

# Analytical evaluation of the forces during fast current quenches in tokamaks

V.D. Pustovitov

*National Research Centre Kurchatov Institute and National Research Nuclear University*

*MEPhI (Moscow Engineering Physics Institute), Moscow, Russia*

**1. Introduction.** Recently a problem of the sideways forces during and because of the plasma disruptions in tokamaks became a hot topic of theoretical debates [1–3]. Discussions of the related aspects and comparison of some predictions with experimental data from JET can be found in [4–6]. The problem belongs to the research priorities of ITER studies because significant uncertainties remain in quantitative modeling of the electromagnetic loads associated with unmitigated disruptions in ITER.

Several mechanisms are discussed [1–6] that can contribute to the force, but the approaches are still fragmentary and incomplete. For example, the opinion that “the repelling force between the plasma and the vessel is of little significance in asymmetric events” [5] is completely opposite to the reasoning in [2]. Also, there is no yet agreement on the key elements of the process and boundary conditions in the task [1–3, 6].

Here the model of electromagnetic interaction between the current carrying systems [5] is re-examined and the estimates of the forces during fast current quenches in tokamaks are presented. In [5], the eddy current forces have been neglected, but here we take them into account and show that their contribution into the integral force can be large.

**2. Formulation of the problem.** Our analysis is essentially based on classical results of the plasma equilibrium theory [7, 8]. In those papers (see also [9]), the force balance has been calculated for a tokamak treated as a system of three interacting elements: plasma, toroidal field coils and poloidal field coils. Here we add the fourth element, the wall with a current either dynamically induced or resulting from the halo currents. Another difference from [7–9], where a static equilibrium has been examined, is that we consider the current quench stage of the tokamak disruptions. This means a fast evolution of the plasma with its inevitable motion. Though this is far from equilibrium, the plasma inertia plays a minor role and can be neglected compared to the electromagnetic forces described by  $\mathbf{j} \times \mathbf{B}$ , where  $\mathbf{B}$  is the magnetic field and  $\mathbf{j} = \nabla \times \mathbf{B} / \mu_0$  is the current density. In ITER, the current quench duration  $\tau_{iq}$  is expected to be about 36 ms [10], while the resistive wall time is estimated as  $\tau_w \approx 0.34$  s [11]. With  $\tau_{iq} \ll \tau_w$ , the fast change in  $\mathbf{B}$  inside the tokamak chamber will be shielded by the currents induced in the chamber wall. Here we assume finite  $\tau_{iq} \partial \mathbf{B} / \partial t$  inside, but with  $\partial \mathbf{B} / \partial t = 0$  in the outer region.

**3. Calculation of the force.** Here we calculate the ballooning force on the wall,

$$F_w \equiv \int_{wall} (\mathbf{j} \times \mathbf{B}) \cdot \mathbf{e}_R dV, \quad (1)$$

where  $\mathbf{e}_R = \nabla R$  is the unit vector along the major radius. This requires  $\mathbf{j}$  in the wall, but the integral nature of (1) and careful account of the interaction between all 4 elements of the system allows estimation of  $F_w$  without knowledge of  $\mathbf{j}$ .

The radial force balance in this toroidal system is

$$F_p + F_w = F_{\Sigma\Sigma} + F_{\Sigma c} + F_{\Sigma t}, \quad (2)$$

where

$$F_p \equiv \int_{plasma} (\mathbf{j} \times \mathbf{B} - \nabla p) \cdot \mathbf{e}_R dV \quad (3)$$

is the force on the plasma (its pressure  $p$  can be neglected at a current quench), and

$$F_{ik} \equiv \int_i (\mathbf{j}_i \times \mathbf{B}_k) \cdot \mathbf{e}_R dV_i \quad (4)$$

is the integral force acting on the element  $i$  (with current density  $\mathbf{j}_i$ ) from the element  $k$  (creating the magnetic field  $\mathbf{B}_k$ ). Accordingly,  $\Sigma$  means wall ( $w$ ) plus plasma ( $p$ ),  $c$  the poloidal coils, and  $t$  the toroidal coils. A force  $F_{ii}$  describes the interaction of the current  $\mathbf{j}_i$  with the field  $\mathbf{B}_i$  produced by the same current.

