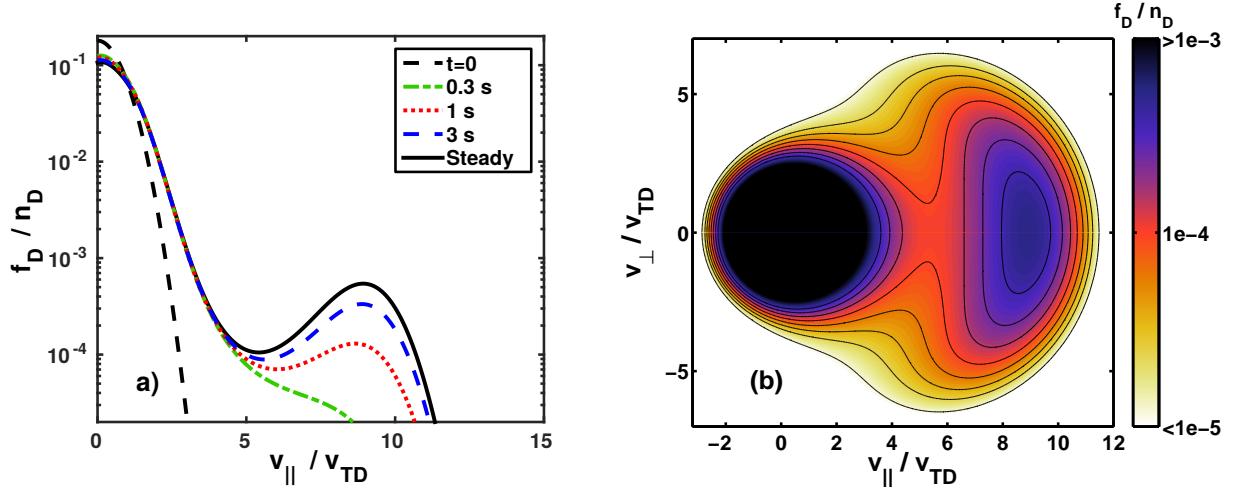


Figure 2: Deuterium distribution function in a plasma characterized by $T = 1 \text{ keV}$ for all particle species, $n_e = 3 \cdot 10^{19} \text{ m}^{-3}$, $Z_{\text{eff}} = 2$ due to fully ionized carbon impurities with $n_C/n_D = 4\%$, and $E = 1.64 \text{ V/m}$. (a) Time evolution of the $\xi = 1$ cut of the distribution and (b) contour plot of the steady state distribution, established after 20 s.

unlikely to occur in operational tokamak plasmas for long enough that an appreciable runaway ion population has time to form.

Large electric fields are induced during tokamak disruptions and are well-known to generate runaway electrons [12]. However, low-frequency magnetic fluctuations in the frequency range $f \approx 60 - 260 \text{ kHz}$ have been observed in disruptions induced by massive gas injection (MGI) [13]. The observed frequency range is consistent with low-mode number Toroidal Alfvén Eigenmodes (TAEs) [14], which can be resonantly driven by fast ions satisfying $v_{\parallel} = \{v_A/3, v_A\}$, where $v_A = B/\sqrt{\mu_0 \rho_m}$ is the Alfvén speed and ρ_m the mass density.

A limited analytic study of the potential for runaway ions to drive such modes was given in Ref. [5]. CODION allows us to investigate the scenario in detail. With the same plasma composition as used in the previous case, but with a lower temperature of $T = 10\text{ eV}$, we may solve for the ion runaway distribution in a post-disruption plasma after a characteristic time scale of 2 ms, assuming a Maxwellian initial ion distribution. The resulting distribution functions are illustrated in Fig. 3 for various electric fields. The resonant velocity is $v_A/3 \sim 30\text{-}40 v_{TD}$, indicating that runaway ions typically have too low energy to drive TAEs, and also require higher electric fields than expected of typical post-disruption fields according to models [15]. Note, however, that fast ion populations present due to heating schemes in use before the disruption may not be completely expelled, and it is therefore not certain that the initial distribution is accurately described by a Maxwellian as assumed here.

Summary A numerical solver of the 2D ion Fokker-Planck equation has been presented, and its usefulness demonstrated in investigating the conditions required for ion runaway in cold and hot tokamak plasmas. It is shown that Alfvénic activity observed in disruption experiments are unlikely to be explained by the runaway mechanism alone.

