Development of the multi-fluid transport code on the tokamak flux coordinates

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Introduction

We have developed a one-dimensional fluid-type transport code TASK/TX [1]. The code is essentially based on a self-consistent two-fluid model, which consists of two-fluid equations (conservation of mass, momentum and energy) plus Maxwell's equations. It also involves the equations for beam ions [2] and neutrals [3]. It differs from conventional diffusive transport codes mainly in that: the quasi-neutrality condition $n_\mathrm{e} = \sum_i Z_i n_i$ and the ambipolar flux condition $\sum_a \Gamma_a = 0$ need not be imposed; the flux-gradient relationship is not used for particle transport; the neoclassical features are self-consistently reproduced. A main drawback is, however, that the governing equations are built on the cylindrical coordinates (r,θ,ϕ), which is equivalent to the large aspect ratio limit of a plasma. In this sense, some physics originating from magnetic geometry such as the Pfirsch-Schlüter flux has been dropped. Furthermore, due to the formulation on the cylindrical coordinates, it is difficult to intuitively understand the models and the results of TASK/TX in comparison with the theory and the models developed on the flux coordinates. Hence we derive the governing equations of TASK/TX on the axisymmetric flux coordinates (ρ,θ,ζ) and then numerically implement them.

Hereafter we will expand plasma parameters in terms of a small gyroradius and take into account their non-perturbed (lowest) part. We assume that the drift ordering is appropriate for the momentum equations whereas the transport ordering for the otherwise equations.

Maxwell's equations

On the axisymmetric flux coordinates, the magnetic field is expressed as $\boldsymbol{B} = \nabla\zeta\times\nabla\psi + I\nabla\zeta$, where $I(\psi) = RB_t$. Maxwell's equations consist of Gauss's law (Poisson's equation), Faraday's law and Ampère's law. From Faraday's law $\boldsymbol{E} = -\nabla\Phi - \dot{\boldsymbol{A}}$, it is found that the covariant components of the electric fields are strongly tied to (the temporal change in) the magnetic fluxes: $E_\theta = \boldsymbol{E}\cdot\sqrt{g}\nabla\zeta\times\nabla\rho = -\dot{\psi}_t, E_\zeta = g_{\zeta\zeta}\boldsymbol{E}\cdot\nabla\zeta = \dot{\psi}(= RE_t)$. Ampère's law relates (the spatial gradient of) the magnetic flux to the current. Taking the scalar product of Ampère's law with $\nabla\zeta$ and the subsequent flux-surface average (FSA) yield (cf. FSA Grad-Shafranov equation)

$$\frac{1}{c^2}\frac{\partial\dot{\psi}}{\partial t} = \frac{1}{V'\langle R^{-2}\rangle}\frac{\partial}{\partial\rho}\left[V'\left\langle\frac{|\nabla\rho|^2}{R^2}\right\rangle\frac{\partial\psi}{\partial\rho}\right] - \mu_0\frac{\langle j^\zeta\rangle}{\langle R^{-2}\rangle}, \quad \frac{\partial\psi}{\partial t} \equiv \dot{\psi}, \tag{1}$$

the latter of which is not only the definition of $\dot{\psi}$ but also one of the governing equations that constitutes Maxwell's equations. This is because it is just alternative form of Faraday's law. Taking the scalar product of Ampère's law with $\boldsymbol{B}$ and then subtracting (1) give the equations for the toroidal flux in the form:

$$\frac{1}{c^2}\frac{\partial\dot{\psi}_t}{\partial t} = V'\left\langle\frac{|\nabla\rho|^2}{R^2}\right\rangle\frac{\partial}{\partial\rho}\left[\frac{1}{V'\langle R^{-2}\rangle}\frac{\partial\psi_t}{\partial\rho}\right] + \mu_0\frac{\langle Bj_\parallel\rangle - I\langle j^\zeta\rangle}{\langle B^\theta\rangle}, \quad \frac{\partial\psi_t}{\partial t} \equiv \dot{\psi}_t. \tag{2}$$

When neglecting the displacement current term that is negligibly small, we readily find that the right-hand sides of (1) and (2) reproduce the expressions of the toroidal and parallel currents. Finally, the Coulomb gauge allows us to simply write FSA Gauss's law as follows:

$$\frac{1}{V'}\frac{\partial}{\partial\rho}\left[V'\langle|\nabla\rho|^2\rangle\frac{\partial\Phi}{\partial\rho}\right] = -\frac{1}{\epsilon_0}\sum_a e_a n_a. \tag{3}$$

Continuity equations

A flux-surface-averaged continuity equation is simply given by

$$\frac{1}{V'}\frac{\partial}{\partial t}(V'n_a)\bigg|_\rho + \frac{1}{V'}\frac{\partial}{\partial\rho}[V'n_a\langle(\boldsymbol{u}_a - \boldsymbol{u}_g)\cdot\nabla\rho\rangle] = S_{\mathrm{n}a}. \tag{4}$$

TASK/TX is distinguished from other transport codes in that $n_a\langle(\boldsymbol{u}_a - \boldsymbol{u}_g)\cdot\nabla\rho\rangle$ is not approximated by the convection-diffusion relationship, but is treated as a dependent variable: The grid velocity $\boldsymbol{u}_g$ and the flux $n_a\langle\boldsymbol{u}_a\cdot\nabla\rho\rangle$ are both self-consistently calculated in the system.

