Tokamak equilibrium reconstruction code LIUQE: implementation, applications, developments

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Equilibrium reconstruction consists of identifying, from experimental measurements, a plasma current distribution that satisfies the pressure balance. Elongated equilibria in TCV impose specific requirements on its reconstruction code LIUQE [2], mainly the stabilisation of the vertical position, the influence of vessel eddy currents and for real time (RT) control a cycle time compatible with the actuator response time. Coupling with RAPTOR [3] is under development, in which 1D transport equations are solved to obtain physics based profiles for the pressure and the current density.

Vertical stabilisation. LIUQE adopts a method based on an iterative solution of the Poisson equation for the poloidal flux ψ, $\Delta^*\psi = -2\pi\mu_0 r j_\varphi$, coupled with a linear parametrisation of the plasma current density j_φ. This algorithm is unstable against vertical motion for elongated shapes. The growth rate is given by $1-\partial_z^2\psi_{eA}/\partial_z^2\psi_A$ [2] where ψ_e is the flux produced by the external coils, and the subscript A tags quantities evaluated on the magnetic axis; this growth rate is thus directly related to the curvature index of the shaping field, and reaches 2 for high elongations. To stabilise the algorithm, it is assumed that the available measurements contain the information necessary to determine the equilibrium vertical position; thus a free parameter δz is added when fitting the plasma current distribution, seeking for solutions of the form $j_\varphi = 2\pi\left(rp'(\psi(r,z+\delta z)) + \frac{TT'(\psi(r,z+\delta z))}{\mu_0 r}\right)$. The specific response to δz of a magnetic measurement located at (r_m, z_m) can be conveniently evaluated using the relation $\int Gj_\varphi(r,z+\delta z)drdz \cong \ldots + \delta z\int G\partial_z j_\varphi(r,z)drdz = \ldots + \delta z\int \partial_{z_m} Gj_\varphi(r,z)drdz$ where $G(r_m, z_m, r, z)$ is the appropriate Green's function. The cumbersome calculation of $\partial_z j_\varphi$, discontinuous on the LCFS, is thus avoided. The Poisson equation can be solved with the unshifted plasma current distribution but with shifted external currents, $\Delta^*\psi_+(r,z) = -2\pi\mu_0 r(j_\varphi(r,z) + j_e(r, z-\delta z))$, which implies a correction on the boundary conditions given by $-\delta z\int \partial_z M(r,z,r',z')j_e(r',z')dr'dz'$ where M is the mutual inductance. The final solution writes $\psi = \psi_+ + \delta z\partial_z\psi_+$. This particular treatment does not impair the algorithm efficiency and accuracy, since it relies only on the smooth derivative $\partial_z \psi_+$ and on precalculated $\partial_{z_m} G$ and $\partial_z M$.

Vessel currents. A vacuum vessel with low resistance designed to enhance passive stabilisation of the vertical position or a conducting shell that stabilises MHD modes give rise to eddy currents with a sizable influence on the equilibrium reconstruction. These are taken into account with dedicated measurements or with an appropriate electromagnetic modelling of the conducting structures. In the case of TCV [2], the vacuum vessel is poloidally divided in 38 segments, each of them being equipped with a flux loop. The current in the segments is estimated with $I_s = -R_s^{-1} U_s$ where U_s is the voltage measured by the loops. The segment resistance R_s is experimentally obtained by applying long current ramps $\frac{dI_a}{dt}$ in the poloidal field coils during which the relation $I_s = -R_s^{-1} M_{sa} \cdot \frac{dI_a}{dt}$ holds. The magnetic measurements X_m can then be used to evaluate R_s, solving $X_m - G_{ma} \cdot I_a = -G_{ms} \cdot R_s^{-1} M_{sa} \cdot \frac{dI_a}{dt}$ in a least square sense.

Error analysis. During the measurement fitting step, the current distribution is parametrised with base functions $g_g(\psi)$ as follows: $j_\varphi = \sum_g a_g r^{\nu_g} g_g(\psi)$, with $\nu_g = \pm 1$ for the p' and TT' terms respectively. A covariance analysis of this procedure shows that, using the available measurements, i.e. magnetic measurements including the diamagnetic flux, the covariance between the coefficients a_g typically reaches 0.99 in absolute value when 3 base functions are used, while it is maintained to 0.80 for 2 base functions. In the latter case, error propagation yields an accuracy of 3% for the central pressure current density. This limit in the number of relevant base functions is compatible with the measurement set, the total plasma current giving a constraint on the base function sum, and the diamagnetic flux on the TT' terms.

