Inertia and equilibrium impurity flow in 3D magnetic surfaces.

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Abstract

We derive a compact and exact expression for the parallel inertia of impurity flows in general 3D magnetic surfaces. The form of the flows is constrained to be $E \times B$ in the perpendicular direction but otherwise general in the parallel one. We present computations of parallel inertia for stellarator geometry and several edge conditions and show that, even for the moderate radial electric fields typical of stellarators, inertia-driven impurity density variations cannot in general be neglected for heavy impurities. This is shown to be due to the form of the incompressible streamlines in 3D magnetic surfaces.

Introduction

Recent works in tokamaks and stellerators have investigated the physical causes of uneven distribution of impurities on flux surfaces [1, 2, 3, 4], and how and to what extent they can alter the radial transport of impurities [5, 6]. Impurities are prone to develop these variations because of their large mass and charge. Inertia, in particular, grows linearly with mass and is known to lead to accumulation in the outer mid-board of rotating tokamaks. In stellarators, their typically much smaller flow over thermal velocity ratio is often invoked to neglect inertial forces in the parallel momentum balance.

A quick order of magnitude estimate of the inertia-driven impurity density variation is obtained by balancing parallel inertia term and pressure gradient terms,

$$\frac{2}{v_{tz}^2}\mathbf{B}\cdot(\mathbf{u}_z\cdot\nabla)\mathbf{u}_z+\mathbf{B}\cdot\nabla\log n_z=0\;. \tag{1}$$

If one considers similar scale length in the two nabla terms and a characteristic impurity velocity u_z such that $M_z = u_z/v_{thz} \ll 1$, one gets

$$\log n_z \sim 2M_z^2 \to n_z/\langle n_z\rangle - 1 \sim 2M_z^2\;,$$

which, for typical $M_z \sim 0.1$, results in a few % relative density variation $\frac{n_z}{\langle n_z\rangle}$. Here $v_{tz} = \sqrt{2T_z/m_z}$, and $\langle A\rangle$ denotes the flux surface average of A. Impurities and main ions are considered to be thermally coupled through collisions so that $T_z = T_i$, and the ion temperature is an approximate flux constant $T_i(\psi)$.

In this work we show that, due to the shape of the streamlines of incompressible flows in stellarators, inertia can locally grow to dynamically relevant values in realistic conditions, particularly so for the large E_r values found at the edge transport barrier in H-modes. In particular, the bending of the icompressible streamlines is such that the velocity variation scale-length can become small locally (smaller than or similar to the minor radius) and the velocity large (several times the perpendicular $E \times B$ velocity) due to the parallel Pfircsh-Schlüter components. The return parallel flows resulting from density variations tend to lower the peak values of inertia on a flux surface, smoothing the bending of streamlines and partially compensating the Pfirsch-Schlüeter flow.

Compact expression for the parallel inertia of impurities

We choose to work in magnetic coordinates (ψ, θ, ϕ) in terms of which the magnetic field can be written as $\mathbf{B} = \sqrt{g}^{-1}\left(\iota \mathbf{e}_\theta + \mathbf{e}_\phi\right)$. The flux label ψ is the toroidal flux over 2π, $2\pi\psi = \Psi_T$. A impurity velocity field that is $E \times B$ in the perpendicular direction and otherwise arbitrary in the parallel direction can be written as

$$\mathbf{u} = \omega(\psi)\mathbf{e}_\phi + K(\psi,\theta,\phi)\mathbf{B}, \quad \omega(\psi) = -\iota^{-1}\frac{d\Phi}{d\psi}\ , \tag{2}$$

where Φ is the electrostatic potential. Using form of the impurity flow the parallel inertia is written in Boozer coordinates as

$$\mathbf{B}\cdot(\mathbf{u}\cdot\nabla)\mathbf{u} = \mathbf{B}\cdot\nabla\left(\frac{K^2B^2}{2} - \frac{\omega^2 g_{\phi\phi}}{2}\right) + B^2\omega\frac{\partial}{\partial\phi}\frac{u_\parallel}{B}\ . \tag{3}$$

