Kinetic Theory of Phase Space Plateaux

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Frequency sweeping signals are attributed to the formation and evolution of phase space holes and clumps in the non-thermal fast particle distribution [1, 2]. Recently their origin was shown to follow from the presence of a nearly unmodulated phase space plateau [3], centered at the wave-particle resonance, which supports a pair of shifted *edge modes* that destabilize due to dissipation in the background plasma and nonlinearly evolve into holes and clumps. The role of the plateau as a hole/clump breeding ground is further substantiated in this article via inclusion of fast particle collisions and sources. Also, it is demonstrated that relaxation of the plateau edge gradients has only a minor quantitative effect and does not change the plateau stability qualitatively.

We consider an electrostatic travelling wave with spatial period λ and wave number $k = 2\pi/\lambda$ in a one-dimensional, uniform plasma equilibrium. The wave carrier frequency is assumed to be high enough that the plasma can be separated into a cold bulk, comprised of electrons and immobile ions, and a low density beam of energetic electrons that may interact resonantly with the wave and therefore need to be treated separately. The cold electrons respond linearly to the electric field $E(x,t)$, so their perturbed velocity v_e satisfies the linear fluid equation

$$\frac{\partial v_e}{\partial t} = \frac{e}{m_e} E - 2\gamma_d v_e \ , \tag{1}$$

where the last term is a linear, dissipative friction force that damps the velocity perturbations exponentially in time. The fast electrons are described kinetically in terms of their distribution function $F(x,v,t)$, which evolves according to the kinetic equation

$$\frac{\partial F}{\partial t} + v\frac{\partial F}{\partial x} - \frac{e}{m_e} E \frac{\partial F}{\partial v} = \mathfrak{C}[F] + S(v) \ . \tag{2}$$

The right hand side represents fast particle collisions and sources, whose action is to relax F towards an equilibrium distribution $F_0(v)$ that is taken as a constant, positive slope throughout the wave-particle resonance. It is modeled by the following combination of operators, commonly known as Krook-type collisions, collisional drag and velocity space diffusion,

$$\mathfrak{C}[F] + S(v) = -\beta\left(F - F_0\right) + \frac{\alpha^2}{k}\frac{\partial}{\partial v}\left(F - F_0\right) + \frac{\nu^3}{k^2}\frac{\partial^2}{\partial v^2}\left(F - F_0\right) \ . \tag{3}$$

The system is closed by Ampére's law

$$\frac{\partial E}{\partial t} = \frac{e}{\varepsilon_0}\left[n_e v_e + \int v\left(F - F_0\right) dv\right] \ , \tag{4}$$

where n_e is the unperturbed density of cold electrons. The plasma wave, with frequency ω_{pe}, is driven at linear rate γ_L by the fast electrons and weakly damped at rate $\gamma_d \leq \gamma_L \ll \omega_{pe}$ due to the friction force. All simulations are performed by means of a numerical algorithm, previously described in [4], that solves Eqs. (1) – (4) for the nonlinear evolution of $E(x,t)$ and $F(x,v,t)$.

Holes and clumps are nonlinear structures that extend in both real and velocity space and carry a particle deficit/surplus as compared to the surrounding distribution. Disregarding the effects of fast particle collisions and sources, they arise symmetrically shifted off the wave-particle resonance of a kinetically unstable bulk plasma eigenmode. Once firmly established, they tend to balance dissipative wave damping in the background plasma with the energy tapped by traversing fast particle phase space as coherent entities. Fig. 1 displays snapshots of the spatially averaged fast particle distribution that illustrate the presence of an intermediate plateau before the hole/clump pairs are created. At that stage, the initial mode is damped out and kinetically stable, but there are shifted resonances situated just inside the edges of the flat plateau region, responsible for small modulations of the plateau edge that begin to grow and eventually evolve into a hole/clump pair that detaches from the plateau.

