Nonlinear simulations of the fishbone instability

M. Idouakass[1], M. Faganello[1], H. L. Berk[2], X. Garbet[3], S. Benkadda[1]

[1] *Aix Marseille Université, CNRS, PIIM UMR 7345, 13397 Marseille, France*

[2] *Institute for Fusion Studies, University of Texas, USA*

[3] *CEA, IRFM, 13115, St Paul lès Durance, France*

Abstract

We propose an extension of the precessional fishbone model developped by Odblöm et al. [1], which allows taking into account the MHD nonlinearities around the $q = 1$ surface, and a linear response for the energetic particles, which are well contained within the $q = 1$ surface. The fishbone mode, resulting from the interplay between the trapped energetic particles and the thermal plasma, is an electromagnetic mode characterized by bursts of activity and frequency down-chirping. It can cause enhanced transport and losses of energetic particles, such as ICRH- or NBI-produced particles in present tokamaks, or alpha particles in future fusion devices, and affect their energy deposition into the plasma.

Introduction

The fishbone oscillation was first observed in the PDX tokamak with nearly perpendicular neutral beam injection active [2]. It was observed to have a dominant $m = 1$ poloidal mode number and $n = 1$ toroidal mode number, with a radial plasma displacement profile similar to the internal kink "top-hat" structure. Its frequency was close to the NBI-produced energetic particles precessional frequency, and was observed to decrease by a factor of about 2 during each burst.

This instability was later interpreted analytically in two different ways, corresponding to two different branches. One branch is a fishbone with a frequency close to the precessional drift frequency of the trapped energetic particles [3]. The other branch is a fishbone with a frequency close to the ion diamagnetic frequency [4]. In the case of the diamagnetic branch, there is a gap forming in the Alfvén continuum, within which the mode frequency is contained. In that case, the MHD resonances are essentially eliminated. In the case of the precessional branch, there is no gap forming in the Alfvén continuum, so that there are MHD resonances, leading to nonlinear MHD behavior, in addition to a kinetic nonlinear behavior.

Therefore, a nonlinear description of the precessional fishbone instability is a challenging problem, because of the interplay between the MHD nonlinearities around the $q = 1$ surface, and more precisely at the two Alfvén resonances [1], and the kinetic nonlinearities [5].

Model

In order to study the competition between the nonlinear MHD and kinetic effects, while still being able to understand the origin of the different nonlinear dynamics, the model used is based on a domain separation.

The first domain is the core region centered on the magnetic axis, and extending up to a radius lower but close to that of the $q = 1$ surface. It is consedered that all energetic particles are contained within this region, so that the nonlinear kinetic evolution needs to be studied only in this region. At the same time only a linearized MHD response is kept for the plasma bulk, which behaves in an internal kink-like manner. The second domain is an annular region centered around the $q = 1$ surface, in which there are no energetic particles. It is considered that all MHD nonlinear effects take place in this region.

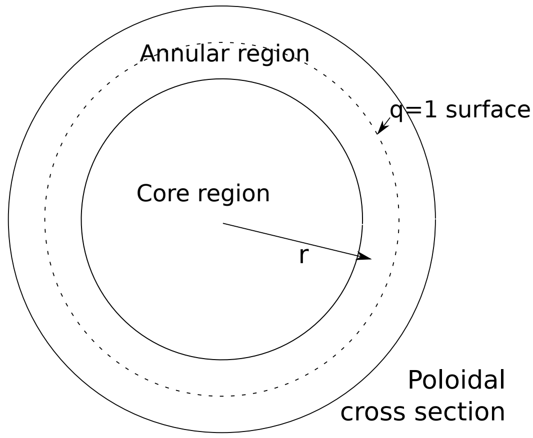

Figure 1: *Scheme of the domain separation*

Annular region

Assuming that this region is a narrow layer centered around r_*, the radial location of the $q = 1$ surface, and considering a single helicity response, it is possible to simplify the geometry to a two dimensional slab. In the annular region, the nonlinear MHD dynamics are described by the two fluid Reduced MHD model, meaning that compressibility and anisotropic effects of the thermal pressure are neglected. The evolution is then given by the frozen-in equation and the vorticity equation :

$$\frac{\partial \psi}{\partial t} = [\psi, \phi] + \eta \Delta_\perp \psi \quad (1)$$

$$\frac{\partial}{\partial t} \Delta_\perp \phi = [\Delta_\perp \phi, \phi] + [\psi, \Delta_\perp \psi] + \nu \Delta_\perp \Delta_\perp \phi \quad (2)$$

where ψ is the magnetic flux, proportional to the parallel vector potential, ϕ is the stream function, proportional to the electric potential, η and ν represent small but finite resistivity and viscosity. The Poisson brackets are defined as $[f, g] = \partial_x f \partial_y g - \partial_y f \partial_x g$, where $x = r - r_*$ and y represents the poloidal variable.

