Theory of the ordinary and extraordinary mode coupling in fluctuating plasmas

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Linear conversion of the ordinary (O) wave to the extraordinary (X) wave plays an important role in excitation of the electron Bernstein waves, which in turn provide an effective way for high-frequency (electron cyclotron) heating and diagnostics of overdense plasma in spherical tokamaks and optimized stellarators [1]. In spite of rather clear theoretical background behind this process, exiting theory tends to overestimate efficiency the O-X coupling in all available experiments. Following Laqua significant discrepancy between the predicted and observed efficiencies is usually explained by turbulent fluctuations of the cut-off layer where the mode conversion occurs [2]. However, only a qualitative model based on essentially one-dimensional theory has been proposed, in which poloidal fluctuations are taken into account by effective broadening of the incident wave spectrum over the poloidal wave numbers. In the present paper we propose a more rigorous approach based on two-dimensional theory of the mode conversion that can adequacy describe fluctuations without any qualitative speculations.

1. Reference wave equations

Following [3], Maxwell equations in a region of the O-X wave coupling can be reduced to

$$\begin{cases} N_{\parallel}\left(i\partial/\partial x-\partial/\partial y\right)F_{\parallel}=\sqrt{2}k_0\left(\varepsilon_{+}-N_{\parallel}^2\right)F_{+} \\ N_{\parallel}\left(i\partial/\partial x+\partial/\partial y\right)F_{+}=\sqrt{2}k_0\,\varepsilon_{\parallel}F_{\parallel} \end{cases}. \qquad (1)$$

Here $k_0=\omega/c$, $N_{\parallel}=k_{\parallel}/k_0$ is the refractive index parallel to the magnetic field, $F_{+,\parallel}=E_{+,\parallel}\exp(-ik_{\parallel}z+i\omega t)$ are slow field amplitudes and $\varepsilon_{+,\parallel}$ are the dielectric tensor components in the Stix frame: $\varepsilon_{\parallel}=1-\omega_{pe}^2/\omega^2$, $\varepsilon_{+}=1-\omega_{pe}^2/\omega(\omega+\omega_{ce})$. Effective coupling occurs when r.h.s. of both equations goes to zero, i.e. $\varepsilon_{\parallel}$, $\varepsilon_{+}-N_{\parallel}^2\rightarrow 0$ (plasma cut-offs). There are three spatial scales in the above equations: the density variation $L_n=n_e/\nabla n_e$, the magnetic field variation $L_B=B/\nabla B$, and the wave coupling length $L_{\nabla}\sim\sqrt{L_n/k_0}$.

Equations (1) have been obtained for a weakly inhomogeneous plasma, $k_0 L_n$, $k_0 L_B >> 1$, so in the tokamak conditions the coupling is highly localised, $L_\nabla << L_n < L_B$.

To study the effect of fluctuations we consider the simplest case in which

$$n_e = (x / L_n)\, n_{cr} + \delta n_e(x, y), \quad \mathbf{B} = (1 + x / L_B)\, B_0\, \mathbf{z}_0, \tag{2}$$

i.e. only density is perturbed, and without this perturbation the problem is reduced to one-dimensional case. Note that x and y represent here the radial and poloidal directions correspondingly, n_{cr} is the plasma cut-off density at which $\varepsilon_\parallel = 0$. Having in mind that O-X coupling occurs on scale L_∇ shorter then all plasma inhomogeneity scales, we retain only linear dependences over x in the magnetic field and the non-perturbed density. Then, the r.h.s. of equations (1) may be simplified noting that

$$\varepsilon_\parallel \propto x' + \delta_s + \delta_a, \quad \varepsilon_+ - N_\parallel^2 \propto x' + \delta_s - \delta_a, \tag{3}$$

where $x' = x / L_\nabla$ is the normalized radial coordinate, and

$$\delta_s = \frac{\delta n_e}{n_{cr}} \frac{L_n}{L_\nabla} - \delta_a, \; \delta_a = \frac{\delta n_e}{2 n_{cr}} \frac{L_n}{L_\nabla} \frac{L_n}{L_B (1 + \omega / \omega_{ce}) - L_n}, \tag{4}$$

represent the “symmetrical” and “asymmetrical” contributions of the density fluctuations. Note that asymmetrical part appears due the regular variation of a magnetic field, and therefore $\delta_a << \delta_s$ for large aspect ratio tokamaks while $\delta_a \sim \delta_s$ for spherical tokamaks.

Study of wave equations (1) can be done after transformation to new field variables $F_+ \propto A^+ + A^-$, $F_\parallel \propto A^+ - A^-$ introduced in such a way that in the WKB limit A^+ and A^- are amplitudes of waves that propagate, respectively, in the positive and negative directions along the radial coordinate:

$$\begin{cases} (i\,\partial/\partial x' + x' + \delta_s) A^+ = (-\partial/\partial y' + \delta_a) A^- \\ (i\,\partial/\partial x' - x' - \delta_s) A^- = (-\partial/\partial y' - \delta_a) A^+ \end{cases}. \tag{5}$$

One can see that the r.h.s. of these equations describes the coupling between counter propagating waves which is affected only by the asymmetrical part of the fluctuations. Opposite to it, the symmetrical part does not disturb the wave coupling however can modify propagation of the separate waves outside the coupling region. So even from a structure of wave equations we can argue that symmetric and asymmetric contributions of density fluctuations act essentially different. Physically it may be understood as δ_s describes the shift of the evanescent region, while δ_a describes the fluctuation of its width, see figure 1.

