Plasma Response in Tokamaks by Helical Current Sheet Application

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ABSTRACT

Resonant magnetic perturbations (RMPs) have been applied in tokamaks to mitigate or even suppress plasma edge localized modes (ELMs) [1]. For some RMPs, it is necessary to considere the plasma response to accurately calculate the perturbed magnetic field inside the plasma. Several models have been used to estimate the plasma response according to references [2-4]. In this work we consider an ergodic magnetic limiter to create a RMP [5], superimposed to large aspect ratio tokamak equilibrium, and a resonant current sheet inside the tokamak to simulate the plasma response. For a given perturbation the plasma response is estimated, as proposed in [3], by choosing the current sheet on the internal surface to produce a null radial magnetic field on that surface. Equilibrium and non-uniform escape of the chaotic field lines are analyzed under the influence of plasma response.

1 – INTRODUCTION

Several works have been used RMPs in tokamaks due to beneficial effects to mitigate or even suppress plasma edge localized modes in high confinement (H-mode) plasmas [1-3]. The literature shows the plasma reacts to RMPs modifying the magnetic field line transport [1].

2 – RESONANT MAGNETIC PERTURBATIONS

Resonant magnetic perturbations (RMPs) are widely employed in order to modify edge instabilities [5, 6]. In order to provide an analytic treatment to the inclusion of RMPs, the cylindrical coordinate system is used in this work, with contravariant components (r,θ,z) and covariant base $(\vec{e}_r,\vec{e}_\theta,\vec{e}_z)$ expressed as a function of Cartesian coordinates (x,y,z) as described in ref. [5]. All equations in this work are derived considering an infinite cylinder with periodic length equal to $2\pi R_0$ along its symmetry axis.

Since the RMPs are originated from helical wires and helical current sheets, it is possible to define the following vector, which expresses the direction of a helix:

$$\vec{e}_h = \frac{\alpha}{\sqrt{b^2\alpha^2+1}}\vec{e}_\theta + \frac{1}{\sqrt{b^2\alpha^2+1}}\vec{e}_z \tag{1}$$

where b is the cylinder radius and $\alpha = d\theta/dz$.

In order to analyze RMPs generated along helices, it is defined the parameter:

$$u_h = m_0\theta - n_0\left(\frac{z}{R_0}\right) \tag{2}$$

which is constant along a helix and defines its winding law, with mode numbers m_0 and n_0.

Considering a RMP that is resonant to the surface $r = r_0$ and its electrical current is located at the surface $r = r_j$, the perturbed magnetic field ($\vec{B}$) is calculated from Maxwell equations and it is associated to a scalar field:

$$\phi(r,\theta,z) = \begin{cases} \sum\limits_{k_z=-\infty}^{\infty}\sum\limits_{k_\theta=-\infty}^{\infty} C^i_{k_z,k_\theta} I_{k_\theta}(k_z\alpha r)e^{i(k_\theta\theta - k_z\alpha z)}, \text{ if } r \le r_j \\ \sum\limits_{k_z=-\infty}^{\infty}\sum\limits_{k_\theta=-\infty}^{\infty} C^e_{k_z,k_\theta} K_{k_\theta}(k_z\alpha r)e^{i(k_\theta\theta - k_z\alpha z)}, \text{ if } r > r_j \end{cases} \tag{3}$$

where $I_{k_\theta}(k_z\alpha r)$ and $K_{k_\theta}(k_z\alpha r)$ are, respectively, the modified Bessel function of the first and second kind with order k_θ and argument $k_z\alpha r$. The constants $C^i_{k_z,k_\theta}$ and $C^e_{k_z,k_\theta}$ must be calculated from the boundary condition expressing the magnetic field discontinuity along the surface $r = r_j$:

$$\Delta\vec{B}\Big|_i^e = \frac{\mu_0 j}{\sqrt{b^2\alpha^2+1}}\left(-b\alpha\vec{e}^z + b\vec{e}^\theta\right) \tag{4}$$

where j is the surface current at $r = r_j$.

3 – MAGNETOHYDRODYNAMIC EQUILIBRIUM

The magnetic field associated to the magnetohydrodynamic (MHD) equilibrium can be obtained from the Grad-Shafranov equation [5], which provides the following covariant components:

$$\begin{cases} B_0^r = 0 \\ B_0^\theta = -\dfrac{\mu_0 I_p}{2r^2\pi}\left[1-\left(1-\dfrac{r^2}{a^2}\right)^{\gamma+1}\right] \\ B_0^z \cong -\dfrac{\mu_0 I_e}{2\pi R_0} \end{cases} \tag{5}$$

where I_p is the plasma current, I_e is the poloidal current and a is the plasma column radius.

Table 1 shows numerical values for variables used in this work, corresponding to the geometry and the operation of the TCABR tokamak [7].

Tab. 1. Parameters values used to perform calculations.