Note that for axis-symmetric systems ($\mathbf{e}_\phi = R\hat{\phi}, g_{\phi\phi} \equiv \mathbf{e}_\phi\cdot\mathbf{e}_\phi = R^2, \partial_\phi = 0$) Equations 2 and 3 reduce to the known tokamak expressions (see e.g. [1]). In order to obtain initial estimates of the impurity density redistribution driven by inertia we start from the constant-density incompressible flow condition:

$$K = \omega(\chi)\left(\frac{\lambda}{B} - \frac{I}{\langle B^2\rangle}\right) + \frac{\langle u_\parallel B\rangle}{\langle B^2\rangle}\ , \tag{4}$$

where

$$\mathbf{B}\cdot\nabla\frac{\lambda}{B} = -\frac{1}{\sqrt{g}}\frac{\partial\sqrt{g}}{\partial\phi}\ , \text{ and } \langle\lambda B\rangle = 0\ . \tag{5}$$

The tokamak solution is $\lambda/B = 0$.

Figure 1 shows the numerical evaluation of the inertia and the equivalent density variation for TJ-II at $\rho = 0.75$. For the sake of testing the correctness of the inertia expression 3 we compare the values obtained along a stream line with a finite differences approximation of the inertia term given by $2B^2\mathbf{u}\cdot\nabla u_\parallel/B$. Large values of inertia are found to be due to large variation of the parallel flow component along the streamline combined with large values of $u = |\mathbf{u}|$ caused

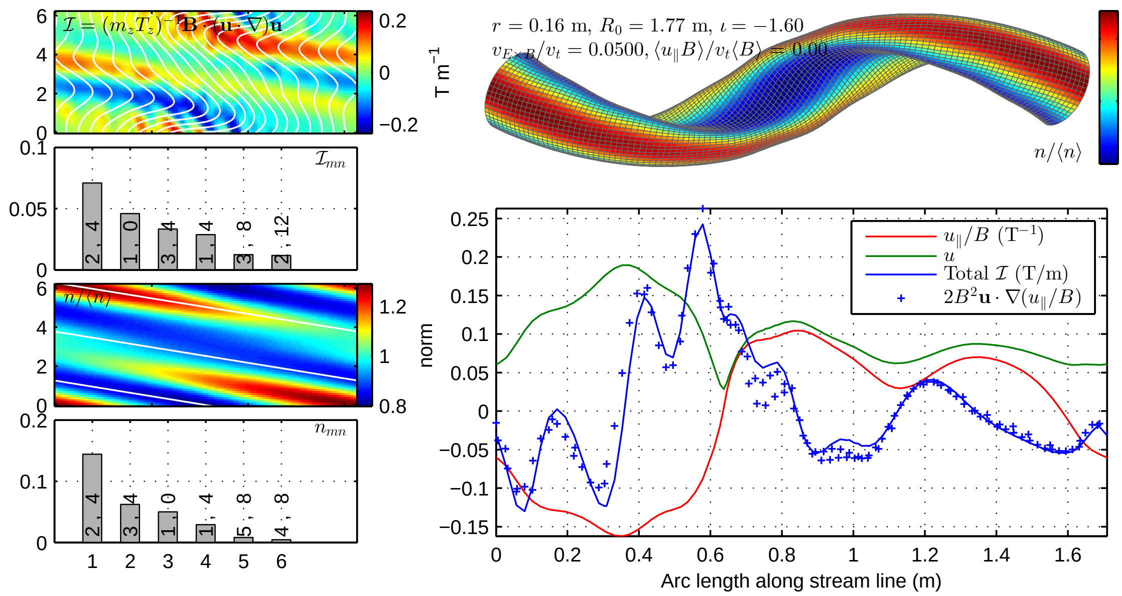

Figure 1: Parallel inertia and equivalent density variation for the $\rho = 0.75$ flux surface of TJ-II. The plot in the lower right corner shows values of $u_\parallel/B, u, \mathscr{I}$ and the finite differences evaluation of approximate inertia expression $2B^2\mathbf{u} \cdot \nabla u_\parallel/B$ along a streamline. Several streamlines of the incompressible flow are shown in the upper-left plot.

by large values of return Pfirsch-Schlüter flows. The equivalent density variation is found by solving the magnetic differential equation 3. To find the actual equilibrium density parallel force balance and particle conservation ($\nabla \cdot n_z \mathbf{u}_z = 0$) need to be solved simultaneously.

