# Impacts of sheared toroidal rotation on magnetic island in 3D MHD equilibria

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## 1. Introduction

In most of situations, tokamak equilibria are analyzed as two-dimensional (2D) systems with the axisymmetry. The nature of this symmetry gives many advantages for its analysis. However, as realistic tokamaks have discreteness of the toroidal field coils, this discreteness yields the toroidal field ripples (TF ripples) and, strictly speaking, realistic tokamaks could not be axisymmetric configurations. In previous work[1)], we pointed out the significance of three-dimensional (3D) effects, which are effects of plasma equilibrium currents along rippled field lines.

On the other hand, in recent tokamak experiments, it is noted that stochastic filed lines reduce strong heat load driven by the edge localize mode (ELM) on the divertor plate. Stochastic field lines are produced by the external helical perturbation and it is called the Dynamic Ergodic Divertor (DED). From the viewpoint of high-β stellarator equilibrium, 3D effects on the stochastic field are very important because finite-β perturbed field produces further stochasticity in the peripheral region. However, in present analysis of DED, 2D MHD equilibrium superimposed vacuum helical perturbed field was still used. In order to consider effects of DED to ELM, considerations of finite-β MHD equilibrium and the impact of 3D effects are critical and urgent issue.

In this study, the fully 3D MHD equilibrium of non-axisymmetric tokamak is solved numerically and the impact of the plasma rotation to the 3D MHD equilibrium is discussed. For this study, we use a 3D MHD equilibrium code HINT[2)], which is widely used to analyze the 3D equilibrium in stellarator researches. Since the HINT uses the real coordinate system, it can treat magnetic island and stochastic field in the computational domain. Thus, as first step, we study the 3D MHD equilibrium including the toroidal rotation. Special attention is the change of the magnetic island due to the toroidal plasma rotation.

## 2. 3D MHD equilibrium calculation including toroidal rotation

At first, we discuss the improvement of the HINT code to include the toroidal rotation. vacuum field in the ITER. The HINT code is a 3D MHD equilibrium calculation code, which is based on the relaxation method. Since the HINT code uses the real coordinate system, which is the cylindrical coordinate, the code can capture the magnetic island and stochastic magnetic field lines in the calculation. The HINT code had been developed for stellarator and heliotron researches and the original version of the code adopted a non orthogonal coordinate system, so-called the rotating helical coordinate system. The HINT code had been updated successfully to the HINT2 code and that code applied to the tokamak calculation with 3D perturbation fields, which are the toroidal field ripple, 3D error field and resonant magnetic perturbation (RMP) fields. However, up to now, the 3D MHD equilibrium is calculated as the magnet static equilibrium. Recently, effects of the plasma rotation to the RMP field, which are shielding and amplification of RMPS, are hot topics in ELM suppression and mitigation experiments. To understand those effects, including the plasma rotation to the 3D MHD equilibrium calculation is urgent issue. In this section, the implementation how to include the plasma rotation is shown.

In this study, only the toroidal rotation is studied for simplicity. The toroidal rotation is prescribed by the function of the toroidal flux and the toroidal flow velocity is defined by the Mach number,

$$M(s) = \frac{v_{\phi(s)}}{v_{th}},$$

where $v_\phi$ is the toroidal flow velocity and $v_{th}$ is the ion thermal velocity. The toroidal flow is prescribed as the function of the normalized toroidal flux, $\mathrm{s} = \Phi/\Phi_{edge}$. The $\Phi_{edge}$ is the toroidal flux on the last closed flux surface.

The HINT code consists of two parts. First part, step-A, is the relaxation process of the plasma pressure with fixed the magnetic field. Second part, step-B, is the relaxation process of the magnetic field with fixed the plasma pressure. The step-A calculates the pressure distribution satisfying the condition, $\vec{B} \cdot \nabla p = 0$. Instead of calculating that condition, the step-A calculates an averaged plasma pressure along a magnetic field lines, because the condition means no variation of the plasma pressure along the magnetic field lines. Details is shown in Ref. For a case of existing the toroidal flow velocity, the pressure distribution shifts to the outward of the torus by the inertial force. In such a case, from the parallel force balance, the pressure

distribution is prescribed by

$$p(s,R) = p\exp(M^2(s)(\frac{R^2}{R_0^2} - 1)).$$

On the other hand, the step-B calculates the time evolution of nonlinear dissipative MHD equations. In these equations, the magnetic field and plasma flow velocity are given by $\vec{v} = \overrightarrow{v_0} + \overrightarrow{v_1}$ and $\vec{B} = \overrightarrow{B_0} + \overrightarrow{B_1}$. Here, $\overrightarrow{B_0}$ is the vacuum magnetic field and $\overrightarrow{B_1}$ is the equilibrium response field. The $\overrightarrow{v_0}$ is a given toroidal flow velocity and $\overrightarrow{v_1}$ is the MHD velocity. Thus, dissipative MHD equations are

