Exploiting genetic algorithms in transport modelling in RFX-mod

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The goal of this work is to gain a deeper understanding of the quality of transport reconstruction from modelling of kinetic profiles in the Reversed Field Pinch RFX-mod. The current procedure for main gas transport modelling in stationary discharges in RFX-mod aims at computing the transport coefficients (*D,U*) (diffusivity and pinch velocity, respectively) by reproducing the experimental data provided by various diagnostics as interferometer, multichannel reflectometer, calibrated edge and core Thomson Scattering. The numerical data are computed via synthetic diagnostics, starting from the radial profile of the generic kinetic quantity *f(r)* obtained solving the model transport equation:

$$0 = \nabla \cdot \left(D \nabla f - U\, f\right) + S \quad (1)$$

Being an instance of an inverse problem, any conclusion drawn is potentially prone to large uncertainties. In RFX-mod a solidly established procedure involves (optimized) lookup tables from which candidate solutions are sequentially picked up and evaluated against data using the standard χ^2 minimization. This approach proved to be fairly effective on the basis of the experience [1]; however, there is room for alternatives, which may promise improvements regard to, at least, two aspects: (I) minimize the human intervention, with the spin-off of improving the degree of objectivity; (II) provide a more accurate recipe for estimating confidence intervals of estimated quantities.

Genetic Algorithms (GAs) are numerical search tools aiming at finding the global optimum of a given real objective function of one or more real variables, possibly subjected to various linear or non-linear constraints. In this work we apply a Differential Evolution version of the GA (details can be found, e.g., in [2]) to the reconstruction of particle density profiles. The desired output is represented by the particle diffusivity *D(r)* and convection *U(r).* In this work the velocity profile *U(r)* will be constrained in agreement with the theory of transport in a stochastic magnetic field: $U(r) = -0.5 * D(r) * \nabla T_i / T_i$ being T_i the ion temperature profile. Such theory, whose details can be found in [1], has proved to be able to describe the transport regime of RFX-mod discharges. The diffusivity is approximated by a piecewise function:

$$D(r) = \sum_i D_i \,, \quad r_{i-1} < r < r_i \qquad (2)$$

The targets are just the amplitudes D_i, but we varied also the number -and accordingly the widths- of the intervals $[r_{i-1}, r_i]$, in order to assess the sensitivity of the results to these parameters. The radial intervals r_i are not equally spaced: they are denser in the edge region, where density gradient and particle source are located.

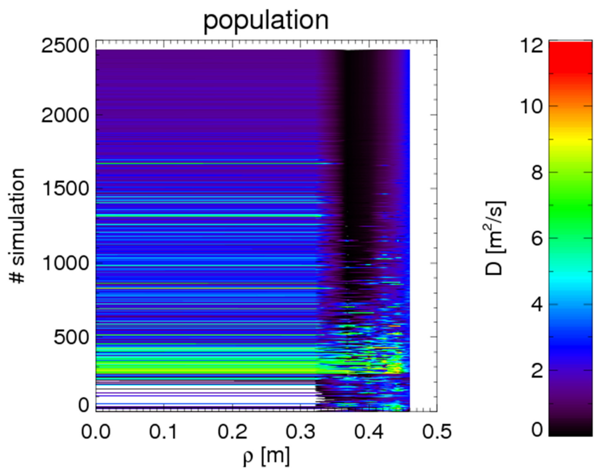

Figure 1: example of evolution of a population: each individual solution is represented by a horizontal line, the colours refer to the D_i magnitude at a specific radius r_i. A new generation starts after every 100 individuals. The convergence towards a common target is suggested by the homogeneity of the population after about 2000 simulations (~200 generations).

