

Model-based reconstruction and feedback control of the plasma particle density in tokamaks

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We present a new model-based approach for real-time reconstruction and feedback control of the plasma particle density. Accurate knowledge and control of the plasma density is necessary to achieve the required plasma pressure, staying below stability limits (e.g. Greenwald limit) and e.g. ECRH/ECE cutoff limits. A control-oriented, physics-based model is presented, which is then employed to design a dynamic profile reconstruction algorithm and a feedback controller. This approach to real-time profile reconstruction is similar to recent work on estimation of temperature and current density profiles [1] and extends this work for the particle density profile.

Physics-based model for control

We present a physics-based model of the tokamak density transport for observer and controller design that is flexible to adapt for multiple devices with multiple diagnostics and actuators and takes physical parameters of the plasma into account. The model is based on a 1D PDE for radial particle transport in the plasma [2] (describing flux-surface averaged electron density $n_e(\rho, t)$, where $\rho = \sqrt{\Phi/\Phi_{LCFS}}$ is the spatial variable and Φ is the toroidal magnetic flux) and two ODEs of the wall and vacuum particle inventories, $N_w(t)$ and $N_v(t)$. See Figure 1 for the modeled transport.

The radial plasma particle transport is governed by

$$\frac{\partial}{\partial t} (n_e V') = \frac{\partial}{\partial \rho} \left(V' G_1 \chi \frac{\partial n_e}{\partial \rho} + V' G_2 v n_e \right) + V' S \quad (1)$$

where $V' = \frac{\partial V}{\partial \rho}$ and G_1, G_2 are geometric terms which depend on the plasma equilibrium. We choose to model transport in an empirical fashion and therefore set the diffusion coefficient χ and drift velocity v as simple functions of ρ . The net plasma particle source S is modeled as

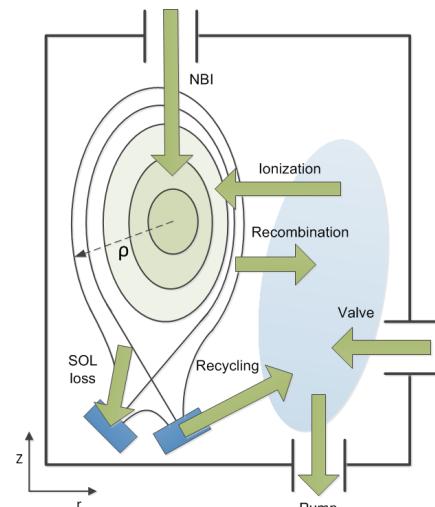


Figure 1: Graphical representation of the plasma, the wall components, the neutral vacuum and the modeled particle flows in the tokamak.

$S = S_{ion\&rec} + S_{inj} - S_{SOL\rightarrow wall}$, and the wall and vacuum inventory balances are modeled as

$$\frac{dN_w}{dt} = \Gamma_{SOL\rightarrow wall}(t) - \Gamma_{recycling}(t) \quad (2)$$

$$\frac{dN_v}{dt} = \Gamma_{valve}(t) + \Gamma_{recycling}(t) - \Gamma_{ion\&rec} - \Gamma_{pump}(t) \quad (3)$$

where $\Gamma_{SOL\rightarrow wall} = \int_{V_p} S_{SOL\rightarrow wall} dV$, $\Gamma_{ion\&rec} = \int_{V_p} S_{ion\&rec} dV$ and $V_p = \int V' d\rho$ is the plasma volume. We model the sources and flows as

$$\begin{aligned} S_{ion\&rec} &= \langle \sigma v \rangle_{ion}(T_e) n_n n_e - \langle \sigma v \rangle_{rec}(T_e) n_e^2 \\ S_{inj} &= \Lambda_{NBI}(\rho) \Gamma_{NBI}(t) + \Lambda_{pellet}(\rho) \Gamma_{pellet}(t) \quad S_{SOL\rightarrow wall} = \frac{n_e}{\tau_{SOL}} \delta_{\rho \in SOL} \\ \Gamma_{recycling} &= \frac{N_w(t) - c_w V_{v,0} n_n}{\tau_{release}} + \frac{N_w(t)}{N_{sat}} \Gamma_{SOL\rightarrow wall} \quad \Gamma_{pump} = \frac{V_{v,0} n_n}{\tau_{pump}} \end{aligned}$$

where n_n is the neutral vacuum density, $\tau_{release}$ is the outward wall diffusion time constant, c_w is a dimensionless balance constant and Γ_{NBI} and Γ_{pellet} are the NBI and pellet fuelling rates.