Core region

In this region, where a kinetic description is used for the energetic particles, the fact that there is a time scale separation between the gyromotion, the bounce motion and the precessional drift of particles, is used to simplify the study. Indeed, the particles gyro-frequency is much higher than the bounce frequency, itself much higher than the precession frequency. Considering that experimentally, the fishbone mode has a frequency close to that of the precessional motion, a gyro and bounce averaging is done. Furthermore, only deeply trapped particles, with a velocity parallel to the magnetic field that is negligeable, are taken into account. This fact allows considering a simplified two dimensional phase-space, that is described in action-angle variables represented by the toroidal momentum P_α and the toroidal angle α respectively. A last simplification is to consider that the energetic particles are characterized by a single value of the magnetic moment : $\mu = \mu_*$. In this case, the Hamiltonian, separated in an equilibrium part and a perturbation, reads :

$$H(\mu, v_\|, P_\alpha, \alpha) = H_0(\mu_*, v_\| = 0, P_\alpha) + H_1(\alpha, P_\alpha) \tag{3}$$

In the Vlasov equation, the equilibrium contribution gives the precession frequency:

$$\frac{\partial H_{eq}}{\partial P_\alpha} = \omega_D \tag{4}$$

The perturbed Hamiltonian is given by the electric potential of the $m = n = 1$ mode, which has a kink-like structure : a rigid displacement of the whole core, characterized by ϕ/r nearly constant. The perturbed Hamiltonian reads

$$H_1(P_\alpha, \alpha) = er\frac{\phi(r_{bound}, \alpha)|_{n=1}}{r_{bound}} \tag{5}$$

where the radius $r = r(P_\alpha)$ is a function of the toroidal momentum and r_{bound} defines the boundary between the core and the annular regions.

Coupling

The coupling between the particle dynamics and the bulk plasma evolution is done including the particle pressure in the linearized MHD response of the core. Integrating in space this response, from the magnetic axis to the boundary between the two domains, we obtain a time dependent boundary condition:

$$\left(\frac{\partial^2}{\partial t^2} + \omega_A^2\right) \frac{\partial \phi}{\partial r}\bigg|_{bound} = \frac{B_T}{r_{bound}^2 \rho_0} \frac{\partial}{\partial t} \int_0^{r_{bound}} drr^2 (b \times \kappa) \cdot \nabla \left(\frac{P_{\perp,h}}{B_T}\right) \tag{6}$$

where $P_{\perp,h} = \int \mu B_T f d^3v$ is the energetic particle pressure.

Linear Results

Linearizing the Vlasov equation and the Reduced MHD equations, while taking into account the boundary condition coupling the two different dynamics, it is possible to obtain a dispersion relation. This is done in a way similar to that of Ödblom et al. [1], but taking different assumptions for the energetic particles distribution function. The differences come from the need to be able to describe, in the future, the nonlinear kinetic dynamics.

Taking a distribution function of the form $F_{eq} = AF(r)\delta(v_{\parallel})\delta(\frac{\mu}{\mu_*} - 1)$, the dispersion relation reads :

$$i = K \int_0^1 \frac{y^2 q(y)^2 \left(-\frac{dF}{dy}\right)}{q - \omega T y} dy \quad ; \quad y = \frac{r}{r_*} \tag{7}$$

where K is a normalized energetic content, and T is a characteristic precession period

$$K = \frac{2\pi^2 \mu_* B_T \omega_c \Gamma}{2(1-q_0)\rho_0 k_\theta(r_*) v_{A,\theta}(r_*)} \quad ; \quad T = \frac{1}{\omega_D(r_*)} \tag{8}$$

Using this model, it is possible to get a threshold condition for the fishbone instability assuming that the distribution is:

- monotonically decreasing with radius, consistently with what would be expected from energetic particles in a fusion reactor
- going to a constant at the magnetic axis, and towards the $q = 1$ surface
- varying rapidly over a radial length small compared to r_*

Furthermore, the q-profile is considered to be nearly equal to 1 over all the core region.

With all these assumptions taken into account, the dispersion relation is simplified to :

$$i \simeq \frac{K y_0^2}{\omega T} \int_{-\infty}^{\infty} \frac{-\frac{dF}{dy}}{\frac{1}{\omega T} - y} dy \tag{9}$$

where y_0 is the normalized radius around which $\frac{dF}{dy}$ is peaked. This simplified dispersion relation gives, for the threshold condition :

$$K_{threshold} \simeq -\frac{1}{\pi y_0^3 F'|_{y_0}} \quad ; \quad \omega_{threshold} \simeq \frac{1}{y_0 T} = \omega_D(y_0) \tag{10}$$

These values are confirmed by numerical simulations based on the model presented in this paper.

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