Coupling with RAPTOR. The goal of coupling LIUQE and RAPTOR is to obtain equilibrium reconstruction using physics based profiles for the pressure and the current density, obtained by solving with RAPTOR 1D transport equations constraint by measurements. This alleviates the arbitrary choice in the base functions and allows, for example, steep or non-monotonic profiles. The flux surface averaged (FSA) equations involve flux contour integrals in the form $Q_{\mu\nu} = (2\pi)^{-\mu} \oint |\nabla\psi|^\mu r^\nu dl_\theta$, a particular case being the safety factor $q = \frac{TQ_{-1,-1}}{2\pi}$. Based on the contour description as the distance from the magnetic axis $r^*(\rho, \theta)$ for a given normalised flux ρ^2 and poloidal angle θ, one can rewrite $Q_{\mu\nu}$ as $Q_{-1,\nu} \sim \oint_0^{2\pi} r^\nu \rho^{-1} \partial_\rho r^{*2} d\theta$ and $Q_{1,\nu} \sim \oint_0^{2\pi} r^\nu \left(r^{*2} + \frac{1}{4} r^{*-2} (\partial_\theta r^{*2})^2 \right) \rho (\partial_\rho r^{*2})^{-1} d\theta$. Near the magnetics axis, a second order Taylor expansion of the flux yields the analytical formula given in [2]. For a divertor equilibrium, the limit values on the separatrix are $Q_{-1,\nu}/Q_{-1,1} = r_X^{\nu-1}$ and $Q_{1,\nu} = 0$, r_X being the radial position of the active X point.

In between an efficient determination of the flux contours is necessary. It is based on a bilinear interpolation of the flux $\hat{\psi}$ in the grid cell. A Newton method is used to search for the intersection of the flux contour at level ψ_ρ and the radius at angle θ, iteratively applying the update: $r^* \leftarrow r^* - (\hat{\psi} - \psi_\rho)/(\nabla\hat{\psi} \cdot \nabla r^*)$ where $\hat{\psi}$ is evaluated at $(r_A - r^* \cos\theta, z_A + r^* \sin\theta)$. Each Newton iteration is interleaved with a reconstruction iteration; the latest values for ψ and (r_A, z_A) are used while the cell containing the contour point at the previous iteration is used for the interpolation. Note finally that for an accurate evaluation of the derivative $\partial_\rho r^{*2}$ in $Q_{\mu\nu}$, a non equidistant ρ grid is chosen, especially near an X point.

In its simplest form the coupling between LIUQE and RAPTOR is achieved by interleaving a LIUQE iteration for each RAPTOR iteration: RAPTOR is fed with the 2D equilibrium geometry obtained from LIUQE, while LIUQE uses as base functions for p' and TT' the profiles calculated by RAPTOR using a FSA Grad-Shafranov equation.

Implementation. To facilitate its collaborative development and its application to other devices, LIUQE is coded in MATLAB, with a generic diagnostic description and an interface with the ITM data structure. It has been successfully used to analyse RFX tokamak equilibria.

In its RT implementation, the number of iterations is reduced to one, while the value for the current distribution in the Poisson equation is taken from the previous time sample, assuming that a sufficiently short sampling time is used so that the equilibrium does not change significantly between two samples. The RT version has been implemented in TCV's control system [4], coded in its SIMULINK standard environment [5]. Running on one single core of an INTEL i7 processor overclocked at 5 GHz, a cycle time of 200 μs was achieved for an (r, z) grid of 28 by 65, using all 133 measurements and including the FSA quantities necessary for the RT RAPTOR on a (ρ, θ) grid of 16 by 32. Amongst the choice of numerical methods and code optimisation techniques that lead to this performance, we can cite the followings:

- In fitting the currents to the measurements, the latter are grouped in the measurements Y_e sensitive only to the external currents I_e, those Y_i sensitive only to the plasma current distribution, i.e. the parameters $a_j = \{a_g, \delta z\}$, and those Y_r sensitive to both. The fitting is then obtained by solving in a least square sense the equations $\begin{bmatrix} Y_r - W_{re} \cdot J_e \\ Y_i \end{bmatrix} = \begin{bmatrix} W_{rj} \\ W_{ij} \end{bmatrix} \cdot a_j$ and $\begin{bmatrix} Y_r - W_{rj} \cdot a_j \\ Y_e \end{bmatrix} = \begin{bmatrix} W_{re} \\ W_{ee} \end{bmatrix} \cdot J_e$ where W are the corresponding response matrices. This system can be solved iteratively first for a_j and then for J_e; in doing so the only matrix which depends on the equilibrium and cannot be precalculated and inverted has the size of a_j, at most 4 by 4, and is inverted in RT using an unrolled LDL decomposition.

- Quantities depending on (r, z) are stored with z as the fastest varying index, leading to contiguous indexing for the numerous operations along the z direction.
- For loop intensive operations are coded in C. They pervade the Poisson solver, based on a direct tridiagonal resolution in the r direction and a cyclic reduction in the z direction, the search for the X points and the magnetic axis, and the identification of the plasma domain.
- Matrix and vector multiplications use the *Intel Math Kernel Library (MKL)*.
- Code vectorisation using the specific instruction set of the target processor is automatically achieved with the compiler option *-ax*. This vectorisation is further facilitated by using integer and floating point numbers with the same binary length.
- All floating point operations are carried out in single precision, since none of the algorithm steps depends unduly on rounding errors. The required memory for variables and parameters then fits in the 8 Mbytes of the L3 cache, minimising cache faults.

Conclusion. The equilibrium reconstruction code LIUQE features specific requirements to handle highly shaped equilibria in TCV. It has been translated in MATLAB and SIMULINK to facilitate future collaborative developments and applications to other devices. Its RT implementation runs on the TCV control system with a 200 μs cycle time and serves as a basis for iso-flux shape control [6] and for sawtooth and NTM control using ECH [1]. Coupling with RAPTOR offers a promising way to introduce physics based source profiles, including steep or non monotonic profiles often encountered in advanced scenarios.

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