We have included the influence of the LCFS electron temperature $T_{e,b}(t) = T_e|_{\rho=1}$, plasma current, 2D equilibrium, and operational modes (limited or diverted plasma $c_D \in \{\text{lim, div}\}$, L- or H-mode $c_H \in \{\text{L, H}\}$) on various transport coefficients as follows. We choose $v = v_0(\rho) \frac{I_p}{I_{p,0}}$ to represent the increase of pinch at higher current. An H-mode density pedestal implies a reduction of transport in the plasma edge and is reproduced by lower edge diffusion $\chi(\rho \in \text{SOL})|_{c_H=\text{H}} \ll \chi(\rho)|_{c_H=\text{L}}$ and a lower drift velocity $v_0(\rho)|_{c_H=\text{H}} < v_0(\rho)|_{c_H=\text{L}}$. Furthermore, $\tau_{SOL}(c_D)$ is the time constant of particle loss in the scrape-off layer (chosen to increase for a diverted plasma), $N_{sat}(c_D, c_H)$ is the wall saturation level (chosen to increase on a limiter-to-divertor transition but decrease on a L-to-H transition), and $\tau_{pump}(c_D)$ is the pumping time constant (may depend on divertor strike point locations). We define the time-varying external input parameter $p(t) = [T_{e,b} \ I_p \ V' \ \psi \ c_D \ c_H]$ and we assume that these values are known in real time through diagnostics or real-time equilibrium reconstruction.

The neutral vacuum density outside the plasma is approximated as $n_n \approx \frac{N_v}{V_r - V_p}$ where V_r is the vessel volume. We choose ad-hoc approximations of the spatial dependency of the neutral vacuum density n_n and the electron temperature $T_e(\rho, t)$ inside the plasma. These approximations are parametrized using the vacuum density $\frac{N_v}{V_r - V_p}$ and the LCFS electron temperature $T_{e,b}(t)$, respectively, based on known spatial distributions of ionization and recombination near the edge. We choose the deposition functions $\Lambda_{NBI}(\rho)$ and $\Lambda_{pellet}(\rho)$ empirically based on known deposition locations from detailed physics analysis. We have nominal constants: the nominal vacuum volume $V_{v,0}$ and the nominal plasma current $I_{p,0}$, which are device-specific and define the operational point of the model.

Using a spatial discretization of n_e and a time discretization, the system (1)-(3) is written as

$$x_k = f(p_{k-1}, x_{k-1}) + Bu_{k-1} \quad (4)$$

where $x_k = x(t_k)$ consists of $N_w(t_k)$, $N_v(t_k)$ and a parameterization of $n_e(\rho, t_k)$, the inputs are $u_k = [\Gamma_{\text{valve}}(t_k) \ \Gamma_{\text{NBI}}(t_k) \ \Gamma_{\text{pellet}}(t_k)]^T$ and $p_k = p(t_k)$. The outputs y_k of the interferometry system are the line-integrated electron density along multiple chords and depend on the 2D equilibrium

$$y_k = \left[\int_{L_1} n_e(\rho, t_k) dL \cdots \int_{L_N} n_e(\rho, t_k) dL \right]^T \quad (5)$$

A forward model of the line-integrals is given by

$$y_k = C_k(p_k)x_k \quad (6)$$

Dynamic state observer for density profile reconstruction

We design a dynamic state observer, or Extended Kalman filter to estimate the density profile in real-time. It recursively fuses data of multiple diagnostic channels with model information. More precisely, the estimate \hat{x}_k of the state x_k at time t_k is a linear combination of (a) a one-step ahead prediction \hat{x}_k^p given the previous estimate \hat{x}_{k-1} (using (4)) and (b) the innovation $z_k = y_k - C_k(p_k)\hat{x}_k^p$, which is the difference between measurements (5) and predicted line-integrated density (using (6)). Furthermore, the observer is employed to estimate systematic modeling errors and disturbances as slow-moving deviations from the one-step predictions.

In Figure 2, offline reconstruction results of the observer on interferometry and equilibrium reconstruction data of TCV are shown. The conformity of measurements and line-integrated density predictions (see Figure 2(f)) indicates accurate estimation of the profile.