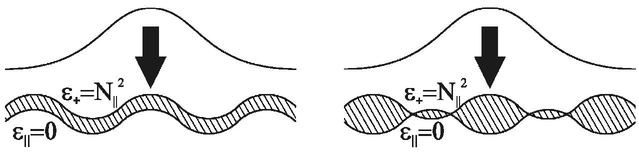

Fig. 1. Schematic of the symmetric (left) and asymmetric (right) contributions of density fluctuations. Dashing shows the region where studied electromagnetic modes are evanescent.

2. Coupling efficiency for plasma with harmonic fluctuations

Equations (5) were investigated in details in paper [3]; the main results of these studies may be summarized as follows.

For radial harmonic fluctuation, $\delta_{s,a} = a_{s,a}\cos(\kappa x' + \varphi)$, the problem is reduced to the essentially one-dimensional case, $\partial/\partial y' \to ik_y$, that may be treated in a standard Born approximation. In for case of effective mode coupling, $k_y << 1$, one can calculate the transmission coefficient of incident plane wave averaged over a random phase φ,

$$T_{1D} = 1 - \int_0^{2\pi} \frac{|A^-(-\infty)|^2}{|A^+(-\infty)|^2}\frac{d\phi}{2\pi} = \pi k_y^2 + \tfrac{1}{2}\pi a_a^2 - \pi^2 k_y^2 (a_a^2 + 4a_s^2 k_y^2/\kappa^2)\sin^2(\kappa^2/4) + \ldots \quad (6)$$

For poloidal harmonic fluctuation, $\delta_{s,a} = a_{s,a}\cos(\kappa y' + \varphi)$, the symmetric contribution (a_s) results in coupling of co-propagating harmonics, so one can neglect the reflection (take the r.h.s. of Eq.(5) equal to zero). In this case an exact analytic solution may be obtained, that describes the slow modification of the poloidal spectrum of an incident beam as it approaches the plasma cut-off. This is actually well-known diffusion in k-space induced by fluctuations [4]. It is important to note, that this effect is collected on a much longer path than the localized reflection region where the incident wave couples with a counter-propagating wave. Inside this zone, the symmetric contribution is of minor importance for all meaningful parameters being investigated. The asymmetric contribution (a_a) couples counter-propagating harmonics close to the plasma cut-off. The phase-averaged transmission coefficient for the plane wave with $k_y = 0$ may be estimated as

$$T_{2D} \approx \exp[-\tfrac{1}{2}\pi(1+\pi\kappa^2)^{-1}a_a^2]. \quad (7)$$

3. Discussion

Basing on the developed theory we revise the role of fluctuations in the OXB experiment. Note that our definitions of fluctuation parameters (4) contain product of small and large terms. As a result, in a fusion experiment the symmetric contribution δ_s may be of order of unity even though the relative density perturbations are small. Nevertheless, study of the typical parameter space shows that the impact of symmetric contribution of both radial and poloidal fluctuations on the O-X coupling remains small even for unrealistically large fluctuation level (δT ~1% for $\delta n_e / n_e \sim 20\%$). However, poloidal fluctuations can modify the amplitude and phase distributions in the incident quasi-optical beam far outside the coupling region located near the cut-off layer, what affect the consequent mode-conversion process (δT ~10% for the path 10 cm and $\delta n_e / n_e \sim 2\%$). The asymmetric contribution resulted from a magnetic field inhomogeneity typically can be neglected in large aspect ratio devices but not in compact tori since $\delta_a / \delta_s \sim L_n / L_B \sim a / R$. Therefore, in a conventional tokamak fluctuations result only in distortion of a wave beam on its way towards the mode coupling region but have no influence on the efficiency of the mode interaction inside the coupling region. In a spherical tokamak with $\delta_a \sim \delta_s$ the asymmetrical part of density perturbations may be significant inside the coupling region resulting in degradation the transmission efficiency by up to 25% for realistic conditions.

Summarizing, we find that plasma fluctuations can not be fully responsible for the low efficiency of OXB heating of overdense plasma observed in present-day experiments. We guess that the main factor impeding the tunneling of the electromagnetic waves through the plasma cut-off in these experiments may be associated with a curvature of toroidal magnetic surfaces [5].

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References

[1] H. P. Laqua, PPCF, 2007, 49 R1–42.
[2] H. P. Laqua et al., Phys. Rev. Lett., 1997, 78, 3467.
[3] A. G. Shalashov, E. D. Gospodchikov, PPCF, 2014, 56, 125011
[5] E. V. Sysoeva et al., PPCF, 2013, 55, 115001.
[6] E. D. Gospodchikov et al., PPCF, 2012, 54, 045009.