Ion-acoustic rogue waves in a magnetized plasma with a q-nonextensive electron velocity distribution: Application to solar wind plasma.

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1. Introduction

Ion-acoustic rogue waves (IARW) are addressed in a two-component plasma composed of positively charged fluid ions, as well as electrons obeying the q-nonextensive distribution function in the presence of the external magnetic field **B** with application to the solar wind. The basic set of hydrodynamic equations is presented, and the nonlinear Schrödinger (NLS) equation is derived by using the reductive perturbation method. The electron nonextensivity, the obliqueness and the magnitude of the magnetic field are found to modify the properties of the rogue waves. Our results may aid to explain and interpret the nonlinear oscillations (IARW) that may occur in the solar wind plasma.

2. Theoretical model

Let us consider a collisionless, magnetized plasma having ions fluid, nonextensive electrons of densities n_i and n_e, respectively. The ion fluid velocity $\mathbf{V_i}=V_{ix}\,\mathbf{e_x}+V_{iy}\,\mathbf{e_y}+V_{iz}\,\mathbf{e_z}$ and the external magnetic field $\mathbf{B}=B_0\cos\theta\,\mathbf{e_x}+B_0\sin\theta\,\mathbf{e_z}$. We assume that the wave length $\lambda \ll$ the s-species particle gyro- radius $\rho_s=Vts/\Omega_s$, (s=e,i) where $V_{ts}=(T_s/m_s)^{1/2}$ and $\Omega_s=eB_0/m_s$ is the thermal velocity and the gyration frequency of s- species particle, respectively. The nonlinear dynamics of the ion- acoustic waves (IAWs), whose phase speed is much smaller (larger) than the electron (ion) thermal velocity, is governed by the normalized basic equations

$$\frac{\partial N_i}{\partial T}+\frac{\partial (N_i V_{ix})}{\partial X}=0 \tag{1}$$

$$\frac{\partial V_{ix}}{\partial T}+V_{ix}\frac{\partial V_{ix}}{\partial X}=-\frac{5\sigma}{3}N_i^{-\frac{1}{3}}\frac{\partial N_i}{\partial X}+\omega_{ci}V_{iy}sin\theta-\frac{\partial \Phi}{\partial X}. \tag{2}$$

$$\frac{\partial V_{iy}}{\partial T}+V_{ix}\frac{\partial V_{iy}}{\partial X}=-\omega_{ci}[V_{ix}sin\theta-V_{iz}cos\theta]. \tag{3}$$

$$\frac{\partial V_{iz}}{\partial T}+V_{ix}\frac{\partial V_{iz}}{\partial X}=-\omega_{ci}V_{iy}cos\theta. \tag{4}$$

$$\frac{\partial^2 \Phi}{\partial X^2}=N_e-N_i. \tag{5}$$

With [1] $$N_e = [1 + (q-1)\Phi]^{\frac{1}{q-1}+\frac{3}{2}}. \quad (6)$$

The parameter q stand for the strength of electron nonextensivity. $\sigma=T_i/T_e$, T_i and T_e are the ion and electron temperatures, respectively. The density and velocity are normalized by the unperturbed ion density n_{i0} and ion-acoustic speed $C_{si}=(k_BT_e/m_i)^{1/2}$, respectively. The electrostatic potential Φ is normalized by the thermal potential k_BT_e/e; e is the magnitude of the electron charge. The time T and the space X variables are normalized by the ion Debye length $\lambda_{Di}=(k_BT_e/4\pi e^2n_{i0})^{1/2}$ and the ion plasma period $\omega_{pi}^{-1}=(m_i/4\pi e^2n_{i0})^{1/2}$, respectively. To examine the modulation of the ion-acoustic waves (IAW) propagating in our plasma, we employ the reductive perturbation technique to derive the nonlinear NLS equation. The independent variables [2]- [6] $\xi=\varepsilon(X-V_gT)$ and $\tau=\varepsilon^2T$ where the small parameter ε is to be regarded as a measure of the strength of the nonlinearity and V_g is the group velocity of the wave to be determined later. Furthermore, all physical quantities appearing in Eqs. (1- 6) are expanded as $F(X,T) = F_0 + \sum_{m=1}^{\infty} \varepsilon^m \sum_{l=-\infty}^{l=\infty} F_l^{(m)}(\xi,\tau)e^{il\Theta}$, where $\Theta=kX-\omega T$, $F_l^{(m)} = [N_{il}^{(m)}, V_{ixl}^{(m)}, V_{iyl}^{(m)}, V_{izl}^{(m)}, \Phi_l^{(m)}]$ and $F_l^{(0)} = [1,0,0,0,0]^T$. Here k and ω are real variables representing the fundamental (carrier) wave number and frequency, respectively. $F_l^{(m)}$ satisfies the reality condition $F_l^{(m)}= F_l^{(m)*}$ and the asterisk denotes complex conjugate. Substituting these expression along with stretching coordinate into Eqs. (1)- (5) and collecting terms of the same power of ε. The first order approximation (m=1) with the first harmonic (l=1) yields the following dispersion relation

