A quantitative model for heat pulse propagation in Large Helical Device plasmas using a travelling wave transformation

H Zhu[1], R O Dendy [2,1,3,4], S C Chapman[1,5,6] and S Inagaki[3,4]

[1]*Centre for Fusion, Space and Astrophysics, Department of Physics, University of Warwick, Coventry CV4 7AL, UK*

[2]*CCFE, Culham Science Centre, Abingdon, Oxfordshire OX14 3DB, UK*

[3]*Itoh Research Center for Plasma Turbulence, Kyushu University, Kasuga 816-8580, Japan*

[4]*Research Institute for Applied Mechanics, Kyushu University, Kasuga 816-8580, Japan*

[5]*Max Planck Institute for the Physics of Complex Systems, Dresden, Germany*

[6]*Department of Mathematics and Statistics, University of Tromsø, Tromsø, Norway*

1. Introduction The transport of energy across magnetically confined fusion plasmas, and the storage of energy within them, reflect a wide range of turbulent and nonlinear phenomenology. There is extensive experimental evidence for the transport phenomena that are non-diffusive and may be non-local. Here we focus on heat pulse experiment[1-6]. These heat pulses are typically initiated by injection into the edge plasma of ice pellets or supersonic molecule beams, by gas puffing, and by laser ablation. The zero-dimensional model is know to work at the best-diagnosed spatial location, capturing the time evolution of the pulse there, where $t = 0$ is defined to be the local arrival time of the initial impulsive perturbation. Therefore we conjecture that the zero-dimensional model should work at each location across the radial domain of the plasma within which the same physical processes determine the behaviour of the pulse. From this we infer that the model ought to apply in a frame that is co-moving radially with the heat pulse across this region. We then apply a travelling wave transformation by replacing t by $\xi = x + v_0 t$; here v_0 is to be considered as proxy of the pulse propagation velocity in the radial direction x, and the sign convention adopted for t assists consideration of inward propagation.

2. Model description The normalised zero-dimensional model[4] examined below is constructed in terms of the three key physical quantities that were measured[4] so as to characterise pulse propagation in LHD. These quantities are the deviation from steady state of the electron temperature gradient $\delta \nabla T_e$, the excess turbulent heat flux δq_e, and the deviation δT_e of electron temperature from its steady-state value. The dimensionless counterpart of these variables are denoted by x_1, x_2 and x_3 respectively, as defined in Ref.[4]. The model of Ref.[4] shows quantative agreement between its outputs and experimental measurements of the time evolution. However, for given parameters and initial conditions, the model only simulates the time evolution of a passing heat pulse at a specific radius. The model equation (5) to (7) of Ref.[4] are as follows

$$\frac{dx_1}{dt} = \kappa_{T0}x_2 + x_1x_2\frac{\partial\kappa_T}{\partial x_1} + x_2x_3\frac{\partial\kappa_T}{\partial x_3} - \gamma_{L1}x_1$$
$$\frac{dx_2}{dt} = -\kappa_{Q0}x_1 + x_1^2\frac{\partial\kappa_Q}{\partial x_1} - x_1x_3\frac{\partial\kappa_Q}{\partial x_3} - \gamma_{L1}x_2$$
$$\frac{dx_3}{dt} = -\frac{1}{\tau_c}\frac{\eta}{\chi_0}x_2 - \gamma_{L2}x_3$$

Central to the present paper is the adoption of travelling wave transformations as a method for generating (x,t)-dependence from the t-dependent model above. We assume that

$$x_i\left(x,t\right) = y_i\left(\xi\right), \xi = x + v_0 t, i = 1,2,3$$

Here x and t are considered to be independent variables, and we refer to v_0 as the pseudo-velocity of the pulse. Then we get the following equations by substitution, leading order approximation and transposition:

