A Model of Recovering the Fast Nonlocal Transport Parameters in Magnetic Fusion Plasmas

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1. Introduction. Superdiffusion formalisms (an integral equation with nonlocal, longer than diffusive, spatial correlations and respective dominance of the long mean-free-path energy carriers) were suggested to describe the anomalous heat transport in magnetized fusion plasmas. These included the steady-state heat transport by the electron/ion Bernstein waves [1] and the electron cyclotron waves [2]; perturbative heat transport by the electron Bernstein waves [3], which appeared to be described by the Biberman-Holstein (B-H) equation known from the theory of excitation transport in atomic/ionic spectral lines; fractional diffusion model [4] for perturbative transport (see more references in the survey [5,6] of fast nonlocal transport interpretations). The increasing evidences for the fast nonlocal transport in magnetic fusion devices (e.g., "cold pulse" experiments in tokamaks and stellarators) suggest further diversification of perturbative models to identify possible physics mechanisms of nonlocality.

Here we suggest a model, which modifies and extends the approach [7]. We formulate an inverse problem for the recovery of main features of the kernel of the heat transport equation from the space-time dependence of the observed temperature perturbations. The developed model is applied to interpreting the data from the "cold pulse" experiments in tokamak TFTR [5, 8] and stellarator LHD [9].

2. A model of fast nonlocal heat transport (FNHT). The model aims at recovering the main parameters of unknown fast carriers of the heat via solving an inverse problem. The model is based on the following assumptions. (i) We assume that at the initial stage of the FNHT events, like those caused by the "cold pulse" or similar perturbations (see [5]), the observed unexpected dependence of electron temperature T_e (namely, the immediate, in the diffusion time scale, rise of T_e in the central plasma in the case of fast cooling of the edge plasma) is fully determined by the unknown fast carriers (presumably, of electromagnetic origin), i.e. their intensity (heat flux) exceeds that of any other heat transport mechanism at this stage. Also, their group velocity is assumed to be high enough to neglect the retardation effects in the course of the carrier's travel between the spatial points of emission ("birth") and absorption ("death") of the carrier. (ii) The energy density of the carriers is small as compared to that of plasma. (iii) The carriers are (almost) fully reflected from the plasma boundary or the vacuum chamber wall. This makes their intensity spatially homogeneous and isotropic in the group velocity angles and determined by the current parameters of the plasma at each time instant. (iv) To exclude possible contribution of the diffusion processes to the observed evolution of T_e, the inverse problem is solved only in the central plasma, to satisfy the limitations of the model (see the first item) both in the space and time. Correspondingly, we treat the impact of the fast cooling of the edge upon the temperature dynamics in the central plasma (namely, approximately linear growth of Te at initial stage of the event, see [5, 8]), whereas the source function in the central plasma is assumed to be not disturbed by the T_e perturbation in the edge. This enables us to express the time derivatives of T_e in the core in terms of the abrupt change ("jump") of the T_e profile in the edge plasma. (v) The source function, $S(\omega,T_e(\mathbf{r}))$, which gives the distribution of the carriers' source in their energy (described by the wave frequency ω), is assumed to be determined by the local value of T_e, and taken a Gaussian with unknown, sought-for dependence of three parameters, namely, the total power density Q(T), the mean energy/frequency $\overline{\omega}(T)$, and the energy dispersion (the width of the "line shape"), $\Delta(T)$, on the temperature. The dependence of these parameters on the other parameters, e.g., plasma density, is neglected.