Momentum equations

The parallel motion of plasma species regulates neoclassical properties in axisymmetric systems. Using the non-conservative form of the momentum equation, we have up to $O(\delta)$ The lowest order of the viscous term is the neoclassical viscosity. The viscous term and the friction term can be calculated according to the moment approach [4]. Reproducing the driving of poloidal flows, resistivity and the shielding factor of the beam driven current, equations for heat flows have to be simultaneously solved, as is usual for neoclassical transport solvers. Therefore, defining $\langle B\hat{q}_{a\parallel}\rangle = 2\langle Bq_{a\parallel}\rangle/(5p_a)$, the following equations should be solved:

$$m_a n_a\frac{\partial\langle Bu_{a\parallel}\rangle}{\partial t} = -\hat{\mu}_1^a(\langle Bu_{a\parallel}\rangle - BV_{1a}) - \hat{\mu}_2^a(\langle B\hat{q}_{a\parallel}\rangle - BV_{2a}) + \sum_b \ell_{11}^{\mathrm{ab}}\langle Bu_{a\parallel}\rangle - \sum_b \ell_{12}^{\mathrm{ab}}\langle Bu_{a\parallel}\rangle$$
$$+ e_a n_a\langle BE_\parallel\rangle, \tag{5}$$

$$\frac{5}{2}m_a n_a\frac{\partial\langle B\hat{q}_{a\parallel}\rangle}{\partial t} = -\hat{\mu}_2^a(\langle Bu_{a\parallel}\rangle - BV_{1a}) - \hat{\mu}_3^a(\langle B\hat{q}_{a\parallel}\rangle - BV_{2a}) - \sum_b \ell_{21}^{\mathrm{ab}}\langle Bu_{a\parallel}\rangle + \sum_b \ell_{22}^{\mathrm{ab}}\langle B\hat{q}_{b\parallel}\rangle. \tag{6}$$

Note that source and sink terms have been omitted here and hereafter.

The toroidal momentum equation is important for TASK/TX because it includes the $\boldsymbol{v}\cdot\nabla\rho$ term that provokes a particle flux as well as a $\boldsymbol{j}\times\boldsymbol{B}$ torque once losses of beam ions produce the non-ambipolar flux. In conventional transport codes the particle transport coefficients evaluated by external modules are directly substituted in the particle transport equation, whereas in TASK/TX coupling of the equations self-consistently brings about particle transport through the continuity equation. While the first-order viscous stress term vanishes due to the CGL form, the second-order term can be expressed as a combination of a convective momentum flux, a.k.a. inward pinch, and a diffusive one plus a residual stress. We have to add a turbulent force F_a^{QL} that drives a turbulence-induced quasilinear particle flux. The toroidal momentum equation is finally given by, where $\langle\mathcal{L}_a\rangle \equiv m_a n_a \langle R u_{a\zeta}\rangle$,

$$\frac{1}{V'}\frac{\partial}{\partial t}(V'\langle\mathcal{L}_a\rangle) = -\frac{1}{V'}\frac{\partial}{\partial\rho}V'\left[\langle|\nabla\rho|\rangle v_{a\zeta}\langle\mathcal{L}_a\rangle + (u_a^\rho - u_g^\rho)\langle\mathcal{L}_a\rangle - \langle|\nabla\rho|^2\rangle\chi_{a\zeta} m_a n_a \frac{\partial\langle R u_{a\zeta}\rangle}{\partial\rho}\right.$$
$$\left. + \langle\Pi_a^{\mathrm{res}}\rangle\right] + \sum_b \ell_{11}^{\mathrm{ab}}\langle R u_{b\zeta}\rangle - \sum_b \ell_{12}^{\mathrm{ab}}\frac{I}{\langle B^2\rangle}\langle B\hat{q}_{b\|}\rangle + e_a n_a\langle R E_\zeta\rangle + e_a\frac{\partial\psi}{\partial\rho}n_a u_a^\rho. \quad (7)$$

