Analytical model of wall force produced by kink modes combined with plasma vertical displacement

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1. Introduction. This task is related to evaluation of the electromagnetic forces on the conducting structures during plasma disruptions [1–7]. Such forces are observed on Joint European Torus (JET) [2] accompanying with the vertical displacement events (VDEs). In [6], an expression for the force produced by kink modes together with VDEs was derived using Eq. (43) from [4]. The latter is shown to be incorrect in [7]. This is a sufficient reason for revision of the predictions made in [6].

Here we calculate the non-axisymmetric (sideways) wall force produced by kink modes combined with VDEs. We use a model described in [7], but extend it here by considering three modes and their nonlinear beating.

2. Formulation of the problem. As in [4, 6, 7], we consider a cylindrical plasma with minor radius r_{pl} surrounded by a coaxial resistive wall of uniform conductivity σ, radius r_w and thickness d_w. The plasma-wall gap and space behind the wall are treated as a vacuum. The magnetic field is described as $\mathbf{B} = \mathbf{B}_0 + \mathbf{b}$, where $\mathbf{B}_0$ is the axisymmetric equilibrium field ($\partial \mathbf{B}_0 / \partial t = 0$) and $\mathbf{b}$ is the perturbation. The latter induces the eddy current $\mathbf{j} = \sigma \mathbf{E}$ in the wall, where $\mathbf{E}$ is the electric field governed by $\nabla \times \mathbf{E} = -\partial \mathbf{b} / \partial t$. Then, a force with a volume density $\mathbf{f} = \mathbf{j} \times \mathbf{B}$ will act on the wall. A lateral (sideways) force is defined by

$$F_X \equiv \int_{wall} \mathbf{f} \cdot \mathbf{e}_X dV = R_0 r_w d_w \oint f_r \cos\theta \cos\zeta \, d\theta d\zeta \,, \tag{1}$$

where $\mathbf{e}_X$ is the unit vector along a fixed horizontal direction, R_0 is the major radius, θ and ζ are the poloidal and toroidal angles, respectively, and

$$f_r \equiv \frac{1}{d_w} \int_{in}^{out} (\mathbf{j} \times \mathbf{B}_0 + \mathbf{j} \times \mathbf{b}) \cdot \mathbf{e}_r dr \quad \text{so that} \quad d_w f_r = (B_{0\theta} b_\theta + B_{0\zeta} b_\zeta)\Big|_{out}^{in} + \frac{1}{2} b_\theta^2 \Big|_{out}^{in} . \tag{2}$$

In the quadratic term here, we disregarded a small contribution from b_ζ. Equation (1) shows that only $f_r \propto \cos\theta \cos\zeta$ can give a non-zero sideways force. Therefore, $m/n = 1/1$ mode can contribute into linear term of Eq. (2). This case was discussed in [7]. In quadratic term, we need $b_\theta^2 \propto \cos\theta\cos\zeta$. This can be produced by the beating of m/n and $m\pm1/n\pm1$ modes. Here we consider the perturbations in form

$$b_\theta = |b_{\theta,11}|\cos(\theta-\zeta+\delta_{11}) + |b_{\theta,VDE}|\sin\theta + |b_{\theta,21}|\sin(2\theta-\zeta+\delta_{21}) \quad (3)$$

with δ_{11} and δ_{21} that are toroidal phase shifts of the modes. VDE denotes $m/n = 1/0$ mode.

With such b_θ for ideal conducting wall, Eqs. (1)–(2) give a sideways force

$$F_X = F_1 + F_2 = F_l \frac{|b_{\theta,11}|}{B_{0\theta}} + F_q \frac{|b_{\theta,21} b_{\theta,VDE}|}{B_{0\theta}^2}, \quad (4)$$

where $F_l = -F_{Xm}\left(1 - \frac{r_{pl}^2}{q_{pl} r_w^2}\right)\cos\delta_{11}$ was calculated in [7], $F_q = F_{Xm}\frac{1}{2}\frac{r_{pl}^2}{q_{pl} r_w^2}\cos\delta_{21}$, $F_{Xm} \equiv \frac{\pi r_w}{2} J B_{0\zeta}$, b_θ and $B_{0\theta}$ are taken at the wall, q_{pl} is the safety factor $q \equiv r B_{0\zeta}/(R_0 B_{0\theta})$ at the plasma boundary, J is the total current in the plasma. With ITER parameters [8]: $J = 15$ MA, $B_{0\zeta} = 5.3$ T, $r_{pl} = 2$ m, $r_w / r_{pl} = 1.3$, we have $F_{Xm}^{ITER} = 325$ MN. A large plasma vertical displacement was observed on JET at $q_{pl} = 1.1$ [9]. Using the latter and ITER data the term $F_1 \geq 37.5$ MN requires $|b_{\theta,11}| / B_{0\theta} > 0.25$, while the ITER is designed for 48 MN [9]. However, this limit can be exceeded by the contribution from quadratic term of Eq. (4), which gives $F_2 \geq 22$ MN at $|b_{\theta,21} b_{\theta,VDE}| / B_{0\theta}^2 > 0.25$. On the other hand, $F_X = 0$ at