The above model allows the derivation of an equation in the frame of the B-H approach (namely, assuming the complete redistribution over energy/frequency within the spectral line shape of the carriers' source during the process of the absorption and subsequent emission of the carrier in a given spatial point). However, preliminary results of solving such an inverse problem revealed too much freedom for the final solutions. At this stage we decided to restrict ourselves to evaluating the possibility to find the spectrum-integrated source function, Q(T), that formally corresponds to a monochromatic ("one-energy-group") transport. This finally gives the following equation:

$$\kappa\big(T_0(\rho)\big)\int_0^1 S\big(T_0(\rho_1)\big)\rho_1 d\rho_1\left[\frac{1}{\int_0^1\kappa(T_1(\rho_2))\rho_2 d\rho_2}-\frac{1}{\int_0^1\kappa(T_0(\rho_2))\rho_2 d\rho_2}\right]=\Pi(\rho),$$
$$0<\ \rho<\rho_{max}, \tag{1}$$

where $\Pi(\rho)$ is the time derivative of plasma energy density, which is taken from experimental data, ρ is the normalized minor radius of plasma in the 1D transport model, the source function S is related to the absorption coefficient κ with the Kirchhoff law (in the Rayleigh-Jeans limit) at a certain frequency,

$$S(T)\equiv\frac{Q(T)}{4\pi}=const\ T\ \kappa(T)\ , \tag{2}$$

the profiles of temperature before the “cold pulse”, $T_0(\rho)$, and after that, $T_1(\rho)$, differ only in the edge plasma (the profile $T_1(\rho)$ enters Eq. (1) only as an integrand so that its evolution after abrupt jump may be neglected within time of approximate constancy of $\Pi(\rho)$). The inverse problem is formulated as a minimization of the deviation from the equality in Eq. (1) with respect to sought-for function Q(T). This function is assumed to be the sum of several terms of the power-law expansion, including the half-integer power.

The inverse problem solutions give quite different results for stellarator LHD (Figs. 1-2) and tokamak TFTR.

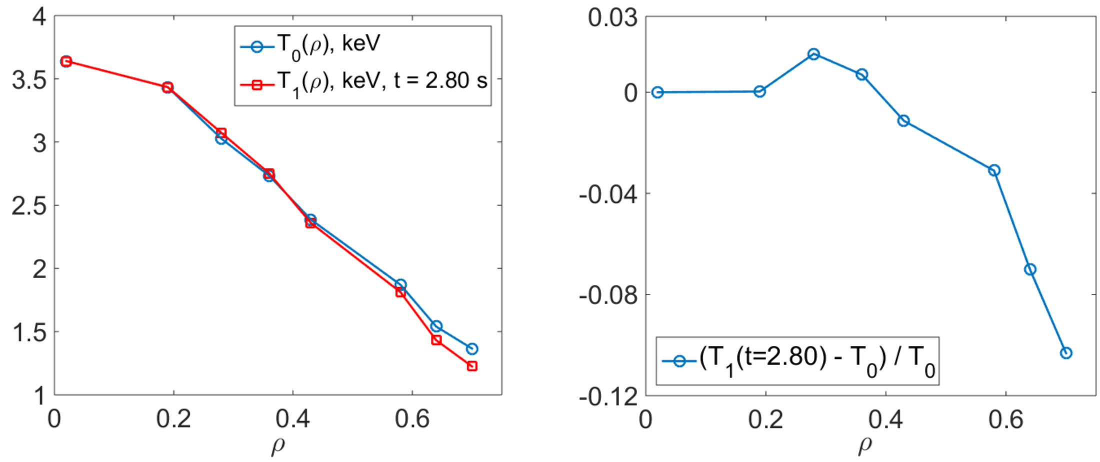

Figure 1. Left: the profiles of electron temperature before, T_0, and after, T_1, pellet injection when $\Pi(\rho)$, the time derivative of Te, is approximately constant in time in the central plasma (see Fig. 1 in [9]). Right: the relative difference of the above-mentioned temperature profiles.