It is clear that (with or without plasma-wall contacts)

$$F_{\Sigma\Sigma} = F_{pp} + F_{ww} \quad (5)$$

because always  $F_{pw} + F_{wp} = 0$ . If  $\partial \mathbf{B} / \partial t = 0$  in the outer region, the forces on the coils must remain constant during the quench. Therefore

$$F_{\Sigma c} + F_{\Sigma t} = F_{pc}^0 + F_{pt}^0, \quad (6)$$

where  $F_{pc}^0$  and  $F_{pt}^0$  are the values of  $F_{pi}$  before the disruption. In the initial state we have

$$F_p^0 = F_{pp}^0 + F_{pc}^0 + F_{pt}^0 = 0. \quad (7)$$

Combination of (2), (5), (6) and (7) yields

$$F_w = F_{pp} + F_{ww} - F_{pp}^0 - F_p. \quad (8)$$

According to [7–9]

$$F_{pp} = \frac{\mu_0 J^2}{2} \left( \ln \frac{8R_0}{a} - 1 + \frac{\ell_i}{2} + \frac{\beta_p}{2} \right), \quad (9)$$

where  $J$  is the net plasma current,  $a$  is the plasma minor radius,  $R_0$  is its major radius,

$$\ell_i \equiv \overline{B_\theta^2} / B_J^2 \quad (10)$$

is the internal inductance per unit length of the plasma column ( $\overline{f}$  stands for the averaged over the plasma transverse cross section),

$$B_J = \frac{\mu_0 J}{2\pi a} \quad (11)$$

is the poloidal field  $B_\theta$  at the plasma boundary,

$$\beta_p \equiv 2\mu_0 \overline{p} / B_J^2 \quad (12)$$

is the ‘poloidal beta’ with  $p$  the plasma pressure and  $\mu_0 = 4\pi \times 10^{-7}$  H/m the vacuum magnetic permeability (SI units are used here).

The same logic and approach as described in [7–9] can be also applied to a system  $\Sigma$  (plasma plus wall). The result is a natural generalization of (9):

$$F_{pp} + F_{ww} = \frac{\mu_0 (J + J_w)^2}{2} \left( \ln \frac{8R_0}{r_w} - 1 + \frac{\ell_i^\Sigma}{2} + \frac{\beta_p^\Sigma}{2} \right), \quad (13)$$

where  $\ell_i^\Sigma$  and  $\beta_p^\Sigma$  are defined similarly to  $\ell_i$  and  $\beta_p$ , but now with averaging  $\langle f \rangle$  over the volume inside the vessel ( $r < r_w$ ) instead of  $\overline{f}$ ,  $r_w$  is the minor radius of the wall, and  $J_w$  is the net current in the wall. At the current quench, the plasma pressure contribution is negligible. Then  $\beta_p^\Sigma = 0$ , and we need only  $\ell_i^\Sigma$  here.

For a circular large-aspect-ratio plasma, we have in the plasma-wall vacuum gap

$$B_\theta \approx B_J a / r \quad (14)$$

with  $B_J$  defined by (11) and  $r$  the radius counted from the plasma centre in the perpendicular cross-section. The integration over the volume inside the vessel yields

$$\int_{p+g} B_\theta^2 dV = V_p B_J^2 \left( \ell_i + 2 \ln \frac{r_w}{a} \right), \quad (15)$$

where  $V_p = 2\pi^2 a^2 R_0$  is the plasma volume. In the geometrically thin wall, the magnetic energy must be small. Disregarding it, but allowing for  $B_\theta$  variation across the wall, we obtain

$$\ell_i^\Sigma \equiv \langle B_\theta^2 \rangle / B_\theta^2(r_{w+}) = \frac{J^2}{(J + J_w)^2} \left( \ell_i + 2 \ln \frac{r_w}{a} \right), \quad (16)$$

where  $r_{w+}$  means the outer side of the wall.

