Propagation of radio frequency waves through fluctuations in plasmas

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Introduction

The scrape-off layer in a tokamak fusion plasma is replete with coherent fluctuations in the form of blobs or filaments [1, 2, 3, 4]. Radio frequency (RF) waves, excited by an external antenna structure, have to propagate through this turbulent region on their way toward the core of the plasma. Since the plasma permittivity inside the fluctuations is different compared to the background plasma, the characteristic properties of the incident RF waves can change during their transit through the fluctuations. It is common to study these changes using the geometric optics approximation (see [5] and references therein). In this approximation, rays, representing plane waves, are refracted due to changes in the plasma permittivity. The domain of validity of this approximation is quite limited; it requires that $\delta n \equiv |n_f - n_b|/n_b \ll 1$ (n_f is the density of electrons inside the fluctuation and n_b is the background density). However, from experimental observations, typically, $\delta n \gtrsim 1$ in the scrape-off layer. Consequently, a study of the scattering process requires a more general approach that is not limited by the bounds of the geometric optics approximation.

In this paper, we present a theoretical model for the scattering of RF waves by density filaments using the full-wave Maxwell's equations. The model is based on the conventional Mie theory that is used for scattering of vacuum electromagnetic waves by spherical dielectrics [6]. Our model applies to magnetized plasmas for which the permittivity is given by a tensor, rather than a scalar, and is derived from our previous study on the scattering of RF waves by spherical blobs [7, 8]. The present model pertains to scattering off a cylindrical filament with its axis aligned along the toroidal magnetic field line. It assumes that the background plasma, as well as the plasma in the filament, is cold and has uniform density. However, it is not restricted to small density fluctuations – δn is completely arbitrary. The theoretical framework applies to plasma waves of any frequency; it is valid for ion cyclotron, lower hybrid, and electron cyclotron waves.

Beyond their respective domain of validity, there are three primary differences between the full-wave theory and the geometric optics approximation. First, whereas geometric optics is used for describing refractive changes in the ray propagation, the full-wave model also includes reflection and diffraction. Second, in geometric optics, the character of the wave does not change as the RF ray propagates through fluctuations. For example, an incident ordinary wave in the electron cyclotron range of frequencies will remain an ordinary wave during its encounter with fluctuations. In the full-wave model, the fluctuations can couple power to other plasma waves. Thus, for the example considered, the ordinary wave couples power to the extraordinary wave. This is not nonlinear parametric coupling; it is linear coupling facilitated by the fluctuations. Third, due to diffraction of waves by the filament, the scattered waves propagate in all radial directions relative to the magnetic field line. Consequently, fluctuations can scatter some of the incident wave power to surface waves which do not propagate into the core plasma. This effect cannot be described within the geometric optics approximation.

Basic equations and boundary conditions

We assume that the axis of the cylindrical filament is aligned along the magnetic field. The center of the coordinate system is taken to be the center of the filament. The filament is assumed to be stationary and of infinite extent in the axial direction. For a cold fluid plasma, described by a linearized set of continuity and momentum equations for electrons and ions, Faraday's and Ampere's equations in Maxwell's system of equations can be combined [9] to yield the following equation for the spatial variation of the electric field,

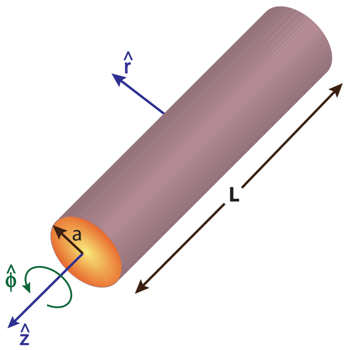

Figure 1: *Coordinate system with respect to the cylindrical filament*

$$\nabla\times\nabla\times\mathbf{E}(\mathbf{r})-\frac{\omega^2}{c^2}\overleftrightarrow{\mathbf{K}}(\mathbf{r}).\mathbf{E}(\mathbf{r}) = 0, \tag{1}$$

where ω is the angular frequency, c is the speed of light, and $\overleftrightarrow{\mathbf{K}}(\mathbf{r})$ is the plasma permittivity tensor. We have assumed that the plasma equilibrium is time independent, while the linearized perturbed electromagnetic fields and the plasma density have a time dependence of the form $e^{-i\omega t}$. In the cylindrical coordinate system, in which the ambient magnetic field $\mathbf{B}_0 = B_0\hat{\mathbf{z}}$ is aligned along the z-axis [9],

$$\overleftrightarrow{\mathbf{K}} = \begin{pmatrix} K_\rho & -iK_\phi & 0 \\ iK_\phi & K_\rho & 0 \\ 0 & 0 & K_z \end{pmatrix}, \tag{2}$$

