

## On the generation and saturation of the Zonal Flows

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### Introduction

In fusion plasmas heat transport is deleterious, therefore transport barrier (TB) generation is a key performance indicator. The spontaneous generation of a transport barrier is due to the interplay between turbulence and zonal flows (ZF). Turbulent eddies can transfer energy via non linear coupling to ZF that tend to store it for long time scale. On the other hand the efficiency of the energy transfer between turbulence and zonal flows can vary and different transport regimes have been observed. Two opposite regimes have been identified: (1) fully turbulent, where one can retrieve an homogeneous turbulence distribution and ZF are weak, (2) zonation, where the ZF dominate and the turbulence is damped. In many dynamical systems one can observe these various regimes, from oceans to planetary atmosphere [1].

We present here an analytical work on the interaction between turbulence and ZF reducing our model to a limited number of modes. The results are then compared with those obtained via a 2D fluid code. The role of non linear coupling in the ZF generation and saturation is defined and appears crucial in the transition from turbulent to zonation regime.

### Zonal flow source and sink: study of three modes coupling

Departing from the charge conservation equation, in an isothermal plasma, in the cold ions limit and in the case of absence of interchange forcing, the vorticity equation is derived as following

$$\partial_t W + [\phi, W] = \nu_\perp \Delta_\perp W + J \quad (1)$$

where  $W$  is the vorticity  $W = \Delta_\perp \Phi$ ,  $x$  and  $y$ , the space coordinates corresponds to the poloidal angle and radial direction,  $J$  is the parallel current loss. The convective turbulent transport  $[\Phi, W] = \partial_x(W(-\partial_y \Phi)) + \partial_y(W(\partial_x \Phi))$  competes with the small scale diffusive transport with coefficient  $\nu$ . The loss term is defined as  $\langle J \rangle_y = 0$ . Averaging along the flux surfaces, there is no current loss in parallel direction and the zonal flows  $V_z = \partial_x \langle \Phi \rangle_y$  evolution can be rewritten as

$$\partial_t V_z + \partial_x R S - \nu \Delta_x V_z = 0 \quad (2)$$

where  $RS(x) = \langle \partial_x \tilde{\Phi} \partial_y \tilde{\Phi} \rangle_y$  is the Reynold stress term and  $\tilde{\Phi} = \Phi - \langle \Phi \rangle_y$  is the potential fluctuations. The collisions term, in this case simplified through the viscosity term, acts as sink for ZF [3]. Conversely the RS term can act as source or sink of the zonal flows, according to the predator-prey scheme[2]. Namely, two possible scenarios have been identified: (a) Turbulence gives energy to ZF, i.e. the ZF are excited and turbulence is damped. ZF govern the large scale transport (in this case RS acting as source). (b) Once ZF source is drained, ZF start decaying and turbulence can grow until a turbulent burst can go through the barrier (in this case RS is a ZF sink). Once the turbulent transport becomes dominant, the cycle restarts. The RS term in the two different regimes has been studied in order to understand the mechanism that let the system cycle between both regimes (a) and (b). The interplay between zonal and turbulent modes can be addressed in the framework of non linear three mode coupling: the zonal flow, the streamer and the more homogeneous turbulent mode, respectively  $\Phi_z(\kappa, 0)$ ,  $\Phi_s(0, k_s)$ ,  $\Phi_t(\kappa, k_s)$ . Two cases of study are proposed in this letter, comparable to the regimes previously mentioned:

- (a) The condensation of the ZF, in the case the streamer  $\Phi_s$  represents the equilibrium profile and  $\Phi_z, \Phi_t$  are the small amplitude perturbations, if the growthrate is positive the turbulence is giving energy to ZF.
- (b) The Kelvin Helmholtz (KH) instability, where the ZF modes is the equilibrium profile and  $\Phi_z, \Phi_t$  are the perturbations, positive growthrate means that the ZF is giving energy to turbulence.

