Magnetic Shear and Stability of Golden Magnetic Barrier in Divertor Tokamak

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ABSTRACT: In Hamiltonian chaos the last torus to be destroyed is the golden torus [1]. It has the most irrational winding number called the golden mean. An external perturbation can be designed to create an invariant torus inside chaos in Hamiltonian systems [2]. The required perturbation is an order of magnitude smaller than the perturbation that creates the chaos. A good magnetic surface can be created inside chaos in magnetically confined plasmas [2-7]. This is called a magnetic barrier. A magnetic barrier with golden value of winding number will be the most stable magnetic surface. Effect of magnetic shear on the stability of golden barrier is studied. Magnetic shear is an important parameter in magnetic confinement. A map for magnetic field lines with variable shear is used [5-6]. The question addressed is - how does the largest magnetic perturbation against which the golden barrier is stable vary with shear. How does the shear affect the confinement of plasma having the golden barrier is investigated. This work is supported by the US DOE grants DE-FG02-01ER54624 and DE-FG02-04ER54793. This research used resources of the NERSC, supported by the Office of Science, US DOE, under contract DE-AC02-05CH11231.

Recently, we used a control method and our method of maps to create invariant magnetic surface inside chaotic regions in magnetically confined plasmas [3-7] by adding to the poloidal flux an external magnetic perturbation that is an order of magnitude smaller than the perturbation that creates the chaotic regions. This invariant surface is called magnetic barrier. It effectively acts as “barrier” against field-line dispersion and, indirectly, particle diffusion, and can be beneficial for plasma confinement. The feasibility, in the DIII-D tokamak, of these magnetic barriers is confirmed in [7] using experimentally constrained EFIT [8] equilibria and vacuum field perturbations exerted by the internal I-coils, taking into account their actual geometry. We also introduced a new area-preserving logarithmic map in magnetic coordinates that calculates magnetic field line trajectories in single-null divertor tokamaks with variable shear. The map parameter denoted by s controls the magnetic shear and is referred to as shear parameter. This map preserves the symplectic topological invariance of the Hamiltonian system. We used this map to study the resilience of golden magnetic surface in tokamaks against resonant magnetic perturbations [5-6]. In this paper, we use this map to study the effects of magnetic shear on the stability of magnetic barrier with golden value of safety factor with and without adding second order control term to the poloidal flux. We call this barrier the golden barrier. For magnetic perturbation, we choose two tearing modes with mode numbers $(m,n)=\{(4,3),(2,1)\}$, and $\chi_1(\theta,\phi)=\delta[\cos(4\theta-3\phi)+\cos(2\theta-\phi)]$. δ is the amplitude of the perturbation. The safety factor profile q is given by $q(\psi)=1-s\ln(1-\psi)$. Here ψ is normalized by ψ_{SEP}, the toroidal flux inside the separatrix surface. The value of safety factor at the magnetic axis, q_0, is equal to unity and $q\to\infty$ at the separatrix. We characterize the magnetic configuration by the value of edge safety factor, q_{95}, at the magnetic surface that encloses 95% of the poloidal flux compared to the poloidal magnetic flux inside the separatrix. Fig. 1 shows q_{95} as a function of the shear parameter s. q_{95} scales as $e^{2s/3}$. From Fig. 1, we also see that for s=1.0, q_{95}=3.1.

As the magnetic shear parameter s is varied, the location of the golden magnetic surface changes. For each value of s, the magnetic perturbation given above is applied to these surfaces with increasing amplitude; and the critical levels of the perturbations, δ_{crit}, when the surfaces break down are calculated. We found that as the shear parameter s increases, δ_{crit} scales as $e^{-2s/3}$. Thus, δ_{crit} scales as $1/q_{95}$. See Fig. 2. Following [2-4], we add a second order magnetic perturbation denoted by χ_2 to the poloidal flux $\chi=\chi_0+\chi_1$. χ_0 is the equilibrium poloidal flux and χ_1 is given above. We refer χ_2 as the second order control term and we denote its location as ψ_b. Figure 3 shows the effects of χ_2 on the phase portrait. Fig. 3a depicts the phase portrait when $\delta = 2\times10^{-3}$ and s= 1. This Fig. shows the m=4 islands and m=2 islands. The perturbation influences mainly the region in the vicinity of the resonant surface ψ_{43}. A large chaotic zone appears around this surface. The golden surface is broken into eight islands and there are no internal transport barriers within the chaotic region. The magnetic field lines close to the magnetic axis $\psi = 0$ and the separatrix are regular. Fig. 3b shows the phase portrait with the same values of δ and s as in Fig. 3a but with the second order control term χ_2 added to the poloidal flux χ. In this case, the phase portrait is more regular, the chaotic region around the resonant surface ψ_{43} has disappeared and the golden magnetic barrier is created. Fig. 4 shows δ_{crit} as a function of the shear parameter s with and without the second order control term χ_2.