with

$$K_\rho = 1 - \frac{\omega_{pe}^2}{\omega^2 - \omega_{ce}^2} - \sum_\iota \frac{\omega_{pi}^2}{\omega^2 - \omega_{ci}^2},$$
$$K_\phi = -\frac{\omega_{ce}}{\omega}\frac{\omega_{pe}^2}{\omega^2 - \omega_{ce}^2} + \sum_\iota \frac{\omega_{ci}}{\omega}\frac{\omega_{pi}^2}{\omega^2 - \omega_{ci}^2}, \tag{3}$$
$$K_z = 1 - \frac{\omega_{pe}^2}{\omega^2} - \sum_\iota \frac{\omega_{pi}^2}{\omega^2},$$

where ω_{pe} (ω_{pi}) and ω_{ce} (ω_{ci}) are the angular electron (ion) plasma frequency and cyclotron frequency, respectively, and the index ι represents all the ion species in the plasma. The plasma and cyclotron frequencies can, in general, be functions of space. The permittivity tensor of the background plasma and of the filament are expressed in terms of their respective densities and ion compositions.

A critical component in the description of full-wave scattering is that the electromagnetic fields satisfy boundary conditions at the interface between the filament and the background plasma. The boundary conditions which follow from Maxwell's equations, with the requirement that there be no free surface charges or currents at the interface, are

$$\Delta(\hat{\mathbf{r}}.\mathbf{B})_{r=a} = 0, \qquad \Delta(\hat{\mathbf{r}}.\mathbf{D})_{r=a} \equiv \Delta\left(\hat{\mathbf{r}}.\overleftrightarrow{\mathbf{K}}(\mathbf{r}).\mathbf{E}\right)_{r=a} = 0, \tag{4}$$

$$\Delta(\hat{\mathbf{r}}\times\mathbf{E})_{r=a} = 0, \qquad \Delta(\hat{\mathbf{r}}\times\mathbf{B})_{r=a} = 0. \tag{5}$$

Here $\mathbf{B}(\mathbf{r})$ is the wave magnetic field, $\mathbf{D}(\mathbf{r})$ is the displacement electric field, a is the radius of the filament, and Δ is the difference, at $r = a$, of the enclosed quantity evaluated inside the filament and in the background plasma. Of these six scalar conditions, it can be shown that only four are independent.

Results

In order to demonstrate the capability of the theoretical model, we consider the scattering of the slow lower hybrid wave [9] by a filament. The background plasma and the plasma inside the filament has deuterium ions with electron densities 2×10^{19} cm^{-3} and 2.5×10^{19} cm^{-3}, respectively. The ambient magnetic field is 4.5 T, and the incident plane wave has a frequency of 4.6 GHz with a parallel refractive index of 2. Figure 2 shows the magnitude of the Poynting flux (normalized to the Poynting flux of the incident wave) in the $x-y$ plane. The incoming plane wave is incident from

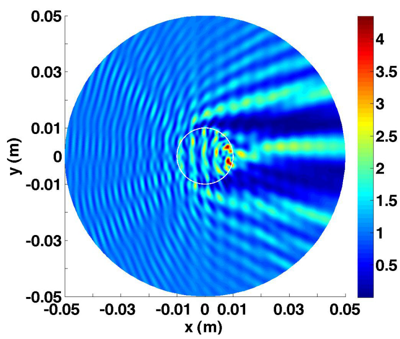

Figure 2: *The magnitude of the total Poynting flux, normalized to the Poynting flux of the incident wave, in the $x-y$ plane.*

the left hand side and the cylindrical filament, outlined in white, has a radius of 0.01 m. The pattern on the left hand side indicates a beating between the incident and the back-scattered wave. The enhanced flux in the forward direction, near the boundary of the filament, is a consequence of large electric and magnetic fields of the wave packet – a result of Gauss' law. The large amplitude fields can be the seeds for nonlinear parametric processes. The scattering in the forward direction includes diffraction and shadowing due to the filament. The detailed wave pattern emphasizes the point that the wave power propagating into the core of the plasma has a more complicated structure than a plane wave. Indeed, a spectral analysis of the electric fields in the forward direction points to a broadening of the wave vector spectrum along the magnetic field line. The spectrum is centered around the wave number of the incident wave. The side scattering of the lower hybrid wave is evident as there is power propagating in the y-direction. The incident Poynting vector is in the $x-z$ plane. For the parameters used to obtain Fig. 2, the fast branch of the lower hybrid wave is evanescent in the background plasma. The slow branch is the only propagating wave outside the filament. Inside the filament, both the slow and fast branches of the lower hybrid wave are propagating. This affects the properties of the scattered waves due to coupling, through the boundary conditions, inside the region separating the filament from the background plasma.

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