$$\frac{\partial \overrightarrow{v_1}}{\partial t} = -\overrightarrow{v_0} \cdot \nabla \overrightarrow{v_0} - \nabla p + \overrightarrow{J_1} \times (\overrightarrow{B_0} + \overrightarrow{B_1}) + \nu \nabla^2 \overrightarrow{v_1},$$

$$\frac{\partial \overrightarrow{B_1}}{\partial t} = \nabla \times ((\overrightarrow{v_0} + \overrightarrow{v_1}) \times (\overrightarrow{B_0} + \overrightarrow{B_1}) - \eta(\overrightarrow{J_1} - \overrightarrow{J_{net}})),$$

$$\overrightarrow{J_1} = \nabla \times \overrightarrow{B_1}.$$

The spatial derivation is approximated by 4th order central finite difference scheme and time marching is calculated by the 4th order Runge-Kutta-Gill scheme. Calculating those two steps iteratively, a steady-state solution is obtained.

**3. Model Calculation of an ITER-like plasma**

In this study, we study a tokamak with a D-shape cross section, which is a comparable to the ITER. In that configuration, the major radius $R_0$ is 3 m and the plasma minor radius $a$ is 1 m.

To study effects of the plasma rotation to the magnetic island, *n*=3 perturbed field is superposed to the plasma. In this configuration, important rationals are *m*/*n* = 2/1 and 3/1. Figure 1 (a) shows a puncture map of magnetic field lines for the *M*=0 case, which is zero plasma rotation. *m* = 3, 4 and 5 island chains appear in the plasma. Special notice is a phase of *m*=3 islands. O-points of *m*=3 islands appear at $\rho$ = 0.78.

On the other hand, figure 1 (b) shows a puncture map of magnetic field lines for the *M*=0.3 case. In this study, the toroidal rotation is approximated by a function, $v_\phi(s) \propto (1 - s^7)$. The toroidal rotation stretches the magnetic island. Widths of *m*=4 magnetic islands are almost identical to the *M*=0 case. However, the phase of *m*=3 islands slip poloidally compared with the *M*=0 case. In addition, the width a little bit shrinks. From these results, the shielding of the RMP is very weak. These results are comparable to other results. Similar studies were done by ANIMEC[3] and NIMROD[4] code. In that result, the shielding of the plasma rotation is very weak but the phase slip appeared in the NIMROD simulation. These are preliminary results but

further results will be discussed in near future.

*(a) M=0* *(b) M=0.3*

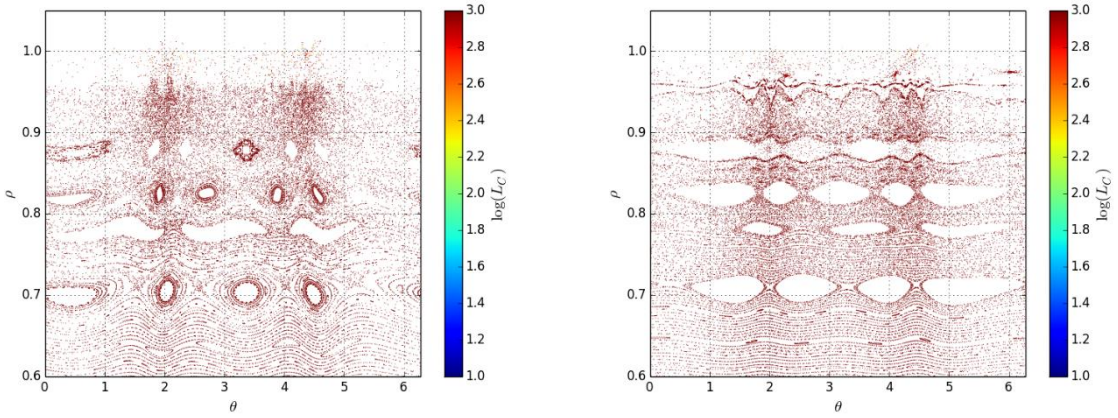


*Figure 1 Puncture maps of magnetic field lines for (a) M=0 and (b) M=0.3 are shown. Pressure and current density profiles are same in both case.*

## 4. Summary

The 3D MHD equilibrium including the plasma rotation is studied for the simple tokamak with circular cross sections. To calculate the 3D MHD equilibrium with the plasma rotation, the 3D MHD equilibrium calculation code, HINT, is improved. In this study, impacts of the toroidal rotation to the magnetic island are studied. If n=3 perturbed field superposes, m=3 and 4 islands appeared. For the M=0.3, the width of magnetic islands is decreased comparing with the zero rotation case. However, m=4 islands slip poloidally but the width is almost identical. Detailed discussions will shown in near future.

1) Y. Suzuki, Y. Nakamura and K. Kondo, Nucl. Fusion **43** (2003) 406

2) Y. Suzuki *et al*., Nucl. Fusion **46** (2006) L19

3) W. A. Cooper, *et al*., 40th EPS Conference on Plasma Physics, (2013) P4.124

4) C.Hegna, *et al*., in private communication.