The Kelvin Helmholtz instability is the primary focus. We can study such instability in a more general way defining the perturbations as  $\phi_l(k_{xl}, k_y), \phi_{l+1}(k_{xl+1}, k_y)$ , where  $k_{xl+1} = l\kappa$  with  $l = [0, \text{inf}]$ , the dispersion relation can be rewritten as

$$[\gamma + \frac{\gamma_l + \gamma_{l+1}}{2}]^2 = -V_l V'_l k_y^2 \kappa^2 |\Phi_z|^2 - \gamma_l \gamma_{l+1} + \frac{\gamma_l + \gamma_{l+1}}{2}^2 \quad (3)$$

where  $\gamma_i = \nu k_i^2 + \sigma / k_i^2$ ,  $k_i^2 = k_{xi}^2 + k_y^2$  with  $i = l, l+1$  and coupling terms are  $V_l = (k_l^2 - \kappa^2) / k_{l+1}^2$ ,  $V'_l = (k_{l+1}^2 - \kappa^2) / k_l^2$ . The instability condition is then  $-V_l V'_l k_y^2 \kappa^2 |\Phi_z|^2 - \gamma_l \gamma_{l+1} > 0$ . If we define  $R_l = -\frac{V_l V'_l k_y^2 \kappa^2 |\Phi_z|^2}{\gamma_l \gamma_{l+1}}$ , then  $R_l > 1$  becomes a necessary condition in order to have the coupled modes unstable, which lead to define three new variables  $X_l, Y_l, Z_l$  such that  $X^2 = \frac{k_{xl}^2 + k_y^2}{\kappa^2}$ ,  $Y_l = \frac{k_{yl}}{\kappa}$  and  $Z_l^2 = X_l^2 - Y_l^2 = \frac{k_{yl}^2}{\kappa^2}$ . Let be  $\bar{k}$  such that  $\nu \bar{k}^2 = \frac{\sigma}{\kappa^2}$ . In this way we rewrite  $R_l = g^2 F(x, y, l) / G(X, y, l)$  such that

$$F = -V_l V'_l \left( \frac{|\Phi_z|^2 \kappa^4}{k_l k_{l+1}} \right)^2 = (X_l^2 - Y_l^2)(X_l^2 - 1)(X_l^2 + 2Y_l) \quad (4a)$$

$$G = \gamma_l \gamma_{l+1} \left( \frac{\sigma}{k_l k_{l+1}} \right)^2 = \left( \frac{k_l^4}{\bar{k}^4} + 1 \right) \left( \frac{k_{l+1}^4}{\bar{k}^4} + 1 \right); g^2 = |\Phi_z|^2 \frac{\kappa^8}{\sigma^2} \quad (4b)$$

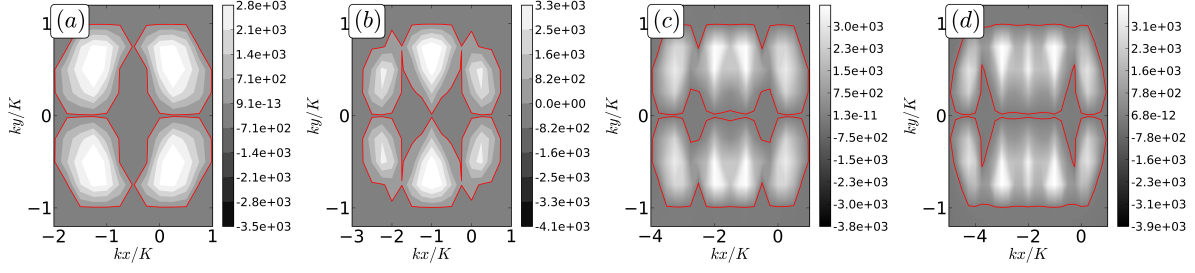


Figure 1: (a) the linear growthrate  $\gamma\tau_{\parallel}$  in function of  $k_y$  and  $k_x$  in case of 2 coupled modes, (b) 3 modes, (c) 4 modes and (d) 5 modes

By definition  $G$  and  $g^2$  are positive and  $X_l^2 \geq Y_l^2$ , therefore  $R_l$  is positive if  $(X_l^2 - 1)(X_l^2 + 2Y_l) < 0$ , which corresponds to two possible cases

$$Y_l^2 + Z_l^2 < 1 \cap (Y_l + 1)^2 + Z_l^2 > 1 \quad \text{or} \quad Y_l^2 + Z_l^2 > 1 \cap (Y_l + 1)^2 + Z_l^2 < 1 \quad (5)$$

In fig.1(a) the growthrate amplitude in function of the perturbation mode is plotted. Additionally, if we take in account more than one couple of modes, the following linear system is derived:

$$\begin{pmatrix} \Omega_1 & V_1 & 0 & \cdots & 0 \\ -V'_1 & \Omega_2 & V_2 & \cdots & 0 \\ 0 & \ddots & \ddots & \ddots & 0 \\ 0 & 0 & \cdots & V'_n & \Omega_n \end{pmatrix} \begin{pmatrix} \Phi_1 \\ \Phi_2 \\ \vdots \\ \Phi_n \end{pmatrix} = 0$$

