High resolution equilibrium and stability calculations of pedestal and SOL plasma in tokamaks

S.Yu.Medvedev, A.A.Martynov, A.A.Ivanov, Yu.Yu.Poshekhonov

Keldysh Institute, Russian Academy of Sciences, Moscow, Russia

Tokamak edge plasma modelling relies upon computational grids aligned to magnetic surfaces because of highly anisotropic transport properties of the plasma with respect to magnetic field [1, 2]. The mapping of magnetic surfaces in the plasma and in the scrape-off layer (SOL) is usually performed based upon free boundary equilibrium data providing rather coarse resolution compared to very narrow pedestal and SOL widths predicted in ITER [3, 4]. The accuracy of the mapping procedure is certainly insufficient for MHD stability calculations and up to now only dedicated runs of nonlinear MHD code were capable to self-consistently treat the pedestal and SOL equilibrium and stability [5]. That is why fast and accurate equilibrium and stability calculations would significantly contribute to integrated modelling of edge plasmas.

1 High resolution equilibrium with SOL A starting point for high resolution fixed boundary equilibrium with plasma outside the separatrix is a free boundary equilibrium with poloidal flux function ψ available outside the separatrix. The values of ψ provide Dirichlet conditions at the boundary of SOL, private flux region and divertor plates for the solution of Grad-Shafranov equation while the position of the X-point and the separatrix are free to adjust. In the modified version of the adaptive grid CAXE code [6] plasma profiles can be independently prescribed in each sub-domain with nested flux surfaces formed by the separatrix branches, including a possibility of discontinuous toroidal current density across the separatrix. The same boundary conditions can be used for computation of a series of high resolution equilibria with varying profiles in the plasma and SOL. For the ITER Scenario 2 15MA plasma adding the high pressure gradient in the narrow SOL region corresponding to the separatrix pressure value $p_{sep} = 4.8$ kPa and the SOL drop-off length of ~ 1.5 mm [5] does not change the equilibrium much (Fig.1a). Note that despite high edge pressure gradient from the SOL side the separatrix angle is close to 90 degree.

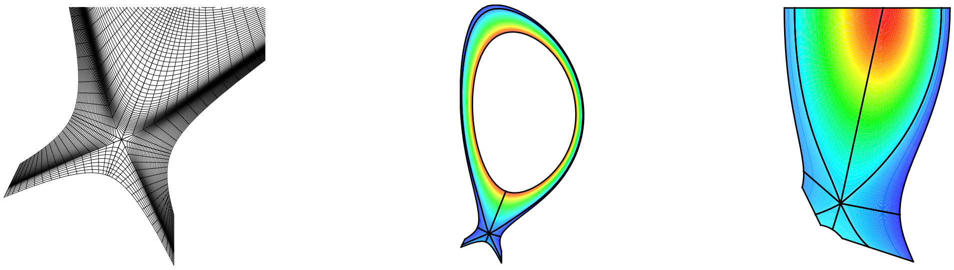

Figure 1. (a) Fragment of the grid adapted to magnetic surfaces near the X-point for ITER 15MA equilibrium; (b) six subdomains for SN CAXE6-SOL version; (c) eight subdomains for DN CAXE8-SOL version.

For a single null (SN) divertor plasma six sub-domains are used in the CAXE code (Fig. 1b). Another version of the CAXE code with eight sub-domains solves for up-down symmetric double null (DN) equilibrium (Fig. 1c).

