Spontaneous excitation of convective cells by kinetic Alfvén waves[*]

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Introduction

Zonal structures such as zonal flows are known to play crucial roles in dynamically regulating plasma transport in tokamak plasmas. The analogues in uniform plasma are the convective cells, which have been extensively studied in the 1970's [1, 2] in the context of cross-field transport [3]. In particular, it is worthwhile mentioning the extensive studies of convective cells excitation by kinetic Alfvén waves (KAW), with applications to flows in the upper ionosphere [4, 5].

It has been recently demonstrated that, generally, electrostatic (ESCC) and the magnetostatic (MSCC) convective cells are excited simultaneously by KAW [6]. Proper theoretical treatment of spontaneous excitation of ESCC and MSCC by KAW must take into account: (i) the finite ion Larmor radius (FILR) corrections to the Reynolds stress; (ii) the finite coupling between ESCC and MSCC. In this work, theoretical predictions are compared with numerical simulation results of a kinetic ion–fluid electron hybrid model [7]. Excellent agreement is obtained for mode structure and generation rate of convective cells (CCs) by KAW, demonstrating that ESCC and MSCC are indeed coupled, and that spontaneous CC excitation is suppressed at long wavelength, showing the crucial role of FILR effects.

As KAW are efficiently excited by mode conversion with the shear Alfvén continuous spectrum, it can be assumed that their perpendicular $\boldsymbol{k}$ spectrum (to the ambient $\boldsymbol{B}$ field) is anisotropic and predominantly directed across magnetic flux surfaces, with negligible impact on cross field transport. Present results imply that CC are not only efficiently excited by KAW via modulational instability at short wavelengths, but also that they isotropize the KAW spectrum as consequence of scattering of the initial KAW spectrum off the CCs. Thus, this effect has potentially great impact on cross field transport in both space as well as fusion plasmas [6, 7].
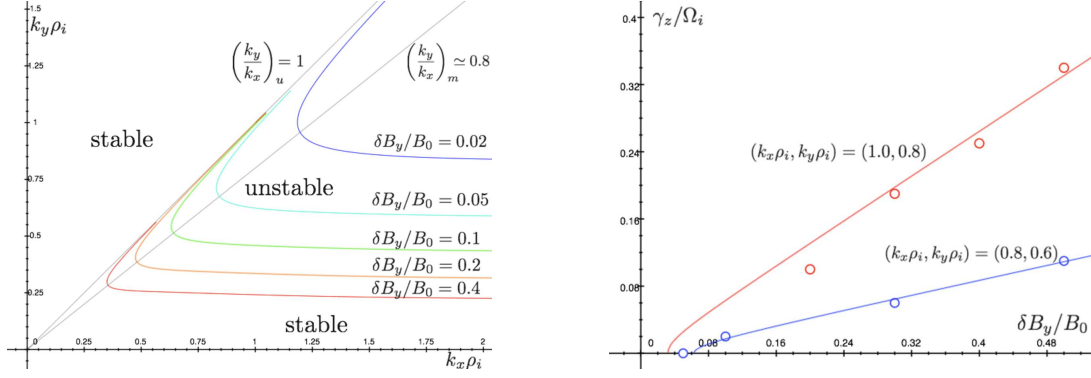
Figure 1: (left) Marginal stability curves in the $(k_x\rho_i, k_y\rho_i)$ plane for fixed $k_{\|0}\rho_i = 0.02$, $\tau = 1$ and $\beta_e = \beta_i = 0.2$ and different values of $\delta B_y/B_0$. (right) Modulational instability growth rate from Eq. (3) (continuous line) vs. $\delta B_y/B_0$ is compared with hybrid simulation results (open circles) for $(k_x\rho_i, k_y\rho_i) = (0.8, 0.6)$ (blue) and $(k_x\rho_i, k_y\rho_i) = (1.0, 0.8)$ (red).

Modulational instability dispersion relation

Consider an infinite, uniform, low-β plasma with $\boldsymbol{B}_0 = B_0\hat{\boldsymbol{z}}$. We adopt $\delta\phi$ and $\delta A_\|$ as the field variables, with $|k_\perp\rho_i|$ formally of $\mathscr{O}(1)$ and ρ_i the ion Larmor radius. Consider also 4-wave interactions among the pump KAW $\Omega_0 = (\omega_0, \boldsymbol{k}_0)$, the convective cell (CC), $\Omega_z = (\omega_z, \boldsymbol{k}_z)$ with $\boldsymbol{k}_z \cdot \hat{\boldsymbol{z}} = 0$; and the KAW sidebands, $\Omega_\pm = (\omega_\pm, \boldsymbol{k}_\pm)$, where $\omega_+ = \omega_z + \omega_0$, $\omega_- = \omega_z - \omega_0^*$ and $\boldsymbol{k}_\pm = \boldsymbol{k}_z \pm \boldsymbol{k}_0$. The pump KAW satisfies the dispersion relation and polarization condition

