Bohm-criterion approximation versus optimal matched solution for a spherical probe in radial-motion theory

Siegbert Kuhn[1], Alif Din[2]
[1] *Institute for Theoretical Physics, University of Innsbruck, Technikerstrasse 21A, A-6020 Innsbruck, Austria*
[2] *Theoretical Physics Division, PINSTECH, P. O. Nilore, 44000, Islamabad, Pakistan*

Abstract. It is demonstrated that in the large-negative-bias regime the probe characteristics obtained for the correct potential distribution ("M solution", cf. A. Din and S. Kuhn, Phys. Plasmas **21**, 103509 (2014)) differs substantially from those obtained by application of the Bohm criterion. This raises questions about the correctness of probe theories relying on merely applying the Bohm criterion.

1. Introduction and overview. The theory of positive-ion collection by a spherical probe immersed in a low-pressure plasma was reviewed and extended by Allen et al. [1]. Numerical extensions were given later by Allen and Turrin [2] and Chen [3]. In all of these treatments, a nonneutral solution (governed by the Poisson equation, potential profile $V_{nn}^{(m)}(r;r_m)$) was matched to the quasineutral solution (governed by the quasineutrality condition, potential profile $V_{qn}(r)$) at some "matching" radius r_m, resulting in the "matched" global solution ("M solution", potential profile $V^{(m)}(r;r_m)$). An efficient numerical procedure for systematically determining the optimal matching radius r_{mo} was presented recently by Din and Kuhn [4]. There it was also demonstrated explicitly that the optimal M solution (i.e., the M solution with $r_m = r_{mo}$) differs substantially from the "B solution" obtained by merely applying the Bohm criterion, which holds at $r = r_{qn,s}$, the singularity point of the quasineutral solution.

In the present work we extend this comparison by comparing the current-voltage probe characteristics ensuing from the B solution and the optimal $(r_m = r_{mo})$ M solution.

2. Comparison of the M and B solutions; applicability of the Bohm-criterion approach
A frequently used method for approximately solving plasma-wall transition problems is the "Bohm-criterion approach" [5], [6], in which the quasineutral or presheath solution is used all the way down to the singularity point, where the Bohm criterion is satisfied and the sheath (which on the presheath scale is infinitesimally thin) is located, cf. Fig. 1(b) of [6]. Let us

now discuss how, for our spherical probe problem, the approximate solution following from this Bohm-criterion approach ("B solution") compares with our correct solution based on the matching approach ("M solution").

Let us start out from the fairly general quasineutrality condition presented in eq. (2.17) of [6], which in our case (spherical symmetry, collisionless cold ions, Boltzmann-distributed electrons) reads

$$\left[m^i\left(u^i\right)^2-kT^e\right]\frac{dn^e}{dr}=\frac{-2m^i\left(u^i\right)^2 n^e}{r}. \tag{1}$$

Since the right-hand side vanishes nowhere, the singularity $dn^e/dr\to-\infty$ clearly occurs where the “cold-ion Bohm criterion”

$$\left|u^i\right|=\sqrt{\frac{kT^e}{m^i}} \tag{2}$$

is satisfied. Applying the normalization scheme $\tilde{Q}=Q/Q_{ref}$ with the reference quantities

$$V_{ref}=kT^e/e,\quad n_{ref}=n_0,\quad r_{ref}=\lambda_{D0}^e=\sqrt{\varepsilon_0 kT^e/\left(n_0e^2\right)},\quad E_{ref}=\sqrt{n_0kT^e/\varepsilon_0},$$

$$v_{ref}=c_s:=\sqrt{kT^e/m^i},\quad I_{ref}=c_s:=4\pi\left(\lambda_{D0}^e\right)^2 en_0\sqrt{2kT^e/m^i}, \tag{3}$$

the normalized forms of eqs. (1) and (2) read

$$\left[\left(\tilde{u}^i\right)^2-1\right]\frac{d\tilde{n}^e}{d\tilde{r}}=\frac{-2\left(\tilde{u}^i\right)^2\tilde{n}^e}{\tilde{r}} \tag{4}$$

and

$$\left|\tilde{u}^i\right|-1, \tag{5}$$

respectively.

In Fig. 1 (taken from [4]) we compare the B and M solutions for $\tilde{I}^i$ = -200. The B solution looks qualitatively like the solution in Fig. 1(b) of [6] and is totally different from our (numerically approximate but conceptually correct) optimal $\left(\tilde{r}_m=\tilde{r}_{mo}\right)$ M solution. In particular, on the presheath scale shown here the sheath of the B solution is infinitesimally thin and for $\tilde{r}_p<\tilde{r}<\tilde{r}_{qn,s}$ cannot reach the probe surface, whereas the nonneutral (“sheath”) region of our M solution obviously is of finite width, extending all the way down to the probe surface.

