Turbulent fluctuations and unstable drift wave in plasma sheared flow

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Introduction

In magnetized plasmas, shear flows reduce the turbulent transport and improve confinement [1–3]. Experimental studies of plasma turbulence in magnetic traps clearly show the relationship of perturbation with drift instabilities [3, 4]: Ion Temperature Gradient (ITG) mode, Trapped Electron Mode (TEM) and, under certain conditions, Electron Temperature Gradient (ETG) mode. Drift-wave turbulence contains spatially coherent structures. A coherent, monochromatic drift wave can drive a band of self-sustaining fluctuating modes. Radially extended convective structures are effective in turbulence spreading. Drift turbulence theories consider decrease of fluctuations as a result of decorrelation of relatively large perturbations under the influence of the stationary or quasi-stationary shear flow [5, 6].

Experimentally established condition for significant reduction in transport is [7]

$$\gamma_s > \gamma, \tag{1}$$

Here γ is the typical growth rate of the instability (ITG mode usually), γ_s is the shear parameter, $\gamma_s = \partial V / \partial x$ in a slab, $\gamma_s = r\partial(V / r)/\partial r$ in cylindrical geometry, V is the flow velocity.

In the plasma with no sheared flow, diffusivity corresponds the estimation $D_{\perp 0} = k_\perp^{-2}\gamma$, where $k_\perp$ is the typical wave number, γ the typical growth rate [8]. The effect of shear flow leads to dependence [9]

$$D_\perp = D_{\perp 0} / [1 + (\gamma_s / \gamma)^2]. \tag{2}$$

Studies of turbulent fluctuations [5, 6] revealed the ratio of the root mean square of the fluctuation level for cases with $\gamma_s \neq 0$ and with $\gamma_s = 0$. In the last case, fluctuation decrease can be explained by the other factors that limit the growth of large perturbations [3].

In the present work, we consider the growth of initial perturbation from the small amplitude harmonic drift wave to the conventional decay. Such decay doesn't mean the disappearance of the wave. It is strong distortion of the wave compared to the original harmonic form, so it causes the non-uniformity of the environment and makes the further existence of the wave impossible. The corresponding amplitude value is typical in terms of limitation of the linear growth of the perturbation and the transition to the nonlinear stage.

The main purpose of the present work is analysis of the decay condition which is adequate to the usually observed turbulence level in magnetized plasmas with the sheared flows.

The calculation model

At the initial state amplitude is assumed to be infinitesimal, but during the growth it is considered as finite. For quantitative estimates we use the linear growth rate of ITG mode [10–12]. The influence of flow shear on the fluctuation level is determined by varying the ratio of shear parameter γ_s to growth rate γ. Wave tilting is not considered, as for the drift waves it means a discontinuity of the medium. It is assumed that the state called as the decay corresponds to the close conditions. Turbulent fluctuation level is associated with amplitude value before the decay stage.

Drift wave is considered in slab geometry. The magnetic field is directed along the z-axis, the wave propagates in the y-direction, the plasma density and temperature decrease in the x-direction, the flow velocity V depends on x. Waveform is assumed to be independent of x within the layer under consideration. Linear dependence $V(x) = \gamma_s x$ approximates flow velocity in a moving coordinate system. At the initial time waveform $X(y, t)$ is a sine function $X = a_i \sin(k_y y)$, where a_i is the initial amplitude, $k_y = k_\perp$ is wave number. Boundary conditions are the following: $X = 0$ at $y = \pm\pi/k_y, \pm 2\pi/k_y, \pm 3\pi/k_y, ...$

The growth of the density perturbations $n^\sim$ is characterized by a constant growth rate as follows: $dn^\sim / dt = \gamma n^\sim$. Deformation field $X(y, t)$ is associated with the perturbed density $n^\sim$ using the relation $n^\sim = -(n_0 / L_n)X$, where $dn_0 / dx = -n_0 / L_n$ is unperturbed density gradient, n_0 is unperturbed density, L_n is gradient scale length. For the electron density perturbation the Boltzmann approximation is performed: $(n^\sim / n_0) = (e\varphi / k_B T)$, where e is the electron charge, k_B is the Boltzmann constant, T is electron temperature. Take into account that drift-wave fluctuations are quasi-neutral.

Since the perturbed electron density and wave potential are proportional to the deformation of the medium, then all of the three values $\delta n \sim \varphi \sim X$ are described by the equation

$$\frac{dX}{dt} \equiv \frac{\partial X}{\partial t} + \gamma_s X \frac{\partial X}{\partial y} = \gamma X + D \frac{\partial^2 X}{\partial y^2} . \quad (3)$$

It is assumed here that $v_y = V(X) = \gamma_s X$. The diffusion operator is introduced in the right-hand side of Eq. (3) to simulate the saturation of the perturbation using effective diffusivity D.

Eq. (3) refers to the same type of equations that the Burgers equation [13], which partial solutions are known. Eq. (3) with zero right-hand side matches the cases of the drift motion of particles. In that form it coincides with the equation of nonlinear drift waves [14]. Such an equation also describes the perturbation in the beam of non-interacting particles [13].