The radial momentum equation or the radial force balance equation is essentially equivalent to the first-order flow within the flux surface. The leading order is $O(1)$ for pressure and Lorentz force terms, and the other terms are practically ineffective. Thus, we obtain

$$0 = -\frac{1}{m_a}\frac{\partial p_a}{\partial\psi}(\langle B^2\rangle\langle R^2\rangle - I^2) - \frac{e_a}{m_a}n_a\frac{\partial\Phi}{\partial\psi}(\langle B^2\rangle\langle R^2\rangle - I^2) + \frac{e_a}{m_a}n_a I\langle B u_{a\|}\rangle - \frac{e_a}{m_a}n_a\langle B^2\rangle\langle R u_{a\zeta}\rangle. \quad (8)$$

Other equations

Additionally the heat transport equation and the equation that connects $\langle R u_{a\zeta}\rangle$ and $\langle u_{a\zeta}/R\rangle$ are solved for each species, but their derivation is omitted. As mentioned above, the three equations for neutrals and the two equations for beam ions, if any, are solved as well.

Numerical results

We here focus on the neoclassical properties in TASK/TX, which implies that the turbulent particle flux is neglected, i.e., $F_a^{\mathrm{QL}} = 0$. For the sake of simplicity, the large aspect ratio plasma with circular cross section is assumed. JT-60U-like plasma parameters are given in the following simulation.

Figure 1 shows the radial profiles of (a) the difference in the charge density and (b) the particle flux for each species, respectively, indicating that automatically the quasi-neutrality is well satisfied and the ambipolar flux condition is satisfied as well. A break of the local charge neutrality in the core region, $(n_\mathrm{e} - n_\mathrm{i})/n_\mathrm{e}$, is of the order of 10^{-7}, which is always neglected in conventional transport codes, but this tiny discrepancy is indispensable to give rise to the radial electric field

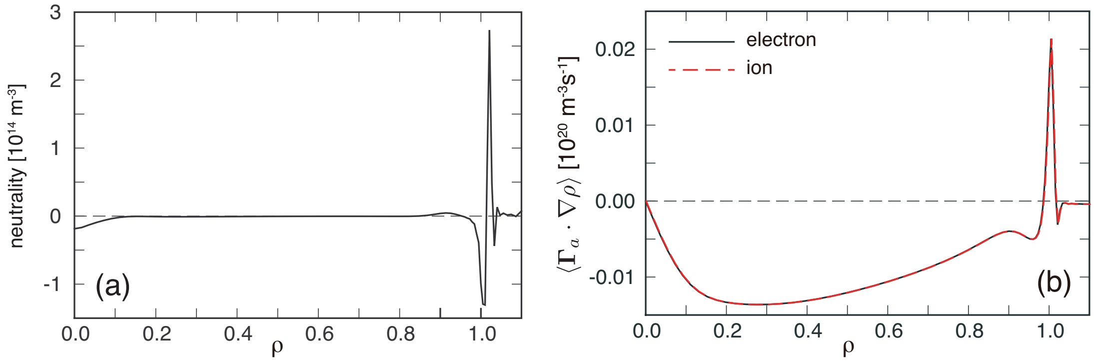

Figure 1: Radial profiles of (a) quasi-neutrality and (b) particle flux for electrons and ions.

E_r through Eq. (3). As obvious in Fig. 1 (b), the cross-field particle flux is automatically ambipolar without imposing ambipolarity. This fact is trivial in the light of neoclassical transport theory because the flux originates from the electric field and friction forces, but the important thing is that it is self-consistently recovered by numerical simulations. The breakdown of the flux cannot be known, but we find that for this case the Ware pinch is dominant over the profile with the aid of NCLASS [5]

Figure 2 displays the comparison of the electron parallel flow $\langle Bu_{e\parallel} \rangle$ calculated in TASK/TX and that by NCLASS: Clearly the good agreement is obtained. In TASK/TX the neoclassical quantities can be internally calculated without external modules such as NCLASS. In this sense, TASK/TX is also a neoclassical transport solver by itself.

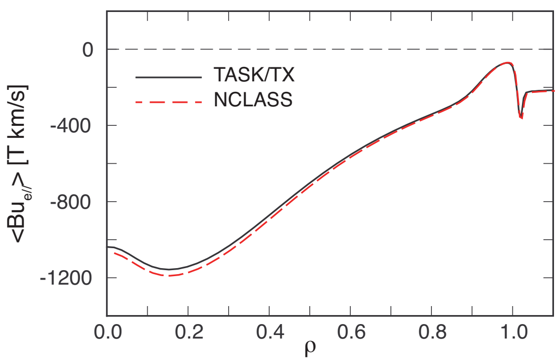

Figure 2: Comparison of $\langle Bu_{e\parallel} \rangle$ calculated in TASK/TX and by NCLASS.

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