Parameter	Value
Plasma current: I_p (A)	7.0×10^4
Poloidal current: I_e (A)	4.0×10^6
Plasma column radius: a (m)	0.18
Cylinder radius: b (m)	0.22
Constant γ	3.0
Characteristic length: R_0 (m)	0.61

The safety factor (q) along the plasma cross section is shown in Fig 1.

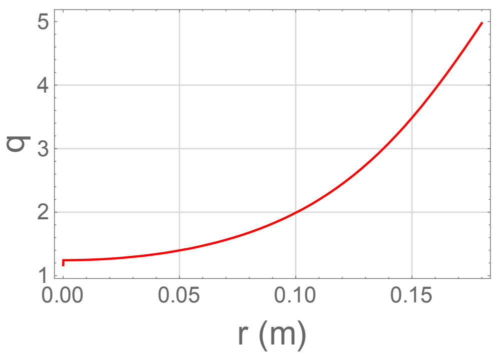
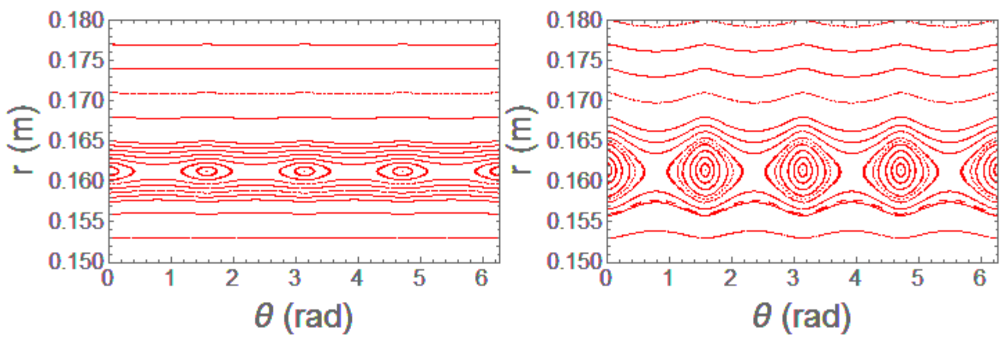


Fig. 1. Safety factor along the plasma cross section.

4 – HELICAL WIRES PERTURBATION

Two helical wires conducting electrical current I_h in opposed senses are placed at the surface $r_j = b$ with the winding laws $0 = m_0\theta - n_0(z/R_0)$ and $\pi = m_0\theta - n_0(z/R_0)$. The resonant surface is located at $r_0 = 0.161\,\mathrm{m}$, corresponding to $m_0 = 4$ and $n_0 = 1$. Figure 2 shows Poincaré sections for $I_h = 0.1\%I_p$ and $I_h = 1.0\%I_p$.



(a) (b)

Fig. 2. Poincaré sections for modal numbers $m_0 = 4$, $n_0 = 1$ and: a) $I_h = 0.1\%I_p$; b) $I_h = 1.0\%I_p$.

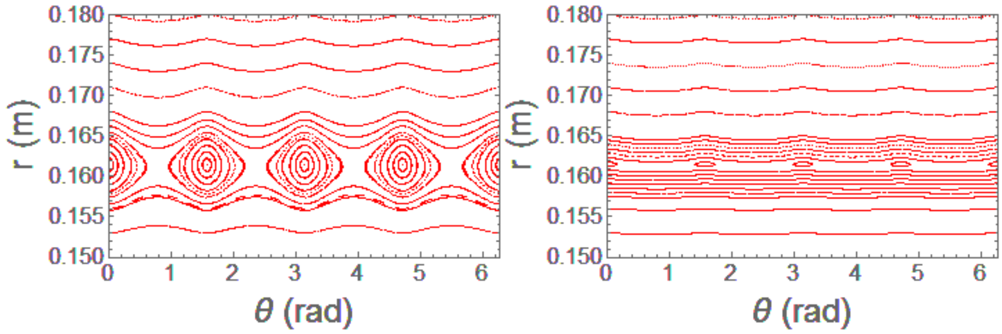
5 – PLASMA RESPONSE

The effect of plasma response can be seen as magnifying or attenuating magnetic islands in Poincaré plots. In order to mimic the situation where the size of magnetic islands is reduced as a consequence of plasma response, it is imposed that the component of perturbed magnetic field orthogonal to the surface where a helical current sheet is located must be zero [3]:

$$\vec{B} \cdot \nabla r = 0 \tag{6}$$

where $\vec{B}$ includes the RMP generated by both the helical windings and the current sheet.

Figure 3 shows the effect of including the plasma response model to the Poincaré sections.



(a) (b)

Fig. 3. Poincaré sections for modal numbers $m_0 = 4$ and $n_0 = 1$ and $I_h = 1.0\% I_p$: a) without plasma response model; b) with plasma response model.

6 – RESULTS AND CONCLUSIONS

Although the plasma response model presented in this work has been proposed to cylindrical plasma, it may be applied to any geometry. The next step to improve the model corresponds to the inclusion of a toroidal geometry through a polar toroidal coordinate system, which is capable of simulation Shafranov shift [5, 6].

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