Effect of 3D Ferromagnetic Materials on Plasma Nonlinear Evolution in Fusion Devices

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1. Introduction

Future magnetic confinement fusion devices, like ITER [1], have such high performances to require a special care in the dimensioning of various components, posing several challenges both from the physical and from the engineering point of view. The electromagnetic interaction of the plasma with the surrounding structures is fundamental for the dimensioning of many critical components (magnets, vacuum vessel, plasma facing components, etc.). This happens both during routine operation (e.g. magnetic control [2]) and in off-normal events (e.g. disruptions [3]).

One of the inherent difficulties in the modelling of such devices is related to the geometrical complexity of the metallic parts surrounding the plasma; another is due to the fact that both non-magnetic conductors and ferromagnetic materials may be present. Indeed, in next-generation devices, where a very high neutron fluence is expected, special reduced activation ferritic martensitic steels will be required for the shielding of the neutrons produced by fusion reactions. Also other applications of ferromagnetic materials in fusion devices can be envisaged, e.g. for the transformer core (as in JET), but also for the so-called ferromagnetic inserts to improve the toroidal field (TF) ripple.

Consequently, reliable computational tools are needed, able to describe such electromagnetic interaction in presence of complex ferromagnetic structures. In particular, recently [4] a novel computational tool has been presented, called CarMa0NL, with the unique capability of simultaneously considering three-dimensional effects of conductors surrounding the plasma and the inherent non-linearity of the plasma behaviour itself. This work reports an extension of this computational tool, namely regarding the inclusion of ferromagnetic materials (previously not considered) into the formulation.

2. Mathematical and numerical formulation

The CarMa0NL formulation, described in details in [4] if no magnetic materials are present, introduces a coupling surface *S* in order to couple the nonlinear evolution of an axisimmetric plasma through equilibrium points, with 3D magneto-quasi-static equations valid in the conductors surrounding the plasma itself. Inside *S*, Grad-Shafranov nonlinear equations are solved in terms of the magnetic flux per radian $\underline{\psi}$ at the nodes of a triangular finite elements mesh, assuming as boundary condition the external flux on *S*, $\underline{\hat{\psi}}_e$:

$$\underline{F}\left(\underline{\psi},\underline{\hat{\psi}}_e\right)=0 \ \ inside\ S \tag{1}$$

Resorting to the numerical formulation described in [4, 5], in the conductors V_c, discretized with a volumetric finite elements hexahedral mesh, the following equations hold:

$$\underline{\underline{L}}\frac{d\underline{I}}{dt}+\underline{\underline{R}}\,\underline{I}+\frac{d\underline{U}}{dt}+\underline{J}_m=\underline{V} \tag{2}$$

where the following terms are also present in the non-magnetic formulation [4]:

$$L_{i,j}=\frac{\mu_0}{4\pi}\int_{V_c}\int_{V_c}\frac{\nabla\times\mathbf{N}_i(\mathbf{r})\cdot\nabla\times\mathbf{N}_j(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|}dV\,dV' \tag{3}$$

$$R_{i,j}=\int_{V_c}\nabla\times\mathbf{N}_i\cdot\eta\cdot\nabla\times\mathbf{N}_j dV \tag{4}$$

$$U_i=\int_{V_c}\nabla\times\mathbf{N}_i\cdot\mathbf{A}_{plasma}\,dV\,,\qquad \underline{U}=\underline{\underline{Y}}\,\underline{I}_{eq} \tag{5}$$

In these equations, $\underline{I}$ is the vector of degrees of freedom of 3D current density, $\mathbf{N}_k$ are the edge elements basis functions, $\underline{V}$ is the vector of voltages applied to electrodes and the quantity $\mathbf{A}_{plasma}$ is the magnetic vector potential due to plasma currents, with the Coulomb gauge, so that $\underline{U}$ is the magnetic flux induced in 3D conductors by the plasma, which is represented by a set of suitable equivalent filamentary currents $\underline{I}_{eq}$ located on *S*. The term $\underline{J}_m$ is instead due to magnetization **M** in magnetic materials V_f, such that $\mathbf{B}=\mu_0\,(\mathbf{H}+\mathbf{M})$, which is discretized in terms of piecewise constant basis functions [5]:

$$\mathbf{M}(\mathbf{x},t)=\sum_k M_k(t)\,\mathbf{P}_k(\mathbf{x}) \tag{6}$$

$$\underline{J}_m=\underline{\underline{F}}\frac{d\underline{M}}{dt},\quad F_{i,j}=\frac{\mu_0}{4\pi}\int_{V_c}\int_{V_f}\frac{\nabla\times\mathbf{N}_i(x)\cdot\mathbf{P}_j\left(x',t\right)\times(\mathbf{x}-\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|^3}dV'\,dV \tag{7}$$