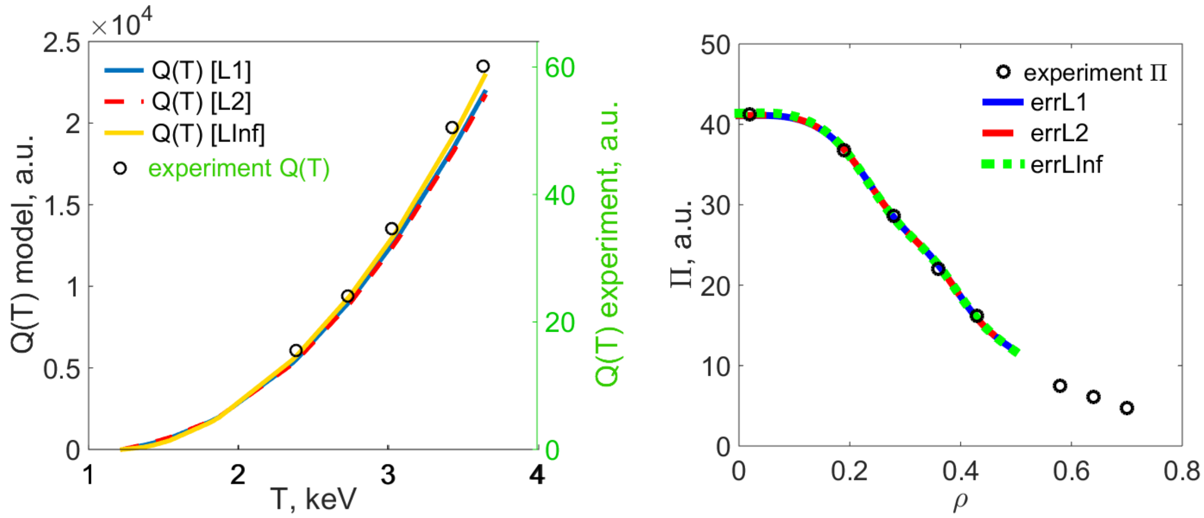

Figure 2. Left: the results of recovering the temperature dependence of the source function for energy carriers from Eq. (1) for the data [9] with ρ_{max}=0.5, for minimization of: the least squares error (L2), the sum of error’s absolute values (L1), the global absolute error (Linf). The circles point on the values of T_e, which correspond to experimental data for $\Pi(\rho)$. The right axis corresponds to the recovery of only the functional form of Q(T) directly from proportionality of $\Pi(\rho)$ and $k(T(\rho))$ in Eq. (1). Right: fitting of the experimental data for $\Pi(\rho)$ for various minimization criteria.

The growth of the absorption coefficient with increasing temperature in the present model explains the effect of the inward heat flux in the cold pulse experiments: the edge plasma after fast cooling absorbs less energy from the circulating flux of energy carriers so that their increasing intensity produced higher absorption in the central plasma. Interestingly, such a mechanism is symmetric with respect to the sign of the fast temperature change in the periphery (the observed effect of "inverse polarity" of the temperature change in the center and the periphery, produced by the fast heating of the periphery, is discussed in [5]).

For tokamak TFTR data [5,8], already the first step of solving the inverse problem, namely the recovery of only the functional form of Q(T) directly from proportionality of $\Pi(\rho)$ and $k(T(\rho))$ in Eq. (1), shows incapability of Eq. (1) to provide reasonable solutions. Our analysis suggests that the present model (i)-(v) should be modified for tokamaks to allow for strong internal transport barriers at magnetic surfaces with rational value of the safety factor q (for reliable identification of these barriers in tokamak RTP see [10]; the models of the MHD equilibria in a sectioned plasma were suggested in [11, 12]). E.g., for the shot in Fig. 1 in [5,8] the peak of $\Pi(\rho)$ is located at q=2 magnetic surface.

3. Conclusions. We suggest a model to interpret the observed fast nonlocal transport events where heat flux is directed opposite to conventional diffusion flux and the response is almost immediate within diffusion time scale. We suggest the following explanation with a hypothetical energy carriers (presumably, of electromagnetic wave origin): the abrupt cooling of the edge plasma leads to an abrupt decrease of the absorption in the edge that, in turn, allows more transparent circulation of the energy carriers and, under condition of almost full reflection from the edge, an increase of the intensity/flux of the carriers. The latter leads to a linear (in time) growth of temperature in the central plasma at initial stage of the event.

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