The influence of electron-electron scattering on the asymptotic behaviour of the strongly degenerate plasma electrical conductivity

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The dc conductivity of fully ionized plasma in the strongly degenerate limit is investigated using the linear response theory. An analytical expression for the temperature dependence of the difference between conductivities of plasma with and without electron-electron scattering is obtained. It is found that this difference is proportional to the fourth power of the degeneracy parameter, if we neglect the ion-ion correlations. Accounting for ion-ion correlations in the approximation of the Debye-Hückel type leads to the third power of dependency above. The result is applicable in constructing interpolation formulas for calculating the plasma conductivity in the wide temperature and density regions on the basis of the Lorentz model.

Introduction

The temperature behaviour of the electron-electron scattering influence on plasma transport properties is interesting both for the description of experiments and the construction of interpolation formulas for the conductivity [1, 2, 3]. The electron-electron scattering makes a significant contribution to the conductivity of low-density non-degenerate plasma [4], where the systematic treatment of it with the dynamical character of dielectric screening is possible on the base of linear response theory [5]. For technical details and asymptotic behaviour of conductivity of low-density non-degenerate plasma see [6]. In strongly degenerate electron systems the electron-ion scattering is dominated and the conductivity is well defined without the electron-electron collisions account [7, 8]. Some works were aimed at the constructions of interpolation formulas, linking the real plasma conductivity with the Lorentz one by introducing the correction factor between them. In particular, two different correction factors were introduced in [9] and [10]. In order to create more suitable correction factor and interpolation formula for conductivity, it would be useful to investigate the possibilities to determine an asymptotic behaviour of it in both limits on degeneracy. In the present work we consider the limit of high degeneracy.

Basic approximations

Following [11], consider a neutral two-component plasma consisting of free singly charged particles with charges e_i (ions) and e_e (electrons, $e_i = -e_e = e$) at temperature T and density $n = n_\mathrm{e} = n_\mathrm{i}$ in the adiabatic limit (the electronic mass is much less than the ionic mass), interacting via Coulomb forces. Introduce degeneracy parameter $\Theta = (2m_\mathrm{e}k_\mathrm{B}T/\hbar^2)(3\pi^2 n_\mathrm{e})^{-2/3}$ and the electron-ion coupling constant $\Gamma = (e^2/4\pi\varepsilon_0 k_\mathrm{B}T)(4\pi n_\mathrm{e}/3)^{1/3}$. For the strongly degenerate plasma $\Theta \ll 1$. Within the linear response theory in the formulation of Zubarev [5], the transport properties are expressed via force-force correlation functions [12, 13, 14]. The resulting expression for conductivity is

$$\sigma = -\frac{e^2}{\Omega \det(d)} \begin{vmatrix} 0 & N_0 \\ \overline{N_0} & d \end{vmatrix}, \tag{1}$$

$$N_n = \begin{pmatrix} N_{n0} & N_{n1} & \dots & N_{nl} \end{pmatrix}, \tag{2}$$

$$\overline{N_n} = \begin{pmatrix} N_{n0} \\ N_{n1} \\ \vdots \\ N_{nl} \end{pmatrix}, d = \begin{pmatrix} d_{00} & d_{01} & \dots & d_{0l} \\ d_{10} & d_{11} & \dots & d_{1l} \\ \vdots & \vdots & \ddots & \vdots \\ d_{l0} & d_{l1} & \dots & d_{ll} \end{pmatrix}. \tag{3}$$

In (1)-(3) Ω - the system volume, N_{mn}, d_{mn} are correlation functions for the thermodynamic equilibrium, N_e - the number of electrons and $\beta = (k_B T)^{-1}$. The dimension of the matrix d coincides with the number of moments in the corresponding relevant statistical operator (see [5]). In the adiabatic limit we can omit the ion flux [15] and obtain for Eq.(3)

$$d_{mn} = d_{mn}^{ei} + d_{mn}^{ee}, \tag{4}$$

$$N_{mn} = N_e \frac{\Gamma(m+n+5/2)}{\Gamma(5/2)} \frac{I_{m+n+1/2}(\beta\mu_e^{id})}{I_{1/2}(\beta\mu_e^{id})}, \tag{5}$$

with $I_\nu(y)$ - the Fermi integrals, μ_e^{id}- the ideal part of the electronic chemical potential.

The correlation functions d_{mn} are evaluated using thermodynamic Green's functions. As in [11], we restrict ourselves to $l = 1$ in (2), (3).

In the adiabatic limit

$$d_{mn}^{ei} = \frac{4m_e^2}{3\pi^2\beta^2\hbar^3} \int_0^\infty dx x^{n+m+2} f_k^e (1 - f_k^e) Q_{ei}(x), \tag{6}$$

$$Q_{ei}(x) = \frac{\beta^2\Omega^2}{16\pi x^2} \int_0^{2k} \left| \frac{V(q)}{\varepsilon_e(q,0)} \right|^2 S_{ii}(q) q^3 dq, \tag{7}$$

$$\varepsilon_e(q,0) = 1 + \Omega V(q)(1 - G_e(q))\chi_e^{(0)}(q,0), \tag{8}$$

with $x = \frac{\beta\hbar^2 k^2}{2m_e}$, $Q_{ei}(x)$ - the transport cross-section for electron-ion scattering, $\varepsilon_e(q,0)$ - the effective static electronic dielectric function, $S_{ii}(q)$ - the ion-ion structure factor, $\chi_e^{(0)}(q,\omega)$ the free-electron polarizability, $G_e(q)$ - the static electronic local field correction.