The nonlinear characteristic of magnetic materials, **M** = G(**B**), can be discretized as [5]:

$$\underline{\underline{D}}\,\underline{M}=\underline{G}(\underline{B}),\quad G_i(\underline{B})=\int_{V_f}\mathbf{P}_i\left(x',t\right)\cdot\left(\sum_k B_k(t)\,\mathbf{P}_k(\mathbf{x})\right)dV,\quad \underline{B}=\underline{\underline{D}}^{-1}\left(\underline{\underline{E}}\,\underline{M}+\underline{\underline{W}}\,\underline{I}_{eq}+\underline{\underline{F}}^T\underline{I}\right)+\underline{B}_c \tag{8}$$

where $\underline{B}_c$ is the magnetic field due to sources constant in time (e.g. TF coils) and

$$D_{ij} = \int_{V_f} \mathbf{P}_i \cdot \mathbf{P}_j dV, \quad W_{i,j} = \frac{\mu_0}{4\pi}\int_S \int_{V_f} \frac{\mathbf{P}_i(\mathbf{x}) \cdot \mathbf{J}_{eq}(\mathbf{x}',t) \times (\mathbf{x}-\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|^3} dS'dV$$

$$E_{ij} = \mu_0 \int_{V_f} \mathbf{P}_i \cdot \mathbf{P}_j dV - \frac{\mu_0}{4\pi}\int_{\partial V_{f_i}} \int_{\partial V_{f_j}} \frac{(\hat{\mathbf{n}} \cdot \mathbf{P}_i(\mathbf{x}'))(\hat{\mathbf{n}}' \cdot \mathbf{P}_j(\mathbf{x}))}{|x-x'|} dS'dS \tag{9}$$

We pose $\underline{M} = \underline{M}_c + \delta\underline{M}$, where $\underline{M}_c$ is the (time constant) magnetization corresponding to $\underline{B}_c$. We suppose that the perturbation around $\underline{M}_c$ is small; a linearization of the magnetic characteristic is performed, defining a local magnetic susceptivity χ and permeability μ:

$$\delta\underline{M} = \underline{\underline{C}}\left(\underline{\underline{W}}\,\underline{I}_{eq} + \underline{\underline{F}}^T \underline{I}\right), \quad \underline{\underline{C}} = \left(\frac{\mu}{\chi}\underline{\underline{D}} - \underline{\underline{E}}\right)^{-1} \tag{10}$$

Substituting (6)-(10) in (2), we have:

$$\underline{\underline{L}}^* \frac{d\underline{I}}{dt} + \underline{\underline{R}}\,\underline{I} + \underline{\underline{Y}}^* \frac{d\underline{U}}{dt} = \underline{V}, \quad \underline{\underline{L}}^* = \underline{\underline{L}} + \underline{\underline{F}}\,\underline{\underline{C}}\,\underline{\underline{F}}^T, \quad \underline{\underline{Y}}^* = \underline{\underline{Y}} + \underline{\underline{F}}\,\underline{\underline{C}}\,\underline{\underline{W}} \tag{11}$$

which is formally analogous to the model without ferromagnetic materials [4], except for a redefinition of matrices, to take into account the presence of the magnetic materials. Similarly, the external flux on S is modified as:

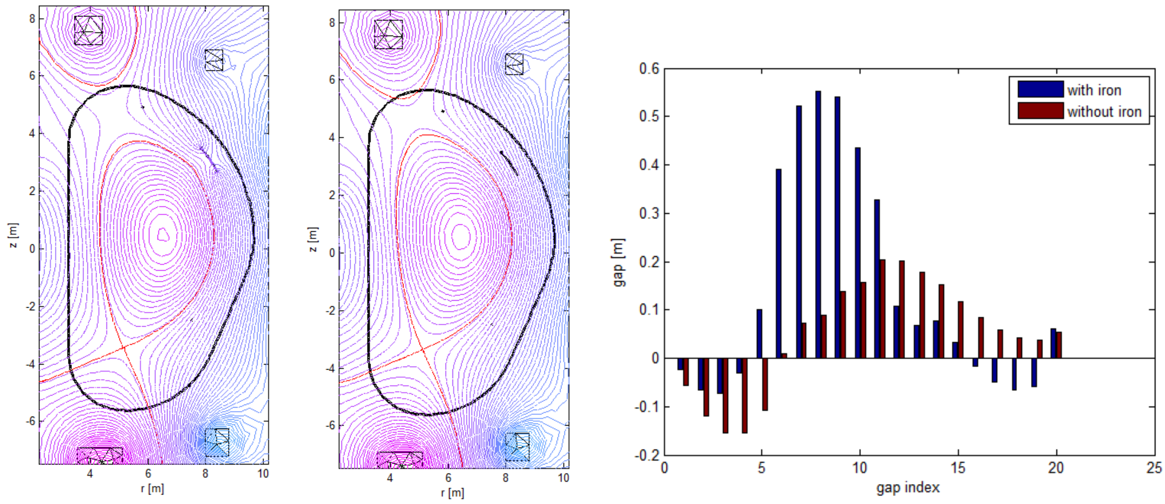
$$\underline{\hat{\psi}}_e = \underline{\underline{Q}}\,\underline{I} + \underline{\underline{Q}}_m \underline{M} = \underline{\underline{Q}}\,\underline{I} + \underline{\underline{Q}}_m \underline{M}_c + \underline{\underline{Q}}_m \delta\underline{M} = \underline{\underline{Q}}_m \underline{M}_c + \left(\underline{\underline{Q}} + \underline{\underline{Q}}_m \underline{\underline{C}}\,\underline{\underline{F}}^T\right)\underline{I} + \underline{\underline{Q}}_m \underline{\underline{C}}\,\underline{\underline{W}}\,\underline{I}_{eq} \tag{12}$$