2 Ideal MHD stability with SOL plasma The sheath compatible MHD boundary conditions on the open field lines [7] are applied to the variational formulation of the ideal MHD stability problem by setting $\xi_{||} \cdot n = 0, \quad \xi_{\perp} \cdot n = 0$, where ξ is the vector of plasma displacement, n is the normal to divertor plates. This formulation is almost identical to the standard variational principle with the exception of only one nonsymmetric term $\delta W_d = -1/2 \int_{S_d} A^* \cdot \delta j_s dS$, where $\delta j_s = n \times < \delta B >$ is a perturbed surface current at the divertor plates, A is the vector potential of the perturbed magnetic field $\delta B_v = \nabla \times A$ in vacuum and the integral is taken over divertor plate surface. Assuming the absence of the surface current on divertor plates (e.g. due to toroidal gaps) we ended up with a traditional variational formulation with the only difference due to a special treatment of the boundary conditions at the divertor plates, where $B \cdot n \neq 0$. In the updated version of the KINX code [8] only the $\xi \cdot \nabla \psi$ component of displacement is free to vary at the divertor plates. It does not mean that perturbed magnetic field normal to divertor plates vanishes and it gives rise to the corresponding perturbation of the magnetic field in vacuum. However the self-consistent vacuum perturbation is not implemented in the KINX yet and $n \cdot \delta B_v = 0$ is assumed at the divertor plates instead, thus making the perturbed vacuum energy smaller and leading to just sufficient stability condition. The sensitivity to the way how the sheath compatible condition is implemented can be checked by setting even more stringent line tying condition $\xi = 0$ at the divertor plates making also $\xi \cdot \nabla \psi = 0$ and $n \cdot \delta B_v = 0$ with the vacuum energy treated consistently. It turns out that the difference between ideal MHD growth rates corresponding to the sheath compatible boundary condition or the line tying boundary condition (the latter leading also to $\delta W_d = 0$) for medium-n kink-ballooning modes localized at the outboard side of the plasma is rather weak.

In the unstructured grid stability code [9] the divertor plate boundary condition treatment is much easier. The ideal MHD stability condition $E \cdot B = 0$ is just replaced by the $E \cdot B \times n = 0$ there. The vacuum treatment is automatically consistent because of the tangential electric field continuity $< E \times n >= 0$ condition at the plasma-vacuum boundary which is natural for the vector Whitney basis functions used in the approximation.

The new versions of the ideal MHD stability codes were applied to preliminary studies of axisymmetric $n = 0$ mode driven by finite current density at the separatrix and to pedestal height limit gradient calculations taking into account high pressure gradients in narrow SOL for the ITER Scenario 2 15MA case.

The calculations of the $n = 0$ mode passive stabilization in tokamak plasma with finite edge current density reveal an existence of the mode localized near the X-points for DN divertor configurations once the global $m = 1$ mode has been stabilized by conducting wall (e.g. for negative triangularity plasmas [10]). Besides the value of current density at the edge, the proximity of the plasma boundary cutoff $\psi_b/\psi_{sep} \to 1$ to the separatrix is destabilizing. On the contrary, the deviation from plasma up-down symmetry in DN, for which perturbed surface current is induced only in case of finite edge current density, to SN configuration, is strongly stabilizing for the localized mode. All these features are consistent with the recent analytic results [11]. The calculation with the MHD_NX code taking into account plasma in SOL demonstrated that the "$n = 0$ peeling" mode is susceptible to the stabilization by even very narrow SOL plasma. The plasma displacement streamlines near the X-point, featuring reverse flow in SOL, are shown in Fig.2.

The issue of the pedestal stability of ITER plasmas taken into account high pressure gradient in SOL was recently revisited in [5] as the scaling [4] predicted very narrow SOL width ~ 1 mm with the corresponding pressure gradient exceeding several times the pedestal values. The dedicated calculations from [5] demonstrated quite expected fact that high but strongly localized pressure gradient cannot drive $n = 40$ mode unless the pedestal region is included in the analysis. This is because the unrestricted mode extends over the whole pedestal region despite high toroidal wave number. The updated KINX3-SOL code allows to estimate quantitative changes in the pedestal height limit due to the presence of the narrow SOL.