$$\cos\delta_{21} = -2\cos\delta_{11}\left|\frac{B_{0\theta} b_{\theta,11}}{b_{\theta,21} b_{\theta,VDE}}\right|\left(1 - q_{pl}\frac{r_w^2}{r_{pl}^2}\right). \quad (5)$$

A result similar to Eq. (4) is described by Eqs. (11)–(15) in [6]. These equations predict a force as a monotone decreasing function of $\gamma\tau_w$ with a maximum at $\gamma = 0$ instead of $\gamma\tau_w \approx 1$ as claimed in [6] for parameters presented in [4], where γ is the kink growth rate and τ_w is the wall penetration time. On the contrary, analysis in [7] shows that F_1 force must be maximal in the ideal wall limit at $\gamma\tau_w \to \infty$. Here we obtain the same result for F_2 and compare it with that in [6].

3. Calculation of the force. We calculate quadratic term of Eq. (2). At $nr << mR_0$ equation $\nabla^2\varphi = 0$ with $\mathbf{b} = \nabla\varphi$ and $\varphi = \sum_{mn} \mathrm{Re}\left[\varphi_{mn}\exp(im\theta - in\zeta)\right]$ gives us

$$\varphi_{mn} = -\frac{r_w}{m} B_m \left[x^{-m} + \frac{\Gamma_m}{2m}(x^{-m} + x^m) \right] \quad (6)$$

in the plasma-wall vacuum gap with $x \equiv r/r_w$ and $\Gamma_m = -2m B_m^{out} / B_m$ (see also [7,10, 11]). Then,

$$b_{\theta,mn}\Big|_{in} = -i(1+\Gamma_m / m) b_{r,mn}(r_w) \qquad \text{and} \qquad b_{\theta,mn}\Big|_{out} = -i b_{r,mn}(r_{w+}), \tag{7}$$

where $b_{r,mn} \equiv \partial\varphi_{mn} / \partial r$ is the amplitude of $\mathbf{b}\cdot\nabla r$ and $r_{w+} = r_w + d_w$. Using these expressions we introduce $d_w f_{r2}^{side} \equiv b_{\theta,21} b_{\theta,VDE}\Big|_{out}^{in}$:

$$d_w f_{r2}^{side} \equiv \frac{1}{4}\mathrm{Re}\left[\Gamma_\Sigma (1+\alpha_2) b^*_{r,VDE}(r_w) b_{r,21}(r_w)\exp(i\theta - i\zeta)\right] \tag{8}$$

that is a force volume density only contributes to sideways force in quadratic term with

$$\alpha_2 = \frac{2}{\Gamma_\Sigma}\left[1 - \frac{b^*_{r,VDE}(r_{w+}) b_{r,21}(r_{w+})}{b^*_{r,VDE}(r_w) b_{r,21}(r_w)}\right] \quad \text{and} \quad \Gamma_\Sigma = 2\Gamma_1^* + \Gamma_2 + \Gamma_2\Gamma_1^*, \tag{9}$$

where superscript * denotes a complex conjugate variables.

In [6], the force is finally expressed through the radial component ξ_r of the plasma displacement $\boldsymbol{\xi}$. To move in this direction we use the consequence of (6)

$$\frac{b_{r,mn}(r)}{b_{r,mn}(r_w)} = \left[1 + \frac{\Gamma_m}{2m}(1 - x^{2m})\right]\cdot x^{-m-1} \tag{10}$$

valid in the plasma-wall gap. If the plasma is ideal as in Ref. [4, 6, 7], we have $\mathbf{b} = \nabla\times(\boldsymbol{\xi}\times\mathbf{B}_0)$, $b_r = \mathbf{B}_0\cdot\nabla\xi_r$ and

$$r_{pl} b_{r,mn}(r_{pl}) = i B_J (m - n q_{pl})\xi_{r,mn} \tag{11}$$

with $B_J \equiv B_{0\theta}(r_{pl})$ and $\xi_{r,mn}$ taken at the plasma boundary. Then, Eq. (8) takes a form of