$$\varepsilon_{A0} = (1 - \Gamma_0)/b_0 - \sigma_0 k_{\|0}^2 v_A^2/\omega_0^2 = 0 \ \text{ and } \ \delta\psi_0 \equiv \omega_0\delta A_{\|0}/(k_{\|0}c) = \sigma_0\delta\phi_0 \ ; \qquad (1)$$

where $\Gamma_0 = I_0(b_0)e^{-b_0}$, I_0 is the modified Bessel function, $b_0 = k_{\perp 0}^2\rho_i^2$, $\rho_i = \Omega_i^{-1}(T_i/m_i)^{1/2}$, $\sigma_0 = 1 + \tau(1 - \Gamma_0)$ with $\tau = T_e/T_i$, and v_A is the Alfvén speed in the considered uniform equilibrium. Due to the interaction of the KAW pump Ω_0 with the CC Ω_z, up and down-shifted (in frequency) KAW sidebands are generated. It is possible to demonstrate that maximum nonlinear coupling occurs for $\boldsymbol{k}_z \perp \boldsymbol{k}_0$ [6], so that $b_\pm = (k_{\perp\pm}^2/k_{\perp 0}^2)b_0 = b_z + b_0$, $\Gamma_+ = \Gamma_-$, $\varepsilon_{A+} = \varepsilon_{A-}$, and

$$b_\pm\varepsilon_{A\pm} = 2\frac{(1 - \Gamma_\pm)/\omega_0}{(1 \pm \omega_z/\omega_0)^2}(\pm\omega_z - \Delta_\pm + \omega_z^2/2\omega_0) \ , \quad \frac{\Delta_\pm}{\omega_0} = \frac{b_\pm\sigma_\pm(1 - \Gamma_0) - b_0\sigma_0(1 - \Gamma_\pm)}{2b_0\sigma_0(1 - \Gamma_\pm)}; \quad (2)$$

with $\Delta_\pm = \Delta$ the frequency mismatch of KAW sidebands from the resonance condition $\varepsilon_{A\pm} = 0$. Here, for simplicity, we consider $\boldsymbol{k}_{\perp 0} = \hat{\boldsymbol{x}}k_{\perp 0}$ and, thus, $\boldsymbol{k}_z = \hat{\boldsymbol{y}}k_z$. Then, field equations for KAW sidebands can be solved for $\delta\phi_\pm$ and $\delta\psi_\pm \equiv (\omega_0 \pm \omega_z)\delta A_{\|\pm}/(k_{\|0}c)$ as a function of $\delta\phi_z$, $\delta\psi_z \equiv \omega_0\delta A_{\|z}/(k_{\|0}c)$, and the KAW pump amplitude. These expressions are then substituted into the evolution equations for $\delta\phi_z$ and $\delta\psi_z$; which, letting $\omega_z = i\gamma_z$, can be cast as [6]

$$\begin{aligned}
(\gamma_z^2 + \Delta^2)\delta\phi_z &= -\alpha_\phi(\delta\phi_z - \delta\psi_z) + \beta_\phi\delta\psi_z \ , \\
(\gamma_z^2 + \Delta^2)\delta\psi_z &= -\alpha_\psi(\delta\phi_z - \delta\psi_z) + \beta_\psi\delta\psi_z \ ,
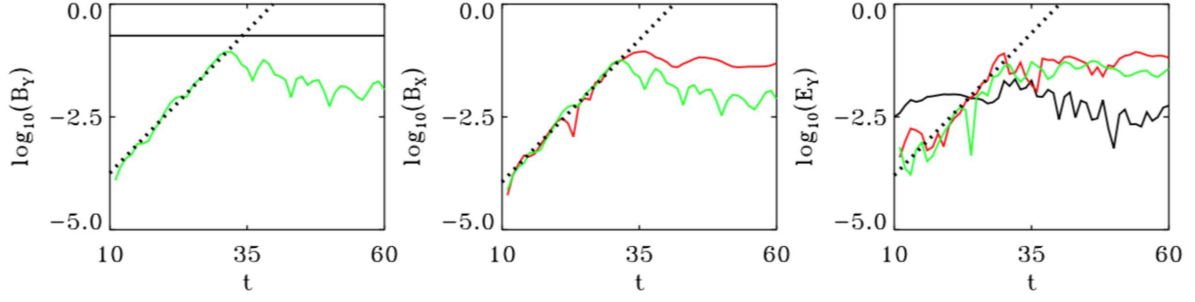\end{aligned} \qquad (3)$$

Figure 2: Time evolution of B_y, B_x, and E_y in the pump KAW (black), daughter convective cell (red), and the matching KAW (green) modes.

which yield CC mode structure and modulational instability dispersion relation. Here, α_ϕ, β_ϕ, α_ψ and β_ψ are known functions of $|\delta B_y/B_0| = |(\sigma_0 c/\omega_0 B_0)k_{\perp 0}k_{\parallel 0}\delta\phi_0|$; and ω_0, $k_{\perp 0}$ and k_z [6]. In the long wavelength limit, $b_0 \ll 1$, we generally have $\delta\psi_z/\delta\phi_z \sim \mathcal{O}(1)$. However, Eq. (3) reduces to $\gamma_z^2 + \Delta^2 \simeq (\alpha_\psi - \alpha_\phi) < 0$. Thus, KAW can not spontaneously excite CC in the $b_0 \ll 1$ limit. Meanwhile, assuming that $\delta\phi_z$ (ESCC) is suppressed, delivers the erroneous claim that MSCC can be spontaneously excited by KAW for $b_0 \ll 1$.