$$D(k,\omega) = 3\omega^4 + \omega^2[-3\omega_{ci}^2 - 5k^2\sigma - 3k^2/(k^2+s)] + \omega_{ci}^2 cos^2\theta[5k^2\sigma + 3k^2/(k^2+s)].$$

Whereas the second order approximation (m=2) with the first harmonic (l=1) gives the expression of the group velocity

$$V_g = \frac{\partial\omega}{\partial k} = \frac{2\alpha^2[5\sigma(k^2+s)^2+3s]}{3(k^2+s)^2[2\omega\alpha^2 + \omega\omega_{ci}^4 sin^2\theta cos^2\theta]}$$

Where s=(3q-1)/2 and $\alpha = \omega^2 - \omega_{ci}^2 cos^2\theta$. The third-order (m=3) with (l=1) gives an explicit compatibility condition from which one can derive the following nonlinear Schrödinger (NLS) equation

$$i\frac{\partial\Phi}{\partial\tau} + \frac{P}{2}\frac{\partial^2\Phi}{\partial\xi^2} + Q|\Phi|^2\Phi = 0. \quad (7)$$

We have denoted $\Phi_1^{(1)}=\Phi$. The dispersion and nonlinearity coefficients denote respectively P and Q are given by

$$P = -\frac{2V_gP_1}{\omega k^2[10\sigma(k^2+s)^2+3(k^2+s)-3V_g(k^2-s)]} \quad (8)$$

$$Q = -\frac{3V_g(k^2+s)^2Q_1}{[10\sigma(k^2+s)^2+3(k^2+s)-3V_g(k^2-s)]} \quad (9)$$

The NLS equation (Eq. 7) has a rational solution that is located on a nonzero background and localized in τ and ξ directions [6] as

$$\Phi = \sqrt{\frac{P}{Q}}\left[\frac{4(1+2iP\tau)}{1+4P^2\tau^2+4\xi^2}-1\right]e^{iP\tau}.$$

3. Numerical results and discussion

For the numerical analysis of the present findings, the typical parameters of the solar wind [7] are be considered.

$r/r_\odot$	215	10	1.5	1.03
n(m^{-3})	7 x 10^3	4 x 10^9	1.6 x 10^{13}	2 x 10^{14}
B(*Tesla*)	5 x 10^{-9}	10^{-6}	0.5 x 10^{-4}	10^{-4}

Here r and $r_\odot$ represents the distance Sun-Earth and the solar radius, respectively, which is $r_\odot \simeq 7\times10^8$ meters. The variation of P/Q as a function of k for different values of nonextensive parameter q is shown in Figure **1**. This figure shows that the instability sets in when $k<k_c$ and the wave remains stable when $k>k_c$. Besides, it can be seen that the critical value k_c, where the instability sets in $P/Q\to\pm\infty$, increases when the nonextensive parameter increases ($q\to1$), i. e., as the electrons evolve toward their thermodynamical equilibrium. The influence of the electrons nonextensivity on the IA rogue waves is displayed in Figures **2** and **3** where we have plotted the wave envelope Φ for nonextensive parameter q=0.9 (Fig. **2**) end q=0.8 (Fig. **3**). Our result depicts that, the amplitude of rogue pulses decreases with the increase of q ($q\to1$), when the electrons evolve toward their thermal equilibrium. The profile of the ion-acoustic rogue waves, which may propagate in solar wind plasma's parameters within the unstable region, for the different physical parameters corresponding to a different distance ($r/r_\odot$) away from the Sun are depicted in the figure **4** for time τ=0. It can be seen that the amplitude of the IA rogue pulses increases with the increase of the distance $r/r_\odot$. As the plasma moves away from the Sun, it would lead to grow the amplitude of the rogue waves.