$$v_0\frac{d^2y_1}{d\xi^2} - \gamma_{L1}\frac{dy_1}{d\xi} + \left(\frac{\kappa_{T0}}{v_0}\frac{\partial\kappa_Q}{\partial y_1} + \frac{\kappa_{Q0}}{v_0}\frac{\partial\kappa_T}{\partial y_1}\right)y_1^2 + \frac{\kappa_{T0}\kappa_{Q0}}{v_0}y_1 = -\frac{\kappa_{T0}\gamma_{L1}}{v_0}y_2 + \frac{\kappa_{T0}}{v_0}\frac{\partial\kappa_T}{\partial y_1}y_2^2$$
$$v_0\frac{d^2y_2}{d\xi^2} - \gamma_{L1}\frac{dy_2}{d\xi} + \frac{\kappa_{T0}\kappa_{Q0}}{v_0}y_2 = \frac{\kappa_{Q0}\gamma_{L1}}{v_0}y_1 - y_1y_2\left(\frac{\kappa_{Q0}}{v_0}\frac{\partial\kappa_T}{\partial y_1} + 2\frac{\kappa_{T0}}{v_0}\frac{\partial\kappa_Q}{\partial y_1}\right)$$

3. Comparison of model outputs with LHD experimental data We recall that y_1, y_2 and y_3 are the dimensionless counterpart of $\delta\nabla T_e$, δq and δT_e. respectively. Solution of our model, embodied in the two coupled nonlinear ordinary differential equations above. The numerical values of parameters carry over identically from Ref.[4], to which we refer for the experimental motivation for these values.

Figure 1 compares time traces of the evolving electron temperature at multiple radial locations, obtained from the model and from experimental data (Te49708) for the R case. Several representative radii are marked by arrows on the right hand side. The model results are able to match experimental data from ρ = 0.450 inward to the core, if we uniformly apply horizontal (+0.01) and vertical (+0.20) shifts, suggesting that the model applies over this broad radial range. It is also clear that electron temperature profiles from ρ = 0.546 outward to ρ = 0.703 are not simulated by the model. This suggests that different physics dominates heat pulse propagation in the outer region of this plasma.

Figure 2 demonstrates the comparison of model results and experimental data (Te49719) for the D case. In common with the R case shown Figure 1, the model results are good from ρ = 0.450 inward to the core, with uniformly applied horizontal (+0.04) and vertical (+0.07) shifts, but not in the outer region of this plasma.

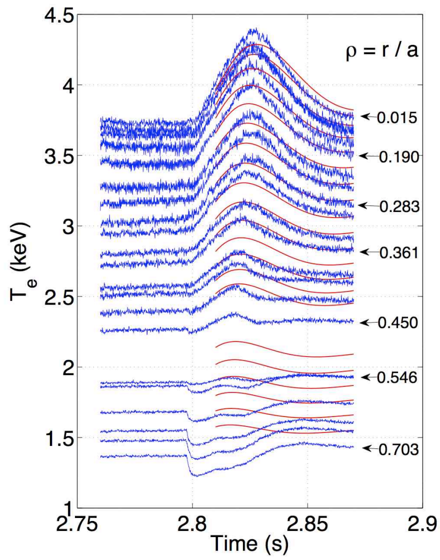

Figure 1. Time evolution of electron temperature at multiple radial locations, derived from LHD data(blue) and the model(red) for the core temperature rise(R) heat pulse propagation experiment in plasma 49708. Radial location rage from edge (ρ = 0.703) to core (ρ = 0.015), where $\rho = r/a$, r is the radial co-ordinate and a ~ 0.6m is minor radius of LHD. Model results match experimental data well from ρ = 0.450 inwards to the plasma core, especially amplitudes and the time structure of pulse decay. The amplitude of model time traces increase from edge to core, as in the measured electron temperature profiles. Model results do not fit experimental data outwards from ρ = 0.546 to ρ = 0.703, implying that different physics applies in the outer LHD plasma. Reproduced from [6].