In the short wavelength limit, $b_0 \gg 1$, we have again that $\delta\psi_z/\delta\phi_z \sim \mathcal{O}(1)$ and Eq. (3) reduces to $\gamma_z^2 + \Delta^2 \simeq \alpha_\phi[\tau - (1+\tau)(b_z/2b_0)]$, with $\alpha_\phi \propto b_z^2$. Thus, $b_z < 2b_0\tau/(1+\tau)$ is requested for modulational instability, and maximum growth rate is obtained for $(b_z/b_0)_m \simeq (4/3)\tau/(1+\tau)$. Meanwhile, $\alpha_\phi\tau > \Delta^2$ is also needed for modulational instability, which poses a lower bound on b_z for effective CC excitation.

Numerical solution of the modulational instability dispersion relation

From Eq. (3), it is generally expected that $\delta\psi_z/\delta\phi_z \sim \mathcal{O}(1)$; i.e., ESCC and MSCC should be excited by KAW simultaneously for $b_0 \sim \mathcal{O}(1)$. This is shown by numerical solution of Eq. (3) in Fig. 1(left), reporting the marginal stability curves in the $(k_x\rho_i, k_y\rho_i) = (k_{\perp 0}\rho_i, k_z\rho_i)$ plane for different values of the KAW pump, $\delta B_y/B_0 = 0.02, 0.05, 0.1, 0.2, 0.4$. Here, fixed parameters are $k_{\parallel 0}\rho_i = 0.02$, $\tau = 1$ and $\beta_e = \beta_i = 0.2$. The amplitude dependent lower threshold in $k_y\rho_i = k_z\rho_i$, and the upper threshold at $k_y = k_x$ for $\tau = 1$ predicted theoretically are clearly visible. The maximum CC growth rate condition $(k_y/k_x)_m \simeq (2/3)^{1/2} \simeq 0.8$ for $\tau = 1$ is also well verified. For the same fixed parameters, Fig. 1(right), meanwhile, shows the modulational instability growth rate vs. the KAW pump amplitude $\delta B_y/B_0$ for $(k_x\rho_i, k_y\rho_i) = (0.8, 0.6)$ and $(k_x\rho_i, k_y\rho_i) = (1.0, 0.8)$, compared with numerical results from hybrid simulations.

Hybrid Simulation

The hybrid simulation scheme is similar to that of *Lin, Johnson, and Wang* [7, 8], in which ions are treated as fully kinetic particles moving in a self-consistent electromagnetic field, and electrons are treated as a massless fluid. The computation scheme is fully nonlinear, but our analysis of the modulational instability focuses only on the early stage of its exponential growth. Fixed parameters are those of Fig. 1, lengths are normalized to ρ_i, and the time to Ω_i^{-1}.

The pump mode is imposed everywhere as a steady driver at each time step, with magnetic field $\delta \mathbf{B}_0 = (0, \delta B_y, 0)\sin(\mathbf{k}_0 \cdot \mathbf{x} - \omega_0 t)$ and specified wave number and frequency, which, in the hybrid simulation, are consistent with Eq. (1). From $t = 0\text{-}10$, the system is filtered to keep only the Fourier mode $\mathbf{k_0}$ of initial pump, which allows a self-consistently development of the pump mode structure. For $t > 10$, more Fourier modes are released in order to examine the excitation of the convective cell modes. Figure 2 shows the time evolution of B_y, B_x, and E_y in the resulting three wave interaction for a case with constant pump amplitude $\delta B_y = 0.4$ at $\mathbf{k}_0 = (1.25, 0, 0.02)$. The black solid curves show the pump mode, for which B_y is constant in time. The E_y component of the pump is two orders of magnitude smaller than E_x (not shown). The excitation of the CC mode, with wave number $\mathbf{k}_z = (0, 0.8, 0)$, is shown with the red curve. Both B_x and E_y increase nearly exponentially with time from $t = 10$, as fitted by the straight dotted line, reaching the saturation levels at $t \simeq 31$. The growth rate is measured to be $\gamma/\Omega_i = 0.29$. No power is excited in the B_x component, consistent with $k_x = 0$ in the CC mode. The green curve depicts the matching KAW mode with $\mathbf{k}_+ = (1.25, 0.8, 0.02)$, in which B_x, B_y, and E_y are all seen to also grow exponentially with $\gamma/\Omega_i = 0.29$.

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