Results of the analysis

Fig. 1 shows the evolution of the perturbations at $D = D_\perp$, where diffusivity $D_\perp$ is calculated according Eq. (2). To consider the effect of the shear separately, we also examined the case $D << D_\perp$. Fig. 2 shows the growth of the amplitude $a = \max(|X|)$ for the two cases: $D = D_\perp$ and $D = 10^{-2}D_\perp$.

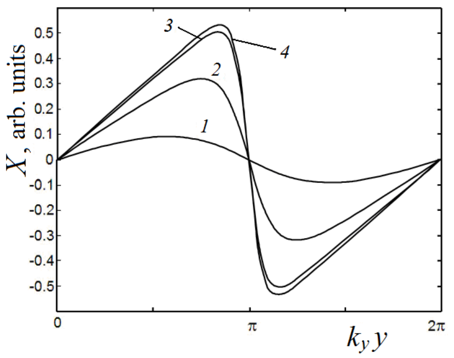

Fig. 1. Temporal evolution of the wave profile at $D = D_\perp$, $\gamma_s/\gamma = 1.5$: 1 – $\gamma t = 3$; 2 – 5; 3 – 7; 4 – 20.

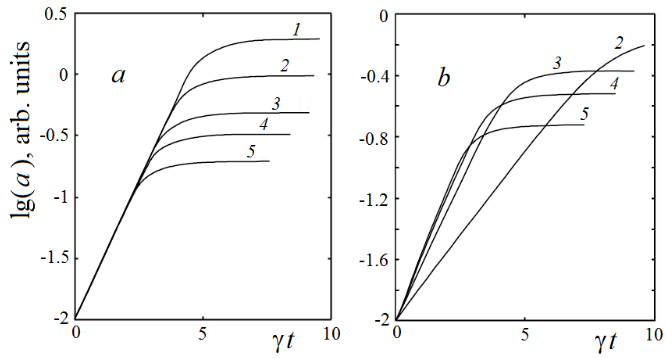

Fig. 2. Amplitude growth at $D = D_\perp$ (*a*) and $D = 10^{-2}D_\perp$ (*b*): 1 – $\gamma_s/\gamma = 0.5$; 2 – 1; 3 – 2; 4 – 3; 5 – 5

At the initial time the amplitude grows linearly. Over time, wave profile tends to a stationary form. It is assumed that the conditions for the existence of the wave no longer hold to the end stage of linear growth. At this time, the amplitude corresponds to the approximate equality of the perturbed and unperturbed gradients [15]:

$$\partial \tilde{n} / \partial y \sim dn_0 / dx. \tag{4}$$

This relationship is considered as a quantitative decay conditions. With no shear flows $k_y\delta n \approx n_0 / L_n$, where δn is perturbed density amplitude. It is associated with a wave amplitude a as follows: $\delta n = (n_0 / L_n)a$. An estimate of the amplitude at $\gamma_s = 0$ is

$$a_0 = (\delta n / n_0)L_n = 1/ k_y, \tag{5}$$

that corresponds to the experimental data [3, 15].

In our model, when $\gamma_s = 0$ perturbation growth is limited by the diffusion operator. The equality to zero of the right-hand side of Eq. (3) corresponds to the relationship $\gamma \approx D / a^2$. Eq. (5) corresponds to diffusivity $D = D_\perp$ calculated by Eq. (2). Using Eq. (2) for $\gamma_s \neq 0$ obtains

$$a^2 / a_0^2 = 1 / [1 + (\gamma_s / \gamma)^2]. \tag{6}$$

The results of calculations using the formulated decay criteria are shown in Fig. 3. As seen from Fig. 3, calculation results for $D = D_\perp$ practically coincide with the Eq. (6). This coincidence favors the diffusion mechanism of dissipation, at the same time as leading to the plasma transport and to a smoothing of the perturbation profile. We considered the case $D << D_\perp$ also.

Conclusions

The analysis showed two regimes with different mechanisms perturbation growth limitation. In the low shear (or no shear) case, dissipative mechanism dominates. It can be presented by the diffusion operator. At strong shear, perturbation growth is limited by deformation of perturbed medium. These mechanisms are in accordance with the Burrell's experimental criterion (Eq. (1)) and result of the theory of Dupree and Itohs (Eq. (2)).

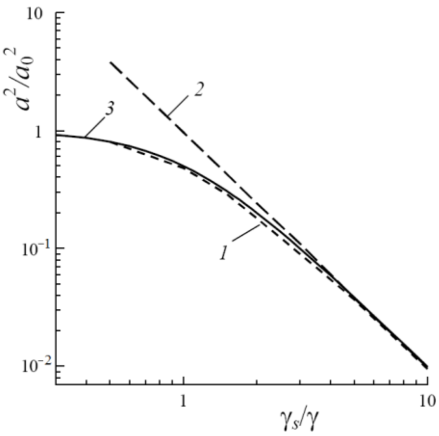

Fig. 3. The relative perturbation amplitude versus the shear parameter: 1 – simulations with $D = D_\perp$; 2 – simulations with $D = 10^{-2}D_\perp$; 3 – Eq. (6)

The analysis is based on the evolution of a single-mode perturbation while really many modes coexist in turbulent medium. The estimates of the perturbation amplitudes (Eqs. (5) and (6)) involve the use of some typical parameters of instability with a certain wave number k_y. Such an estimate gives an average result, not a spectrum.

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