We define the magnetic shear of the magnetic field as $S= 2\psi q'(\psi)/q(\psi)$, and the average magnetic shear as $< S >= \frac{1}{\psi_{95}}\int_0^{\psi_{95}} S(\psi)\,d\psi$. The influences of magnetic shear and the edge safety factor in the stability and improvement of the quality of confinement in magnetically confined plasmas have been firmly established by the results of numerous theoretical and experimental studies using various configurations in the last decades [9-10]. Fig. 5 shows that the average magnetic shear <S> scales as $e^{2s/7}$. As the average magnetic shear increases, we found that the critical levels of the magnetic perturbations δ_{crit} when the golden surfaces break down generally decrease with or without the second order control term χ_2. With the control term χ_2, the values of δ_{crit} fluctuate as $<S>$ increases from 1.1 to 1.3; then decrease with increasing $<S>$. We also found that, for any fixed value of $<S>$, the value of δ_{crit} required for the golden magnetic barrier to break down is about 45% to 50% higher with the second order control term than without. Fig. 6 shows these results.

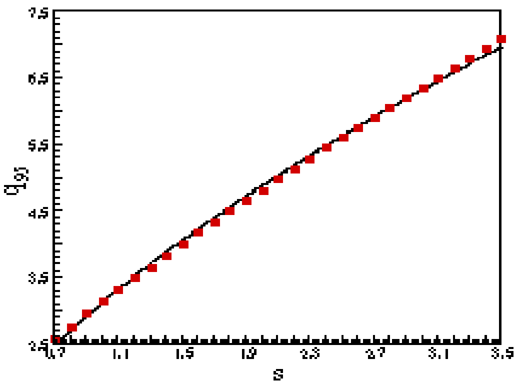


Fig. 1: The edge safety factor q_{95} as a function of the shear parameter s. Curve through data points is a power fit: q_{95} scales as $e^{2s/3}$.

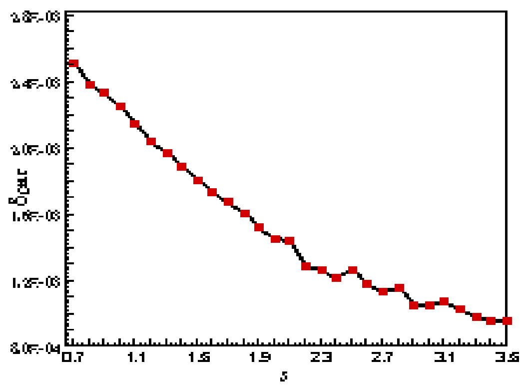


Fig. 2: δ_{crit} as a function of the shear parameter s. Curve through data points is a power fit: δ_{crit} scales as $e^{-2s/3}$.

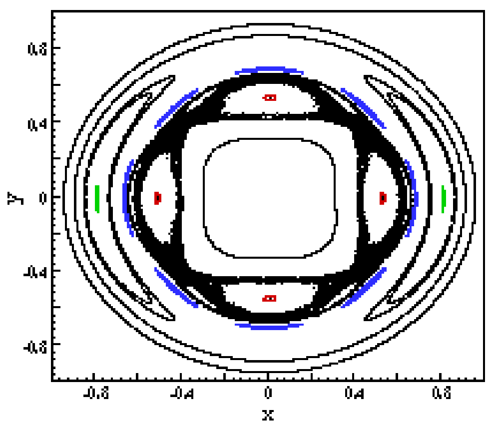


Fig. 3a shows the phase portrait when $\delta=2\times10^{-3}$ and $s=1$ without the control term χ_2 at $\psi=\psi_{golden}$.

