The influence of anode temperature on the length of the anode plasma in a gas discharge under conditions of strong ionization depletion of neutral particles

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Although thrust of electric propulsion thrusters is much smaller than that of chemical thrusters, they can reach much higher propellant exhaust velocities. The most popular type of such thrusters are Hall thrusters that are widely used for correction of orbits of long-life spacecrafts and for controlling their orientation in space today. These thrusters are also perspective for distant space missions because of their ability for reaching high velocities and because of their large efficiency factor of working medium consumption. In electric ion propulsion thrusters of this type the gas discharge in the crossed electric and magnetic fields is used. Working gas is delivered into the discharge channel of the thruster, where it is exposed to ionization depletion under activity of Hall electrons. The ions, formed during ionization depletion, are accelerated by electric field and generate thrust of Hall thruster [1].

In [2,3] the problem of structure of gas discharge anode plasma with strong ionization depletion of neutral particles was considered. There was supposed that mean directed velocity of neutral particles was constant and equal to $\overline{v}/2=\sqrt{2T/\pi M}$, where T is the temperature of neutral particles measured in energy units; M is the mass of neutral particle.

As in the papers [2,3], we will consider only case of collisionless motion of ions in one-dimensional approximation and Boltzmann thermal electron distribution. Motion of neutral particles will be described using kinetic approximation. We will assume that neutral particles have an arbitrary semi-isotropic in velocity space velocity distribution function at anode surface [4].

The potential $\varphi(x)$ at some point of the anode plasma is at its maximum. Let us choose the origin of the coordinate and potential at this point, that is $\varphi(0)=0$, $d\varphi(x)/dx\big|_{x=0}=0$. A neutral gas is fed from the $x=x_a$ ($x_a<0$) anode plane, with the backward flow of neutral particles being absent. All of the ions generated in the region between this plane and the potential maximum turn back, recombine on the x_a surface, and come back to the discharge region in the form of neutral particles. Thus, the flux of neutral particles from the $x = x_a$ plane exceeds the gas flow rate to which the neutral flux density at the potential maximum, q_0, corresponds. The ions are trapped in the region between x_a and the potential maximum in a steady state. Indeed, in view of the conservation of the heavy component $eq(x) + j_i(x) = eq_0$, where $q(x)$ is the neutral flux density, $j_i(x)$ is the ion current density, e is the elementary charge. Since $j_i(0) = 0$, then $q(0) = q_0$.

Let us introduce dimensionless potential $\eta = -e\varphi(x)/T_e$, where T_e is the plasma electron temperature in energy units. Using continuity equation for neutral particles and quasi-neutrality equation we can obtain following expression for dependence of neutral flow density upon potential [2]

$$q(\eta)/q_0 = 1 \mp \varepsilon e^{-\eta}\operatorname{erfi}(\sqrt{\eta})/\sqrt{\pi}, \qquad (1)$$

where sign "-" corresponds to a region $x \le 0$, sign "+" corresponds to a region $x \ge 0$; $\varepsilon = (n_0/q_0)\sqrt{2T_e/M}$ is a depletion parameter; n_0 is slow-electron density at $x = 0$.

It should be noticed that anode surface is usually significantly heated by accelerated ions that move back from anode plasma and electrons. After neutralization at the anode surface these ions come back to the discharge region in the form of neutral particles. So we have two groups of neutral particles that obviously have different temperatures.

Let us consider the case when atoms of neutral gas that are supplied from anode surface and particles that recombine on the $x = x_a$ surface and come back to the anode plasma again as neutral particles have different semi-isotropic in velocity space velocity distribution functions. Let us assume that these distribution functions are characterized by functions of velocity modulus $F_1(v)$ and $F_2(v)$ correspondingly, that satisfy requirements $\int_0^\infty F_i(v)v^k dv < \infty$, $k = 0,1,2,3,...$, $i = 1,2$.