The ideal wall assumption implies that

$$J + J_w = J_0, \quad (17)$$

where  $J_0$  is the plasma current before the quench. Under this constraint, combining (13) with  $\beta_p^\Sigma = 0$  and (16), we obtain from Eqs. (8) and (9)

$$F_w = \frac{\mu_0 J_0^2}{2} \left( \frac{\ell_i - \ell_i^0}{2} - \frac{\beta_p^0}{2} \right) + \frac{\mu_0 (J^2 - J_0^2)}{2} \left( \ln \frac{r_w}{a} + \frac{\ell_i}{2} \right). \quad (18)$$

The first term describes the force that appears due to the thermal quench when  $J_w = 0$  and  $\beta_p$  drops from  $\beta_p^0$  to zero. It is constant at the current quench when an additional force  $\sim (J^2 - J_0^2)$  appears that is related to the magnetic energy redistribution in the inner region.

**4. Discussion.** The latter formula is obtained at  $\partial \mathbf{B} / \partial t = 0$  in the outer region. In reality the disruption duration may be longer than the wall time [5]. In such cases equation (18) can serve as an upper estimate for JET. However, the ideal-wall limit seems to be pertinent at the mentioned ITER expectations [10, 11]:  $\tau_{iq} \approx 36$  ms and  $\tau_w \approx 340$  ms.

Being essentially different from the Noll's formula [2, 4–6], equation (18) represents a mechanism that was not accounted for in [2, 4–6]. Accordingly, the force (8) is an addition to the Noll's and other disruption forces discussed in [1–6, 10]. For tokamaks with a large current, formula (18) predicts quite a large value of  $F_w$ . Thus, this force must be an important contribution of the force balance in ITER.

The presented approach is based on the integral force balance required for a plasma equilibrium in a tokamak. The algorithm outlined by equations (1)–(8) is quite general and can also be used in other tasks where the wall becomes a current carrier. Here the outward penetration of the plasma-produced perturbation is assumed weak because of the skin effect in the wall. For slower events, the wall resistivity has to be properly incorporated.

- [1] H. R. Strauss, R. Paccagnella and J. Breslau, *Phys. Plasmas* **17**, 082505 (2010).
- [2] L. E. Zakharov, S. A. Galkin and S. N. Gerasimov, *Phys. Plasmas* **19**, 055703 (2012).
- [3] H. R. Strauss, et al., *Nucl. Fusion* **53**, 073018 (2013).
- [4] V. Riccardo, S. Walker, and P. Noll, *Fusion Eng. Des.* **47**, 389 (2000).
- [5] V. Riccardo, P. Noll, and S. P. Walker, *Nucl. Fusion* **40**, 1805 (2000).
- [6] S. N. Gerasimov, et al., *Nucl. Fusion* **54**, 073009 (2014).
- [7] V. D. Shafranov, *J. Nucl. Energy, Part C*, **5**, 251 (1963).
- [8] V. D. Shafranov, in *Reviews of Plasma Physics*, Ed. by M. A. Leontovich (Consultants Bureau, New York, 1966), Vol. 2, p. 103.
- [9] K. Miyamoto, *Fundamentals of Plasma Physics and Controlled Fusion*. NIFS-PROC-48 (Toki, 2001).
- [10] T. C. Hender, et al., *Nucl. Fusion* **47**, S128 (2007).
- [11] Y. Gribov and V. D. Pustovitov, 19th IAEA Fusion Energy Conf., Lyon, (2002) IAEA-CN-94/CT/P-12;  
[http://www.iaea.org/programmes/ripc/physics/fec2002/pdf/ctp\\_12.pdf](http://www.iaea.org/programmes/ripc/physics/fec2002/pdf/ctp_12.pdf)