For a given choice for D, we compute the predicted density profile from the transport equation, integrate the solution along the experimental lines of sight and compare it with the interferometer measurements. At each stage within an iterative loop, the GA produces a set ("generation") of potential solutions (2), using as input the previous generations (the starting generation being chosen randomly) and feeds them to the transport code, until an optimum D profile, producing the best data reproduction, is identified. Figure 1 shows an example: each generation consists of 100 different candidate solutions. The first generations contain very different specimens, as highlighted by the wide variations in color between successive generations. Eventually, however, they converge towards a stable, optimal solution. The parameterization (2) is very convenient for a rough modelling of profiles, but has the unfortunate drawback of producing discontinuous profiles for transport coefficients and not penalizing unphysical erratic jumps of D between neighboring intervals (see fig. 2). Thus, we find a possible solution to alleviate this shortcoming. We did not minimize χ^2 itself, but rather the functional:

$$H = \chi^2 + \upsilon\, G \,, \ldots G = \sum_i \left(D_{i+1} - D_i\right)^2$$

where G is a penalty function set to avoid exceedingly oscillating solutions, with its weight υ. Eq. (1) was actually implemented and solved using ASTRA code [3] interfaced with a routine producing D guesses from GA.

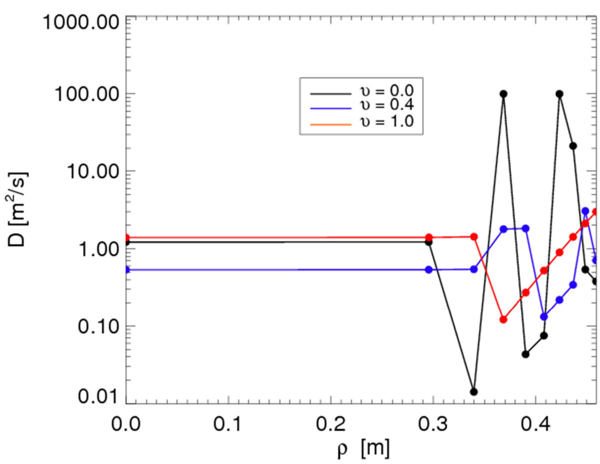

Figure 2: diffusivity profiles for three values of υ.

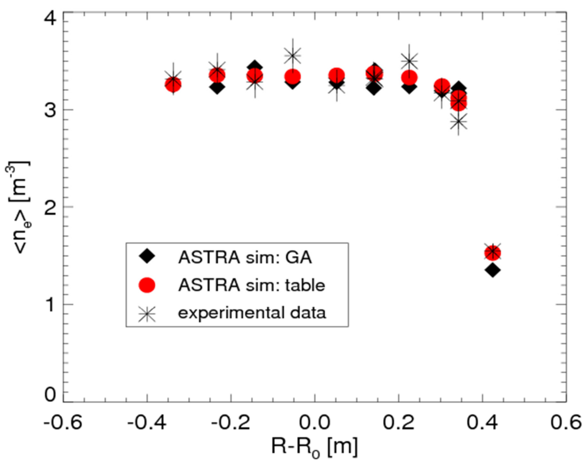

Figure 4: average line integral density: experimental data with corresponding error bars are plotted with asterisks. Black and red full symbols represent respectively the numerical profiles found with the GA and with the lookup-table method

At this stage we performed simulations with different υ, in order to assess its effect and importance. Figure 2 reports three examples of *D* profiles, being smoother and smoother as υ increases. The final χ^2 is comparable between the three cases. Rigorously, υ should be roughly fixed by the constraint that the optimal solution shall have $\chi^2\approx1$. The main outcomes of applying GA's to transport analyses at RFX-mod can be summarized as follows. *(i)* The unsupervised GA method and the supervised traditional look-up table one do yield results of the same quality and provide similar transport level. Figure 3 reports the comparison among the interferometer data of for shot 30056 at 130 ms (black asterisks), the numerical average line integral density for the simulated ASTRA profiles found with the GA (black full diamonds) and the profile obtained with the lookup tables (red full circles): the three sets of data are very close, within the experimental error bars. This is particularly interesting since the two methods do differ also in the spatial structure of the basis functions used to model transport coefficients: the lookup-table method writes (*D,U*) in terms of smooth functions defined globally throughout the whole radius; the GA method employs the piecewise functions (2). (*ii*) Figure 4 shows the *D* profile for data shown in figure 3: the black line represents the profile with the lowest χ^2 computed with the GA, the grey area shows the local confidence intervals (CI's), defined as the envelopes of *D* solutions such that the reconstructed measurements lie within an error bar from the measured ones ($\chi^2 \leq 1$). The CI's are larger in those regions where the true level of transport is not effectively probed due to the lack of sources and gradients: i.e. the core inside about ρ = 0.3 m, in the case of particle density. In the same figure, the red smooth line shows the lookup table diffusivity profile, in good agreement with the GA solution. *(iii)* The conclusion from figure. 4 is that three transport zones may be roughly identified: the core region up to ρ = 0.3 m, a transport