Similar calculations are show in figure 2 for the TJ-II, W7-X and LHD configurations in the radial range from $\rho = 0.3$-0.8 and for $v_{EtimesB}/v_{tz} = 10^{-2}$. The exact values shown in bars are compared with different estimates of the parallel inertia term. Although substantial differences are observed between the configurations, the order of magnitude estimate $\mathbf{B} \cdot (\mathbf{u} \cdot \nabla)\mathbf{u} \sim < u^2 > /r$ is a fair approximation except for interior positions in LHD.

The relative importance of inertia effects in the impurity dynamics in stellarator configurations will obviously depend on the specific profiles, most critically on the ratio of $E \times B$ over thermal impurity velocity. Values of this ratio for Carbon impurity, $T_z \sim 0.1$-2 keV and $E_r/B \sim 1$-10 km/s range from $v_{ExB}/v_{tz} \sim 0.005$-$0.25$. Finally, the bulk parallel rotation has not been considered $\frac{\langle u_\parallel B \rangle}{\langle B^2 \rangle}$ here. This term, for which the analytic solution of equation 3 can be easily found, is much less effective in driving inertial density variations.

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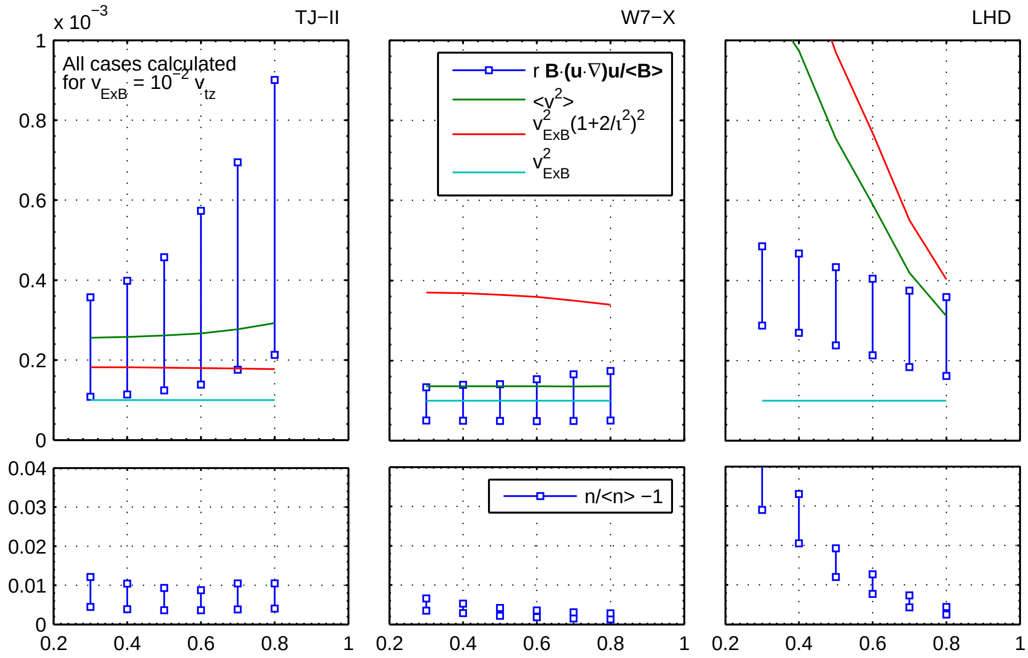

Figure 2: Upper graphs: Typical parallel inertia values for several configurations and radial locations together with different estimates. Lower graphs: equivalent density variations.

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