In [4] expression for neutral gas flow density was obtained that accounts strong ionization depletion of neutral particles. It can be showed that in our case using results of paper [4] and expression (1) we can write for neutral gas flow density following expression

$$q(\eta)/q_0 = 2\int_0^\infty E_3\left(\frac{\varepsilon}{2}\frac{\bar{v}}{v}\left(s(\eta) - s_a\right)\right)\left[\frac{F_1(v)}{\int_0^\infty F_1(v)v^3 dv} + \frac{\varepsilon}{\varepsilon_*}\frac{F_2(v)}{\int_0^\infty F_2(v)v^3 dv}\right]v^3 dv, \qquad (2)$$

where $E_m(x)=\int_0^\infty \frac{\exp(-xw)}{w^m}dw = x^{m-1}\int_x^\infty \frac{\exp(-t)}{t^m}dt$, $m=1,2,3...$ is the integral exponent [5]; $\varepsilon_* = \pi\sqrt{\eta_0}$ is critical value of the depletion parameter at which the cathode plasma boundary goes to infinity and the entire flow of neutrals is depleted in the plasma region [2]; $\eta_0 \approx 0.854$ is potential at the plasma boundaries; $s = x/l$ is dimensionless coordinate; $l=(n_0\bar{v}/2\omega_0 q_0)\sqrt{2T_e/M}$; $\omega_0 = \text{const}$ is constant ionization depletion frequency of neutral particles.

It is easy to notice that if distribution functions of velocity modulus $F_1(v)$ and $F_2(v)$ coincide, then equation (2) takes the same form as equation that was obtained in paper [3].

Expressions (1) and (2) allow us to obtain dependence of potential upon coordinate for different neutral velocity distribution functions $F_1(v)$ and $F_2(v)$. Let us consider in particular cases when absolute values of the initial velocities of neutral particles are equal ($F_i(v)=\delta(v-\bar{v})$) and when neutral particles have Maxwellian velocity distribution ($F_i(v)=\exp(-4v^2/\pi\bar{v}^2)$) for different values of ratio of the anode temperature to the temperature of supplied gas (T_a/T_0).

Dependences of anode (1*a* and 1*c*) and cathode (1*b* and 1*d*) boundary coordinates of anode plasma upon depletion parameter are showed on figure 1 for different values of ratio of the anode temperature to the temperature of supplied gas (T_a/T_0). Figures 1*a* and 1*b* are plotted for kinetic model with equal absolute values of the initial velocities of neutral particles; figures 1*c* and 1*d* are plotted for kinetic model with Maxwellian velocity distribution of neutral particles. It can be seen from these figures that effect of anode temperature increases with an increase of depletion parameter.

It is obvious that decrease of average neutral particle lifetime in the anode plasma leads to increase of length of the anode plasma if depletion parameter is fixed. Therefore, taking into account both kinetic effects of predominant depletion of tangentially propagating and slow moving neutral particles while considering the flow of neutral particles [3] and heating of the anode surface leads to increase of length of the anode plasma.

It should be noticed that coordinate of cathode boundary of anode plasma rapidly increase with an increase of depletion parameter, and if depletion parameter is large enough then length of the anode plasma is approximately equal to distance from point of maximum value of anode plasma potential to its cathode boundary.