$$d_w f_{r2}^{side} \equiv 2\,\mathrm{Re}\left[\frac{\Gamma_\Sigma}{\Gamma_1^*\Gamma_2}\,\frac{B_J^2(2-q_{pl})(1+\alpha_2)\kappa_1^5}{(2/\Gamma_1^* + 1 - \kappa_1^2)(4/\Gamma_2 + 1 - \kappa_2^2)}\,\frac{\xi_{r,21}\xi^*_{r,VDE}}{r_{pl}^2}\exp(i\theta - i\zeta)\right], \tag{12}$$

where $\kappa_m \equiv (r_{pl} / r_w)^m$ and free parameters: α_2 and Γ_Σ are defined by Eq. (9).

4. Comparisons, estimates, and discussion. In [4, 6], a thin shell approximation was used that requires continuity of the normal component of $\mathbf{b}$ (here, $\mathbf{b}\cdot\nabla r$) at the wall. Hence $\alpha_2 = 0$ and Γ_m is related to γ_{mn} by (see [10, 11] and the references therein)

$$\Gamma_m = \gamma_{mn}\tau_w \qquad \text{with} \qquad \tau_w \equiv \sigma r_w d_w. \tag{13}$$

In contrast to [4, 6], expression (12) gives the wall force increases monotonically with γ, starting from $f_{r2}^{side} = 0$ at $\gamma = 0$ and maximal at $\gamma = \infty$. Using $\xi_{r,21}\xi^*_{r,VDE} = \left|\xi_{r,21}\xi_{r,VDE}\right| e^{i\delta_{21}}$ and $\gamma = \infty$, Eq. (12) after integration across the wall volume gives us the sideways force:

$$F_2^{id} = 4F_q \frac{r_{pl}^3}{r_w^3} \frac{(2-q_{pl})}{(1-\kappa_1^2)(1-\kappa_2^2)} \frac{|\xi_{r,21}\xi_{VDE}|}{r_{pl}^2}. \tag{14}$$

The result F_x^{21} described by Eqs. (11)–(12) in [6] at $\gamma = 0$ and at $\gamma = \infty$ differs from one another in 1.33 times for chosen parameters in [6]. Comparing them with Eq. (14):

$$F_x^{21} = 2F_2^{id}(r_w^2 / r_{pl}^2 - 1) \tag{15}$$

at $\gamma = \infty$. With $r_w / r_{pl} = 2$ as in [6] we have $F_x^{21} = 6F_2^{id}$. This shows us that the result described by Eqs. (11)–(12) in [6] overestimates the wall force 6 times. Let us notice that comparison of F_x^{11} in Eqs. (13)–(14) in [6] with $F_{(43)}^S$ obtained after integration across the wall volume of Eq. (43) in [4] at $\gamma = \infty$ gives us

$$F_x^{11} = 2F_{(43)}^S r_w^3 / r_{pl}^3 \ . \tag{16}$$

At $r_w / r_{pl} = 3$ as in [4], $F_x^{11} = 54F_{(43)}^S$ despite the statement made right before Eq. (13) in [6] that F_x^{11} and $F_{(43)}^S$ are the same.

In [2], equation $F_X^{Noll} \equiv 2F_{Xm}\,\xi_z / r_w$ was obtained. It gives a sideways force on the rigid ring with current J in the toroidal field $B_{0\zeta}$ when this ring is tilted about the axis X by a small angle $\alpha = \xi_z / R_0$ so that ξ_z is the amplitude of displacement. With ITER [8] parameters, mentioned in section 2, $F_X^{Noll} \geq 135$ MN requires $\xi_z / r_{pl} \geq 0.27$. At $q_{pl} = 1.1$, $|\xi_{r,11}| / r_{pl} \geq 0.27$ and $|\xi_{r,21}\xi_{VDE}| / r_{pl}^2 \geq 0.073$ the sideways wall force $F_2^{ITER} \geq 40$ MN, while $F_1^{ITER} \geq 16$ MN calculated in [7]. Thus at q_{pl} near 1 the quadratic term F_2 of Eq. (4) dominates over linear, which $F_1 \to 0$ in this case.

5. Conclusion. In contrast to [4, 6], we have shown that the force is monotonically increasing with $\gamma\tau_w$ and maximal at $\gamma = \infty$. With the latter the sideways force F_2 produced by $m/n = 2/1$ kink mode combined with a plasma vertical displacement can be larger than 48 MN that ITER is designed for [9] at $q_{pl} \to 1$ and $\xi_{r,21}\xi_{VDE} / r_{pl}^2 \geq 0.073$, while $F_1 \to 0$ [7].

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