3. Results

A simple test case is presented, in which one equilibrium configuration of ITER is perturbed with a fictitious linear magnetic material placed in correspondence of the inner shell of the vessel (Fig. 1). It is also assumed that the ferromagnetic material is axisymmetric, so that, although a fully 3D mesh is given, a 2D code (CREATE_L [6]) can be used as benchmark. Figure 2 reports the $\underline{\underline{L}}^*$ and $\underline{\underline{Y}}^*$ matrices (11) as computed by CarMa0NL and CREATE_L, for 50 axisymmetric conductors representing the outer vessel shell and for 618 filamentary currents on the coupling surface, showing a good agreement. The perturbation of the matrices is concentrated close to the magnetic material, but the equilibrium configuration is affected globally (Fig 1c). The equilibrium solutions computed by CarMa0NL and CREATE_L differ only of a few percents, confirming the ability of CarMa0NL to correctly describe the effects of ferromagnetic materials on plasma equilibrium configurations.

4. Conclusions

In this paper we have presented an extension of the CarMa0NL computational tool, in order to account for 3D ferromagnetic conductors in the nonlinear evolutionary equilibrium of axisymmetric plasmas. Simple test cases have been solved, in which a one-to-one comparison with axisymmetric models is possible, showing good agreement. Future activity will be devoted to the analysis of the effect of realistic 3D ferromagnetic structures, like the ferromagnetic inserts. This work was supported in part by Italian MIUR under PRIN grant 2010SPS9B3.



(a) (b) (c)

Fig. 1. Reference equilibrium with (a) and without (b) axisymmetric magnetic material and effect on gaps (c)

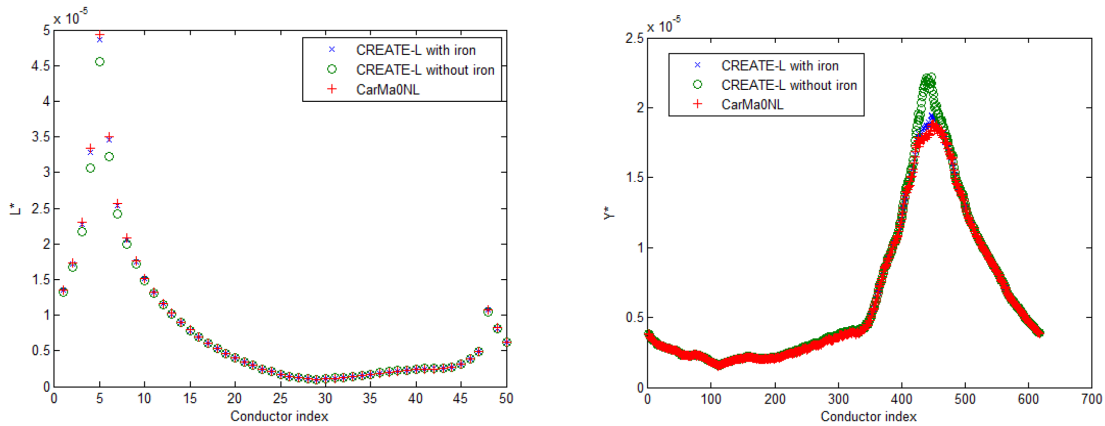


Fig. 2. Comparison of $\underline{\underline{L}}^*$ and $\underline{\underline{Y}}^*$ for test axisymmetric case.

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