The correlation function d_{mn}^{ei} for the electron-electron scattering in the strongly degenerate limit was evaluated in [11]:

$$d_{11}^{ee} = 17.3\Theta^{7/2} k_F d \int_0^{2k_F} \frac{dq}{q^2 \varepsilon_e^2(q,0)}, \tag{9}$$

where $d = \frac{8}{3}\frac{m_e^{1/2} e^4 N_e^2 \beta^{3/2}}{\Omega(4\pi\varepsilon_0)^2}$.

Representing $d_{m,n-m}^{ei} = \int_0^\infty f(E) d((\beta E)^n U_{ei}(E))$, we obtain

$$d_{00}^{ei} = U_{ei}(E_F) + \frac{\pi^2}{6} T^2 U_{ei}^{''}(E_F) + \frac{7\pi^4}{360} T^4 U_{ei}^{IV}(E_F) + ..., \tag{10}$$

$$d_{01}^{ei} = \Theta^{-1}(d_{00}^{ei} + \delta_1\Theta^2 + \delta_2\Theta^4 + ...), \tag{11}$$

$$d_{11}^{ei} = \Theta^{-2}(d_{00}^{ei} + \varepsilon_1\Theta^2 + \varepsilon_2\Theta^4 + ...), \tag{12}$$

$$N_{00} = N_e, N_{01} = N_e\Theta^{-1}(1 + \alpha_1\Theta^2 + \alpha_2\Theta^4 + ...), \tag{13}$$

where E_F is the Fermi energy.

For d_{11}^{ee} we suggest $\varepsilon_{ei} = d_{11}^{ee}/d_{11}^{ei} = o(\Theta^2)$. As the analysis shows, $\varepsilon_{ei} \sim \Theta^4$ for $S_{ii} = 1$, $\varepsilon_{ei} \sim \Theta^3$ for the Debye-Hückel model, and has a tendency to $\Theta^3 ln\Theta$ for HNC procedure.

Results and discussion

Substituting all the expansions into (1), we obtain:

$$\sigma = \sigma_{Lorentz}\left(1 - \varepsilon_{ei}\left(\frac{E_F U_{ei}^{'}(E_F)}{U_{ei}(E_F)} - \frac{3}{2}\right)^2\right). \tag{14}$$

Compare the result obtained with the approximations:

$$\sigma_S = \sigma_{Lorentz}\left(1 - 0.0125\Theta\right), \tag{15}$$

$$\sigma_F = \sigma_{Lorentz}\left(1 - 0.209\Theta^2\right) \tag{16}$$

of [9] and [10], correspondingly.

For $\varepsilon_e(q,0) = 1 + \frac{\kappa^2}{q^2}$ (Thomas-Fermi approximation for the electronic dielectric function, $\kappa^2 = \frac{4k_F}{\pi a_0}$, k_F - Fermi wave number, a_0 - Bohr radius)

$$d_{11}^{ee} = 8.65\Theta^{7/2} d\left(\sqrt{k_s}\arctan\sqrt{k_s} - \frac{k_s}{1+k_s}\right), \tag{17}$$

where $k_s = \frac{4k_F^2}{\kappa^2} = \pi k_F a_0 = \frac{6.029}{r_s}$, $(a_0 r_s)^3 = \frac{3}{4\pi n_e}$.

If $S_{ii}(q) = 1$,

$$d_{00}^{ei} = \frac{3\sqrt{\pi}}{8}\Theta^{3/2} d\left(ln(1+k_s) - \frac{k_s}{1+k_s}\right), \tag{18}$$

and $\varepsilon_{ei} \sim \Theta^4$.

More realistic structure factor is obtained from linearization of HNC procedure, or equivalent contour integral treatment of electron-ion collisions dynamical screening in adiabatic approximation [16]. For the Thomas-Fermi dielectric function (Debye-Hückel model)

$$S_{ii}(q) = \frac{q^2+\kappa^2}{q^2+\kappa^2\left(1+\frac{2}{3\Theta}\right)}. \tag{19}$$

This structure factor approximates well the results of HNC procedure for q up to $2k_F$ and $\Gamma \sim 1$, and it can be used for estimates up to $r_s \sim 2\Theta$.

In this approximation we obtain

$$\varepsilon_{ei} = 17.35\Theta^3 \frac{\sqrt{k_s}\arctan\sqrt{k_s} - \frac{k_s}{1+k_s}}{k_s - ln(1+k_s)}, \frac{E_F U'_{ei}(E_F)}{U_{ei}(E_F)} = \frac{k_s^2}{(1+k_s)(k_s - ln(1+k_s))}. \tag{20}$$

Further analysis can be carried out using the numerical procedures for determining the structure factor. HNC procedure shows the tendency to $\Theta^3 ln\Theta$ for ε_{ei} behaviour. The value $U'_{ei}(E_F)$ is very sensitive to the details of $S_{ii}(q)$ and needs further research.

Unlike (15) and (16), correction factors to Lorentz plasma conductivity show other behaviour on Θ and are dependent on electronic density. These results should be considered in further constructions of interpolation formulas for the conductivity

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