References

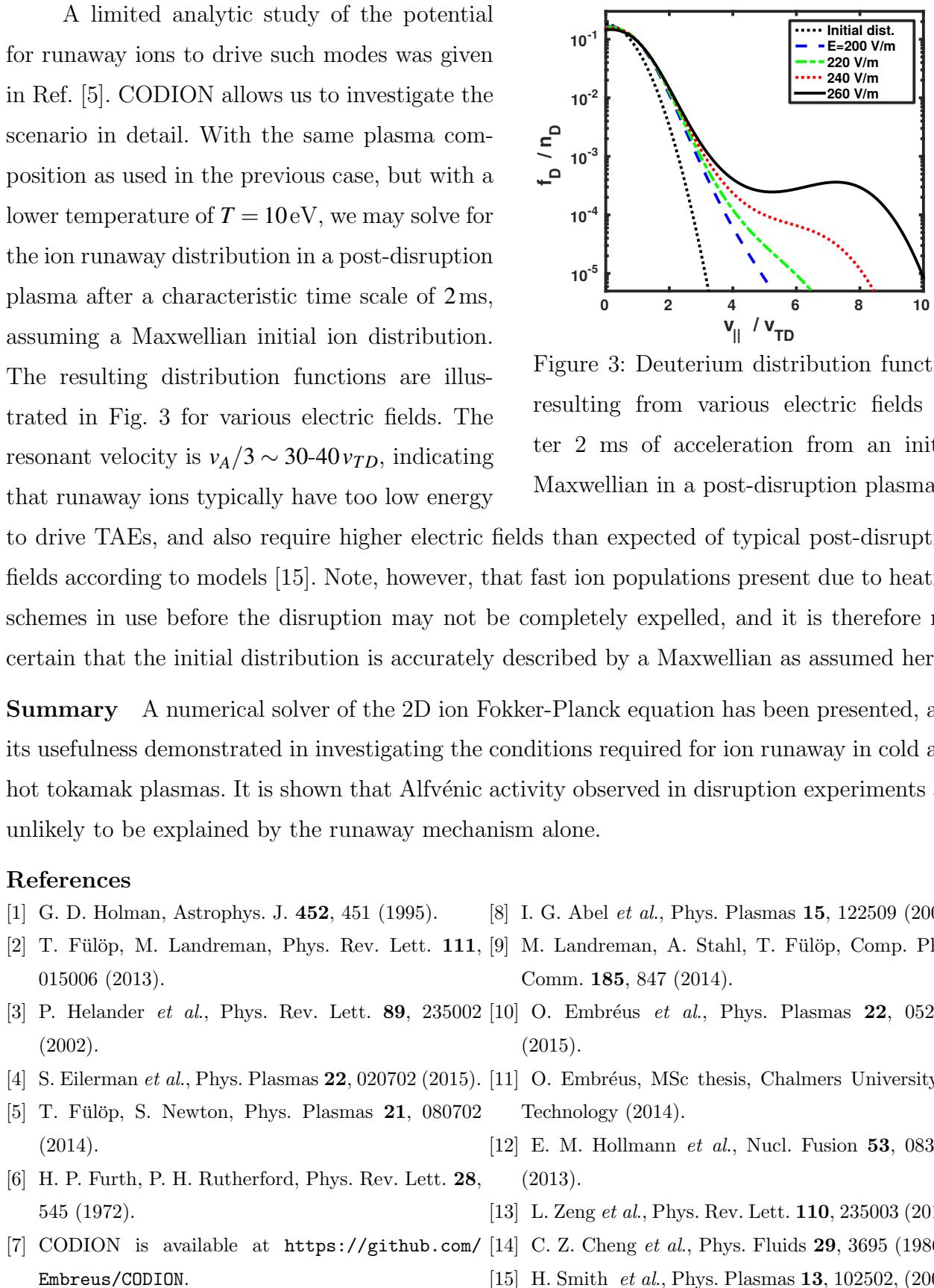


Figure 3: Deuterium distribution function resulting from various electric fields after 2 ms of acceleration from an initial Maxwellian in a post-disruption plasma.

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