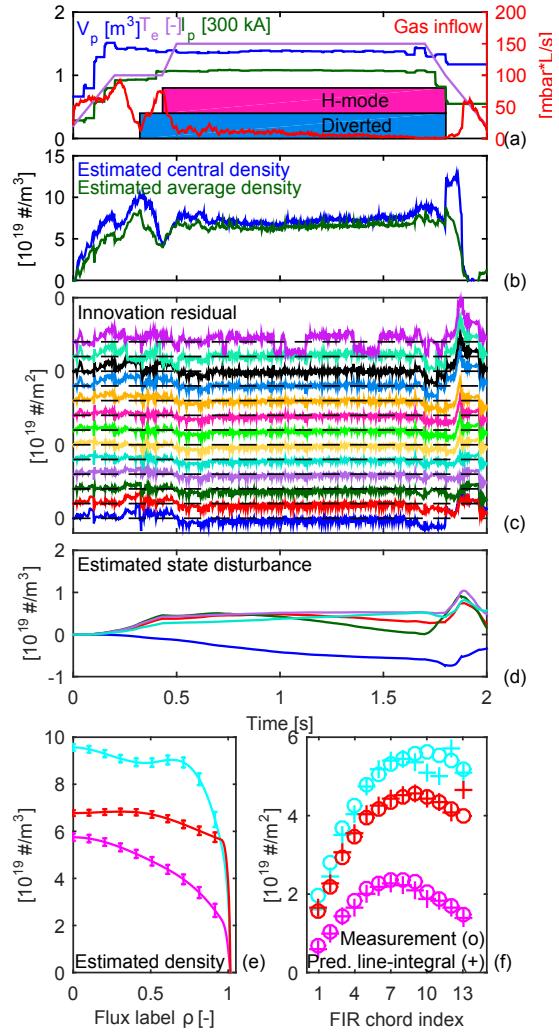


Figure 2: Offline observer results for TCV shot #41953. The gas valve and parameter data are shown in (a). We use artificial temperature data. The estimated central and average density are shown in (b). The elements of the innovation z_k are shown in (c), individually offset at intervals of $10^{19} \#/m^2$. The estimated disturbance is shown in (d). The estimated density profiles at $t = 0.12$ (magenta), $t = 0.3$ (light blue) and $t = 1$ (red) are shown in (e) with confidence bounds. The measurements and predicted line-integrated density at these time instants are shown in (f).

Feedback control design using robust control theory

We use the model (4) for designing feedback controllers for the density that are robust against model uncertainties and disturbances [4]. With the description of dynamics in different modes (4), we design controllers for each mode combination ($c_D \times c_H$) and switch between the controllers according mode transitions. Using the MATLAB Robust Control Toolbox [5], we design a linear switching feedback controller K_{c_D, c_H} to track a predefined reference signal $r(t)$ for the volume-averaged density $\bar{n}_e(t) = \frac{1}{V_p} \int_{V_p} n_e dV$ using the gas valve. Additionally, we apply an anti-windup strategy to prevent the controller from integrating when the gas valve saturates.

$$\Gamma_{\text{valve}} = K_{c_D, c_H} (r - \bar{n}_e) \quad (7)$$

In Figure 3, results of a simulation of the closed-loop interconnection of the plant model ((4) and (6)) with the observer and switching controller (7) are shown. In order to verify the robustness of the observer and controller, various coefficients ($\chi, v_0, \tau_{SOL}, N_{sat}$) have been doubled or halved, whereas the simulation model uses unperturbed coefficients. The controller is able to track representative reference signals, with the performance limited by the inability to rapidly decrease the density.

Outlook on future work

Diagnostic faults such as fringe jumps or misbehaviour of the plasma (preceding a disruption) may be detected by the observer as inconsistencies between measurements and model-based predictions. Also, the method can be used for control of the profile shape, provided that the actuators allow such control.

References

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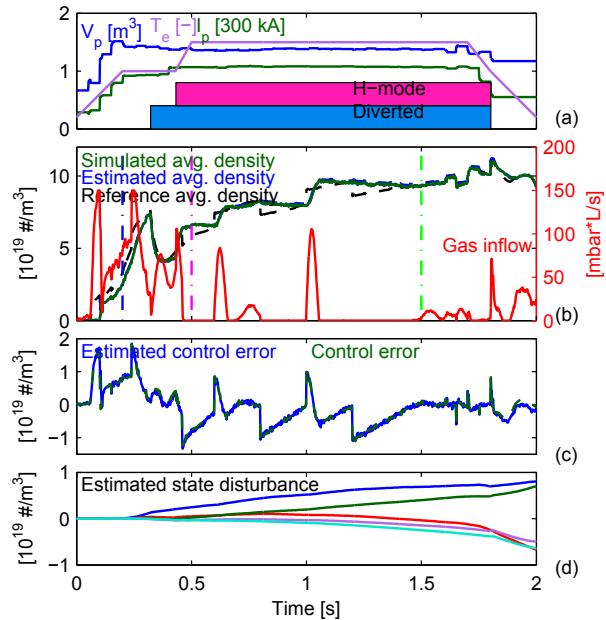


Figure 3: Closed-loop simulation results for a high density H-mode plasma. The parameter data (a) is taken from TCV shot #41953. The controller is able to let the average density (green in (b), obtained by the observer) evolve along the reference trajectory (black in (b)). Negative steps are not tracked: the valve closes and the density decays autonomously.