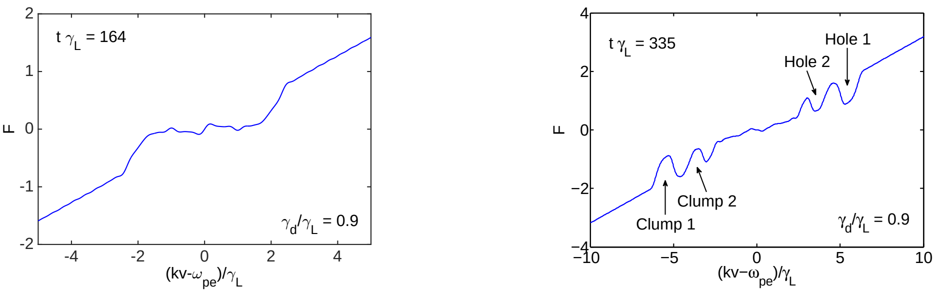

Figure 1: *Intermediate plateau and hole/clump pairs in the fast particle distribution.*

Inclusion of fast particle collisions and sources as given by Eq. (3) has significant impact on the formation (and evolution) of holes and clumps. Collisional relaxation, as mediated by velocity space diffusion or Krook-type collisions, inhibits hole/clump formation from the plasma wave resonance [5, 6]. Slowing down of the fast particles (drag), on the other hand, promotes it, in particular the formation and growth of holes [4].

The role of the intermediate plateau is now further substantiated via linear stability analysis of an unmodulated shelf with continuous edges (see Fig. 2a). Under the assumption that $k\Delta v \ll \omega_{pe}$ the resulting dispersion relation is

$$wz + i\pi\gamma + \frac{1-r}{r}\Big[\log(1+z) - \log(1-z)\Big] - \frac{1}{r}\Big[\log(1-r+z) - \log(1-r-z)\Big] = 0\,, \qquad (5)$$

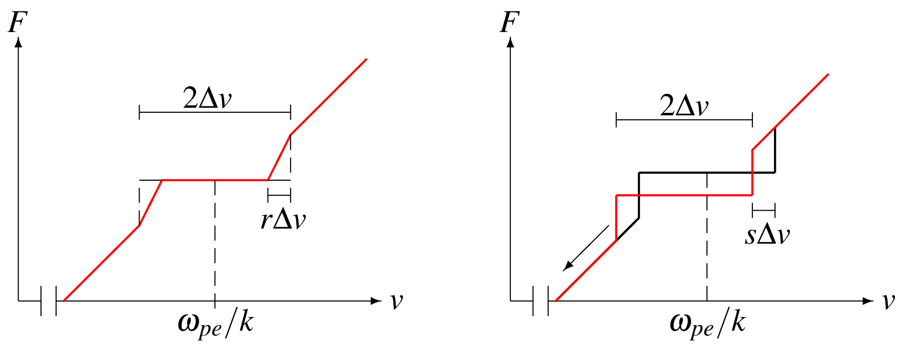

Figure 2: *Fast particle distribution for linear stability analysis of plateau state.*

where r is defined in Fig. 2a, we have introduced the dimensionless variables

$$z \equiv \frac{\omega - \omega_{pe}}{k\Delta v} \;, \quad w \equiv \pi \frac{k\Delta v}{\gamma_L} \quad \text{and} \quad \gamma \equiv \frac{\gamma_d}{\gamma_L} \tag{6}$$

and it has been assumed that $|\mathrm{Re}\,[z]| < 1 - r$. A numerical analysis reveals that Eq. (5) has precisely three complex roots, of which one, dubbed z_c, sits at $\mathrm{Re}\,[z] = 0$ with a negative imaginary part that vanishes at $\gamma = 0$ and whose magnitude increases with γ. It is supplemented by a symmetric pair that bifurcates as w varies, the bifurcation point and precise pattern depend on r as well as γ (see Figs. 3a-c). These modes tend asymptotically to the edges of the flat region, $\pm(1-r)$, as $w \to \infty$ and were hence dubbed *edge modes* (denoted $z_\pm$) in [3]. For a fixed w the modes shift with the edges towards the plateau center as r increases, as shown in Fig. 3d. Their imaginary parts remain almost completely constant, however, meaning that relaxation of the shelf edge within a narrow transition layer has a negligible effect on the plateau stability.