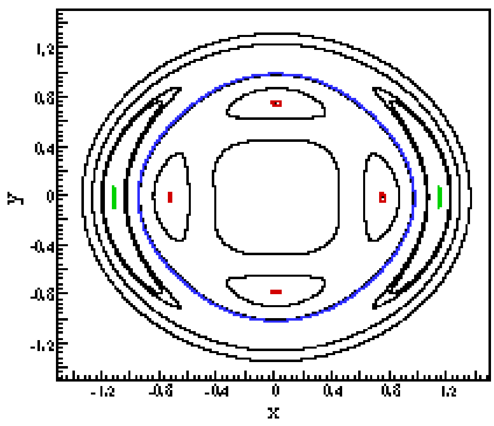
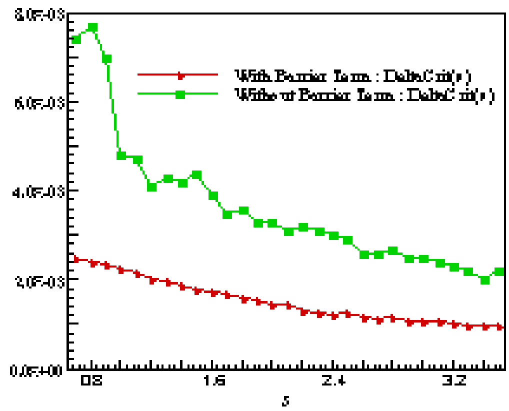


Fig. 3b shows the phase portrait with the same values of δ and s with the control term χ_2 at $\psi=\psi_{golden}$.



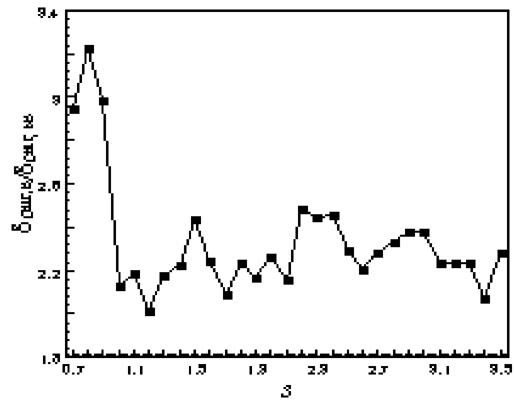
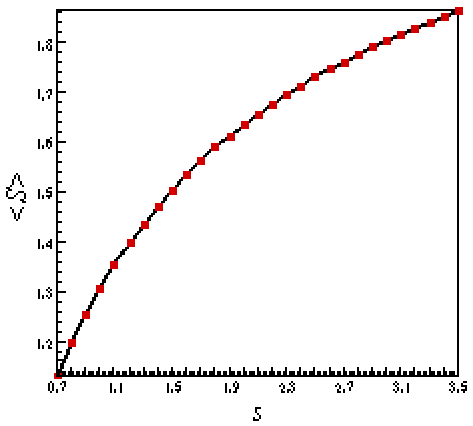


Fig. 4a: $\delta_{crit,}$ as a function of s with and without the control term χ_2 at $\psi = \psi_{golden}$.

Fig. 4b: $\delta_{crit, B}/\ \delta_{crit, NB}$ as a function of s



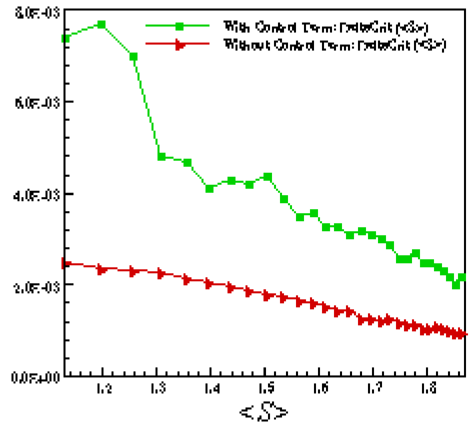


Fig. 5: <S> as a function of the shear parameter s. Curve through data point is a power fit:<S> $\propto e^{2s/7}$.

Fig. 6: $\delta_{crit,}$ as a function of <S> with and without the control term χ_2 at ψ_{golden} added to the poloidal flux χ.

Higher average shear makes the golden surface and the golden magnetic barrier weaker as they break down at lower levels of magnetic perturbation.

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