4. Conclusion

To conclude, we have investigated the modulational instability of the envelope ion-acoustic waves in a two-component plasma composed of positively charged fluid ions, as well as nonextensive electrons in the presence of the external magnetic field B_0, with application to the solar wind. By using the fluid model and employing the reductive perturbation method, a nonlinear Schrödinger equation has been derived. Our results show that the rogue waves may propagate for plasma parameters within the unstable region. Interestingly, we found that the IA rogue waves may be affected by the electrons nonextensivity depending on whether the parameter q. We also found that as the solar wind moves away from the Sun, a significant concentration amount of energy makes the pulses taller. Our investigation may be of wide relevance to astronomers and space scientists working on the solar wind and interstellar plasmas where nonthermal distributions are turning out to be a very common and characteristic feature.

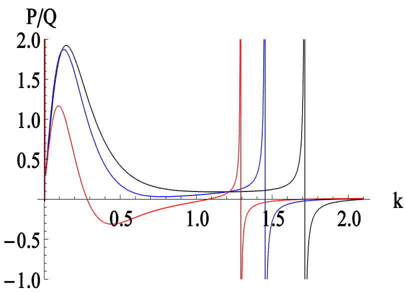

Figure 1: Variation of P/Q with k for q=1 (black line), q=0.85 (blue line) and q=0.7 (red line). For the physical parameters of the solar wind to a distance $r/r_{\odot}=215$.

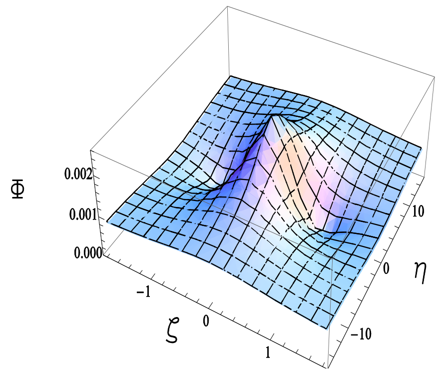

Figure 2: The rogue wave profile Φ vs ξ and τ for q=0.9, σ=0.2, k=1.2, θ=20° and $r/r_{\odot}=215$.

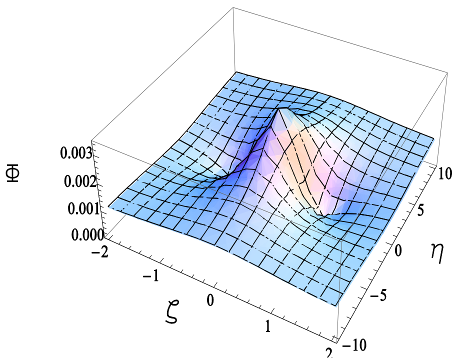

Figure 3: The rogue wave profile Φ vs ξ and τ for q=0.9, σ=0.2, k=1.2, θ=20° and $r/r_{\odot}=215$.

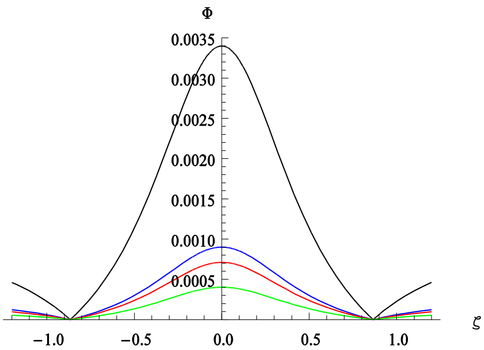

Figure 4: The special profile rogue wave Φ vs ξ for τ=0, q=0.8, σ=0.2, k=1.2, θ=20° for different $r/r_{\odot}=215$ (black line), $r/r_{\odot}=10$ (blue line), $r/r_{\odot}==1.5$ (red line) and $r/r_{\odot}=1.03$ (green line).

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