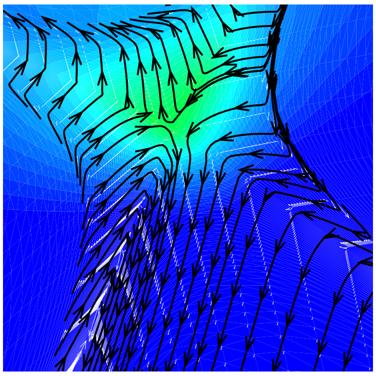

Figure 2. Streamlines and vector plot of plasma displacement for "$n = 0$ peeling" mode in DN configuration. Color map corresponds to E_φ.

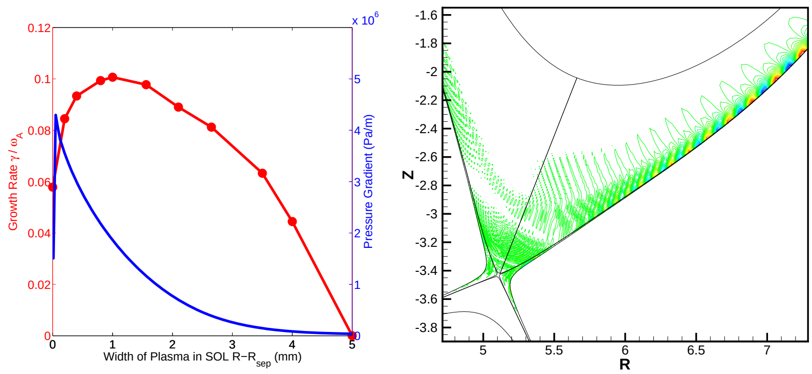

Figure 3. (a) $n = 40$ mode growth rate and pressure gradient at the outboard vs the width of conducting plasma in SOL; (b) level lines of plasma displacement normal to magnetic surfaces, $\gamma/\omega_A = 0.098$, $R - R_{sep} = 1.56mm$.

The same ITER Scenario 2 15MA plasmas with the pedestal height close to the marginal stability of $n = 40$ kink-ballooning mode was investigated. Fig. 3a shows the growth rate of the $n = 40$ mode and pressure gradient distribution (drop-off length 1.5 mm, separatrix pressure

4.8 kPa) versus the width of SOL at the outboard treated as ideally conducting plasma. Only moderate growth rate increase and eventual stabilization with increasing width of conducting plasma in SOL is demonstrated. For comparison the pedestal height limit was computed also with ideally conducting $p' = 0$ plasma in 1.5mm SOL by re-scaling the pedestal on the plasma side, which increases the limit by 2%. Almost the same increase in the pedestal height limit against the $n = 40$ mode was obtained due to re-scaling of p' in SOL of the same width thus providing an evidence that the pressure at the top of the pedestal and not local pressure gradient value is the critical parameter for the pedestal stability.

3 Discussion and future work The developed high resolution equilibrium and stability codes with plasma outside the separatrix and open magnetic field lines at the divertor plates can be applied to a variety of edge and SOL plasma modelling applications. Preliminary results include the stabilization of the $n = 0$ mode driven by finite edge current density taking into account plasma outside the separatrix and estimates of the pedestal height limit set by kink-ballooning modes with high pressure gradient in narrow SOL.

Very localized "$n = 0$ peeling" mode which is unstable in the up-down symmetric DN divertor plasma seems to be susceptible to stabilization by a narrow layer of conducting plasma outside the separatrix.

The pressure at the top of the pedestal in ITER limited by kink-ballooning modes is determined by pressure value at the top of the pedestal also in case of high pressure gradient in SOL in accordance with existing scalings for the pedestal height (see e.g. [12]).

Another possible application of the developed suite of codes is coupling to the SOLPS [1] with a purpose to provide a new flexible interface to the equilibrium increasing the accuracy in magnetic surface aligned grid and to support self-consistent edge transport/equilibrium modelling. Gyrokinetic simulations of tokamak edge micro-instabilities could be also contingent on the uncertainties of the magnetic equilibrium [13].

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