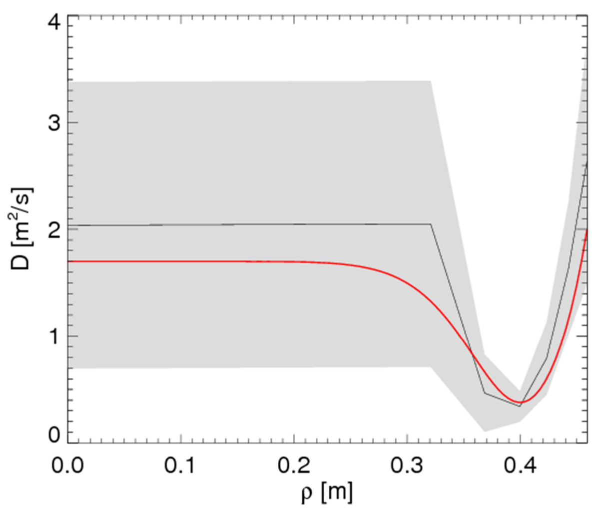

Figure 3: diffusivity profile for shot 30056 at 130 ms computed with υ=0.5, whose experimental data are shown in figure 3. The confidence intervals are shown as grey shaded area. The red smooth line shows the lookup table result.

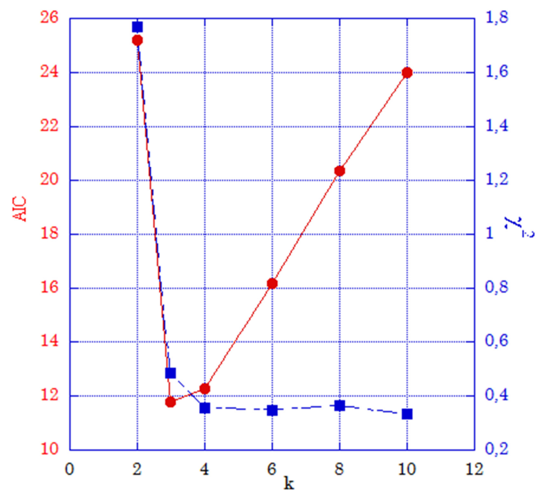

Figure 5: AIC (red symbols) and χ^2 (blue symbols) versus the number of parameters k (Eq. 2).

barrier near to ρ = 0.4 m, a very edge region with high transport. By increasing the number k of intervals $[r_{i-1}, r_i]$ in (2), we refine the spatial resolution of the model, and we may potentially improve the goodness-of-fit. This procedure does not automatically ensure improving the descriptive qualities of the model ,and we use the Akaike Information Criterion (AIC) [4] to identify the optimum number of knots in the solution. The AIC functional is defined as $AIC = 2k + N\chi^2$ (N being the number of interferometer chords). Its minimum value provides the optimal solution as a balance between match of the data (quantified by smaller χ^2) and economy in the description of the model (smaller k). Figure 5 shows the AIC functional and the corresponding χ^2 for different k values: (a) the AIC is minimum for k = 3,4, confirming that a three-zones description is an adequate picture, and (b) above $k \geq 3$ no improvement in the matching of the data is actually meaningful, since $\chi^2 \leq 1$, *i.e.*, data are interpolated to within the experimental errors.

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