Solving the linear system, we observe that the unstable spectrum region is expanded in  $k_x$  if the number of perturbations coupled with  $\phi_z$  increases, while the condition  $\gamma > 0 \Rightarrow |k_y|/|\kappa| < 1$  is always true (see fig.1(b)-(c)-(d), where  $\gamma\tau_{\parallel}$  is plotted in function of  $k_x, k_y$  respectively in case of three, four and five modes contribution). Conversely, if the same procedure is repeated but considering the streamers as equilibrium condition and  $\Phi_z, \Phi_t$  as perturbations (case (a)), the opposite condition is recovered: the region of spectrum of positive growthrate is constrained by  $|k_y|/|\kappa| > 1$ . To verify the analytical observation, the two regimes are now modeled. For (b) case, we assume a equilibrium profile  $\Phi_z(\kappa = 0.024, 0)$  and two streamers are excited with different poloidal size,  $\Phi_{s1}(0, \kappa/2), \Phi_{s2}(0, 7/4\kappa)$ . From fig.2(a)-(b) we can observe that the perturbation  $\Phi_{s1}$  is growing accordingly with the prevision of the linear analysis, i.e. the growthrate is approximately  $\gamma\tau_{\parallel} \approx 2600$ . On the other hand the streamers with poloidal size smaller than the ZF radial width is damped. If we consider the streamers like mode  $\Phi_s(0, k_s = 0.024)$  as equilibrium condition and two zonal flows as perturbations  $\Phi_{z1}(k_s/2, 0), \Phi_{z2}(7/4k_s, 0)$ , we can observe that the zonal flow is excited only if the radial width of the zonal flow is larger than the poloidal width of the streamer, fig.2(c)-(d).

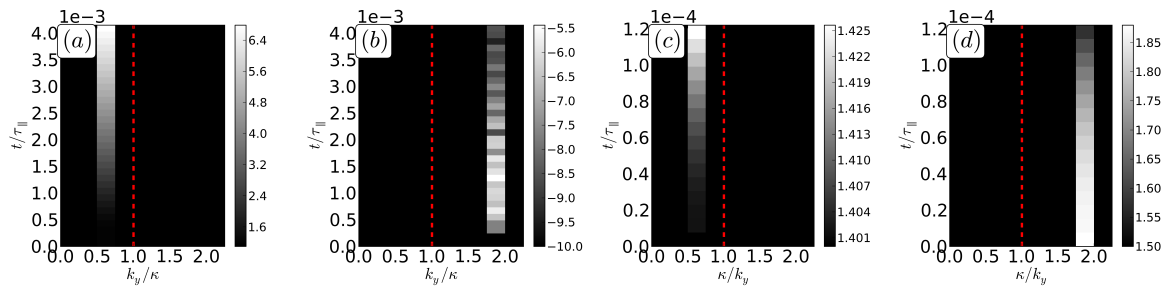


Figure 2: (a) evolution in time of the poloidal spectrum  $k_y$  if a streamer  $\Phi_{s1}$  or (b)  $\Phi_{s2}$  is perturbed, (c) evolution in time of the radial spectrum  $k_x$  if a zonal flow  $\Phi_{z1}$  or (d)  $\Phi_{z2}$  is perturbed.

## Conclusions

We can redefine the turbulent spectrum in two different regions called as (1) 'B-modes region' and (2) 'S-modes region'. (1) 'B' stays for big scales structure. This region of the turbulent spectrum, where the poloidal size of the turbulent structures is bigger than the radial size of ZF, cannot give but only receive energy from ZF through KH instability. Fundamentally this region is not source for ZF. (2) 'S' stays for small scale structures and define the region of the spectrum where the turbulence size is smaller than the radial size of ZF, namely S-modes transfer energy to the ZF and cannot act as a sink for ZF. Inserting the interchange forcing and the density equation we can simulate the selfconsistent zonal flows and turbulence interplay. According to the amplitude of the two regions  $S - B$ , we can recover different transport regimes[4]. If the S-modes dominates we are in the zonation regime, while if the B-modes dominates the system is fully turbulent.

## References

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