In summary, the B solution turns out to be visibly wrong in our case: The Bohm-criterion approach is absolutely inapplicable for $\tilde{r}_p<\tilde{r}<\tilde{r}_{qn,s}$ and yields an approximation to the correct solution which is extremely poor for $\tilde{r}$ slightly above $\tilde{r}_p$ and tends towards the correct solution only for. $\tilde{r} \to \infty$. The fundamental reason for this failure was discussed in Ref. [4].

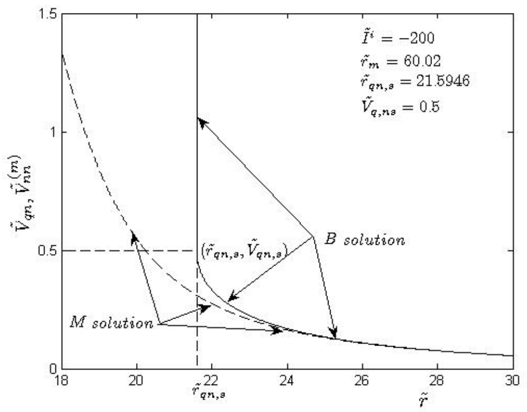

Figure 1: Comparison of potential profiles resulting from our optimal matching approach (M solution) and the Bohm-criterion approach (B solution) [4].

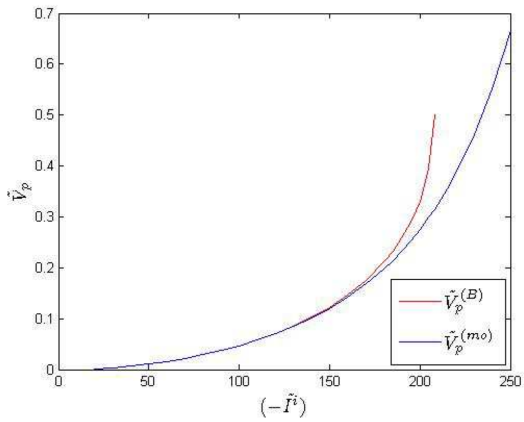

Figure 2: Comparison of the $\tilde{V}_p vs.(-\tilde{I}^i)$ probe characteristics following from the B solution (upper curve) and the optimal M solution for probe radius $\tilde{r}_p = 22$.

3. Comparison of the $\tilde{V}_p vs.(-\tilde{I}^i)$ probe characteristics for the M and B solutions

From potential distributions $\tilde{V}(\tilde{r})$ obtained for various values of $\tilde{I}^i$ (as exemplified by Fig. 1) we can extract the $\tilde{V}_p vs.(-\tilde{I}^i)$ probe characteristics in the large-negative-bias regime for a given value of the probe radius $\tilde{r}_p$ (as exemplified by Fig. 2 for $\tilde{r}_p = 22$) as follows. As a representative example consider Fig. 1, which shows the B and optimal M solutions for $\tilde{I}^i =$ = -200, in which case the singularity of the quasineutral solution occurs at $\tilde{r}_{qn,s}$ = 21,594 and the optimal matching radius has been found to be $\tilde{r}_{mo} = 60{:}02$. For $\tilde{r}_p < \tilde{r} < \tilde{r}_{qn,s}$ the B solution is inexistent and hence yields no value of the probe potential at all, whereas the M solution yields some value of $\tilde{V}_p$ p for any chosen value of $\tilde{r}_p$: In the region of existence of the B solution, $\tilde{r}_{qn,s} \le \tilde{r}_p < \tilde{r}$, the potential distribution corresponding to the B solution always lies above the one corresponding to the Mo solution and tends towards the latter as $\tilde{r} \to \infty$. From this kind of reasoning we obtain characteristics as exemplified in Fig. 2 for . $\tilde{r}_p$ = 22; which (together with the potential distributions exemplified by Fig. 1) show that the Bohm-criterion approximation is absolutely inadequate for $\tilde{r}_p < \tilde{r} < \tilde{r}_{qn,s}$ and very poor for $\tilde{r}$ slightly above but near the singularity point, but becomes more and more acceptable with increasing $\tilde{r}_p$.

4. Conclusion. The fact that the (very approximate) Bohm-criterion approach leads to probe characteristics that differ strongly from the characteristics following from the (essentially correct) matching approach raises questions about the correctness of probe theories relying on merely applying the Bohm criterion. Further investigations into this problem will be conducted by the present authors.

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