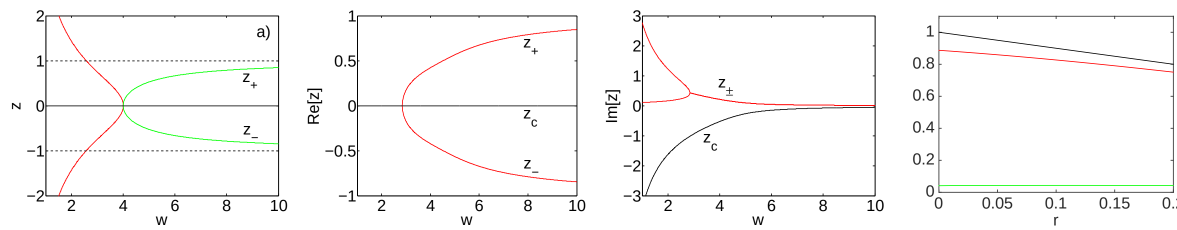

Figure 3: *a) Real (green) and imaginary (red) parts of roots to the dispersion relation* (5) *when* $r = 0$ *and* $\gamma = 0$. *b) Real and c) imaginary parts of roots to the dispersion relation when* $r = 0$ *and* $\gamma = 0.1$. *d) Edge modes (*$\pm\mathrm{Re}\,[z_\pm]$ *in red and* $10 \times \mathrm{Im}\,[z_\pm]$ *in green) when* $\gamma = 0.1$ *and* $w = 4\pi$. *The black line represents the edge of the flat region,* $|1 - r|$.

We proceed to further certify the plateau hypothesis by inclusion of fast particle collisions and sources. The Krook-type and velocity space diffusion operators both inhibit hole/clump production, as expected. For Krook-type collisions this is illustrated in Fig. 4 where hole/clump production ceases above roughly $\beta/\gamma_L = 0.01$ when $\gamma = 0.1$. For diffusive collisions the corresponding rate is $\nu/\gamma_L = 0.1$. Collisional drag, on the other hand, promotes hole/clump formation

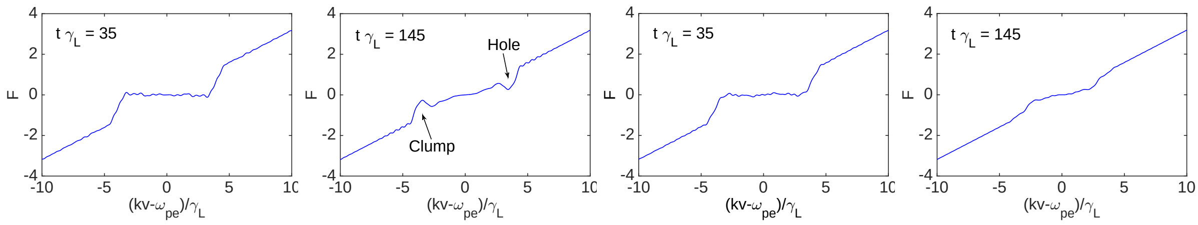

Figure 4: *Fast particle distribution in the presence of Krook-type collisions with rate* $\beta/\gamma_L = 0.005$ *(left two) and* $\beta/\gamma_L = 0.02$ *(right two) and* $\gamma = 0.1$.

and detachment. The effect on the plateau is a convection of the entire shelf down along the ambient linear distribution. Further, linear stability analysis of the shifted plateau in Fig. 2b sheds light on the previous result that collisional drag enhances holes and their sweeping rates but suppresses clumps. For large plateau widths, a power series in $1/w$ reveils the asymmetry

$$\mathrm{Im}\,[z_\pm] = \frac{\pi\gamma}{w^2}\left[1+\frac{1}{w}(2\ln 2w + 1 \pm sw)\right] . \tag{7}$$

These results are further confirmed by nonlinear simulations of an initial plateau as a continuous shelf of tanh-type, cf. Fig. 5, where the plateau experiences a downward shift and the ensuing hole/clump pair evolves asymmetrically with a hole that grows faster than the clump.

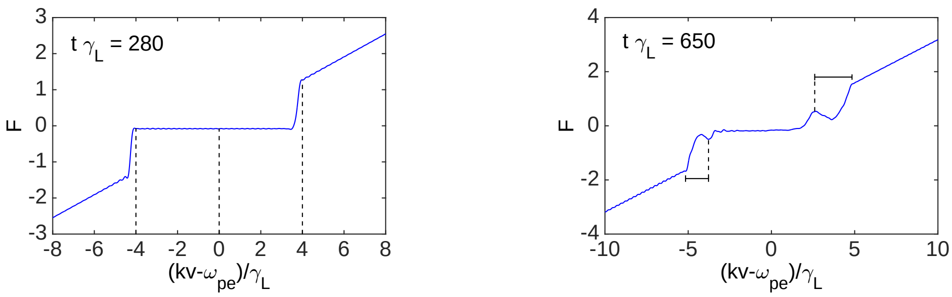

Figure 5: *Evolution of initial plateau when* $\alpha/\gamma_L = 0.03$ *and* $\gamma = 0.1$.

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