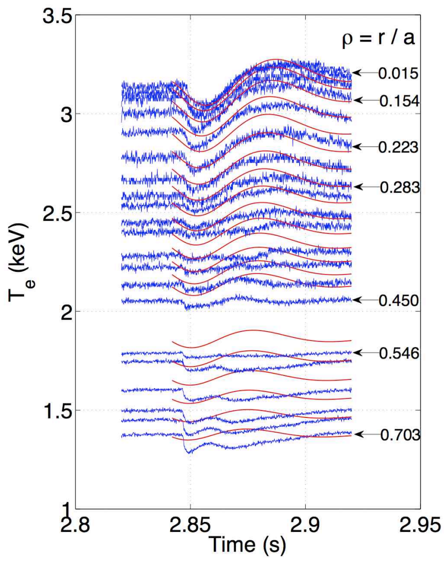

Figure 2. Time evolution of electron temperature at multiple radial locations, derived from LHD data(blue) and the model(red) for the core temperature drop(D) heat pulse propagation experiment in plasma 48719. Radial location rage from edge (ρ = 0.703) to core (ρ = 0.015), where $\rho = r/a$, r is the radial co-ordinate and a ~ 0.6m is minor radius of LHD. As in Figure 1, model results match experimental data well from ρ = 0.450 inwards to the plasma core. Again, model results do not fie experimental data outwards from ρ = 0.546 to ρ = 0.703, reinforcing that different physics dominates in the outer LHD plasma. Reproduced from [6].

4. Conclusions We have derived from first principles a time-dependent model in one spatial dimension, which is able to describe quantitatively the radial inward propagation of heat pulses in the core of two plasmas in the Large Helical Device(LHD). In one plasma the central electron temperature rises, in the other it falls. This new model is derived from a travelling wave transformation of the zero-dimensional model of Ref.[4], which is known to capture the time-evolution of the heat pulse as it passes through a fixed radial location in these two plasmas. Comparison between model outputs and raw experimental data suggests that our model is able to describe heat pulse propagation well, within a broad radial range of the LHD core plasma from $r/a \approx 0.5$ to the centre.

The results of the present paper provide additional support to the physical proposals, which motivate the simple model equations above. Central to these proposals is the conjecture that heat pulses are structures, which involve plasma physics processes which are so strongly nonlinear that heat pulse evolution is primarily determined by the reactions of the perturbed heat flux and the perturbed temperature gradient on each other. In the present model, this is the dominant element of nonlinear physics, whereas turbulent transport plays a relatively minor dissipative role. It is this mutually coupled interaction between perturbed heat flux and temperature gradient that governs the local plasma dynamics of the heat pulse in space, equivalent to the local up-and-down dynamics of a water wave under gravity. We have shown in this paper that this coupling model lends itself readily to a travelling wave transformation, yielding spatio-temporal pulse propagation, and that the pulse velocity that emerges mathematically provides an adequate match to empirical results and expectations. This aspect of the analysis also provides guidance on a previously unanswered question, namely the generic character of the heat pulse: we have shown that it may be closely related to a Korteweg-de Vries-Burgers soliton.

This work was part-funded by the RCUK Energy Program under grant EP/I501045. This project has received funding from the European Union's Horizon 2020 research and innovation program under grant agreement number 633053. The views and opinions expressed herein do not necessarily reflect those of the European Commission. This work was also supported in part by the grant-in-aid for scientific research of JSPF, Japan (23244113) and KAKENHI (21224014) from JSPS, Japan. This work was also supported in part by the Grant-in-Aid for Scientific Research of JSPF, Japan (23360414) and by the collaboration programs of the National Institute for Fusion Science (NIFS) (NIFS13KOCT001) and the Research Institute for Applied Mechanics of Kyushu University.

[1] S Inagaki *et al*, Nucl. Fusion **46**, 133 (2006).
[2] P Mantica *et al*, Phys. Rev. Lett. **82**, 5048 (1999).
[3] S Inagaki *et al*, Plasma Phys. Control. Fusion **52** 075002 (2010).
[4] R Dendy *et al*, Plasma Phys. Control. Fusion **55**, 115009 (2013).
[5] S Inagaki *et al*, Plasma Phys. Control. Fusion **46** A71 (2004).
[6] H Zhu *et al*, submitted to Phys. Plasmas (2015).