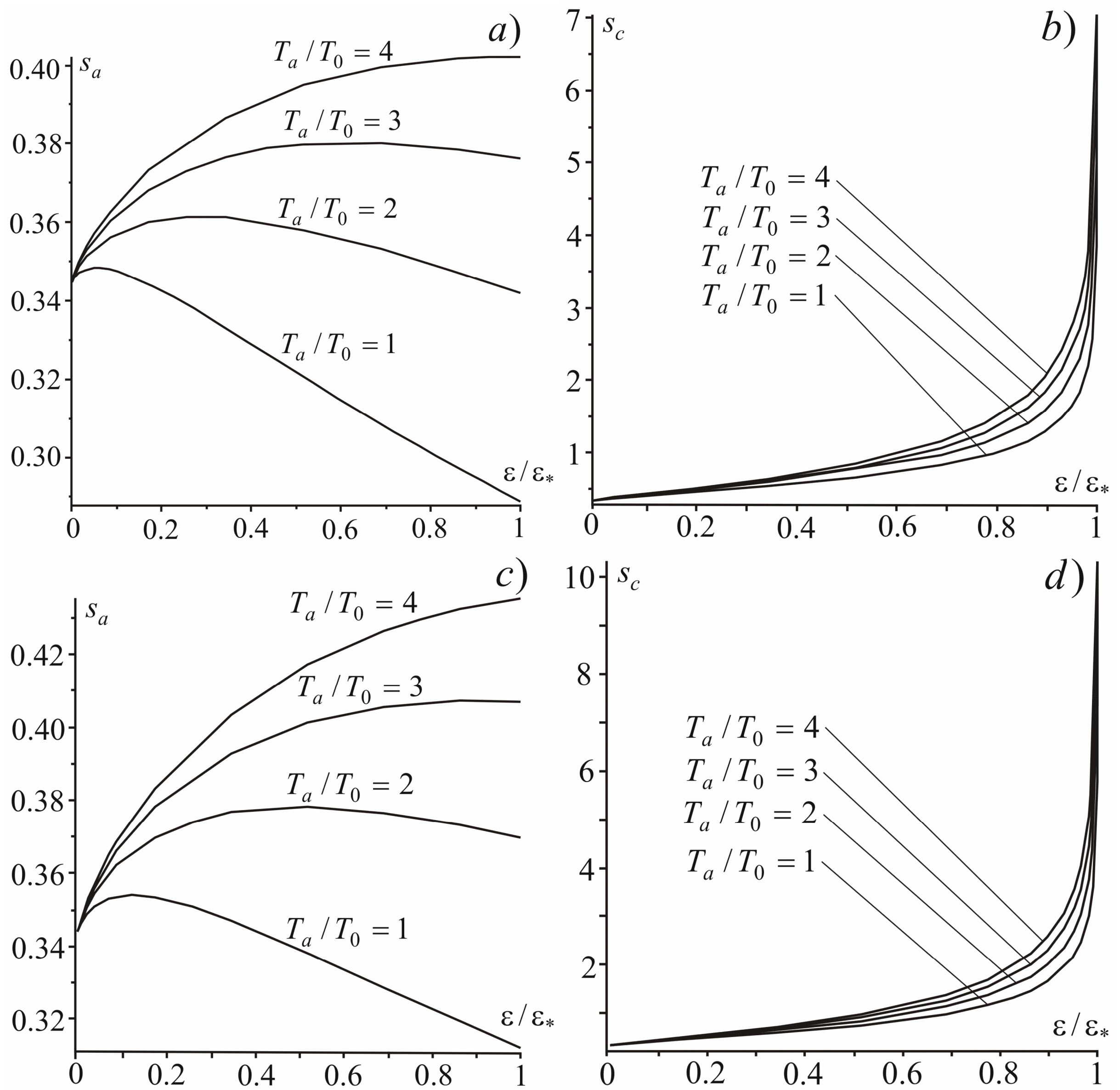

Figure 1. Dependences of anode and cathode boundary coordinates of anode plasma upon depletion parameter for different values of ratio of the anode temperature to the temperature of supplied gas. (a), b) correspond to kinetic model with equal absolute values of the initial velocities of neutral particles; c), d) correspond to kinetic model with Maxwellian velocity distribution of neutral particles).

References:

1. D.M. Goebel and I. Katz. Fundamentals of Electric Propulsion: Ion and Hall Thrusters. Wiley-Blackwell. (2008).

2. A.V. Zharinov and V.P. Shumilin, Journal of Experimental and Theoretical Physics, Vol. 103, №2, P. 324 (2006).

3. V.P. Shumilin, A.V. Shumilin and N.V. Shumilin, Kinetic effects in ionization depletion of neutral particles in the anode plasma of the gas discharge. ESCAMPIG XXII, Greifswald, Germany, July 15-19, 2014, P1-06-05.

4. V.P. Shumilin, A.V. Shumilin and N.V. Shumilin, High Temperature, Vol. 50, №2, P. 178 (2012).

5. A.F. Nikifirov and V.B. Uvarov. Special Functions of Mathematical Physics. Basel (Switzerland): Birkhauser. (1988).