Effect of bremsstrahlung emission on fast electrons in plasmas

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Background Bremsstrahlung radiation is one of the most important energy loss mechanisms for fast electrons in plasmas. At a few tens of MeV, the energy loss associated with the emission of bremsstrahlung radiation can dominate the collisional energy loss [1, 2].

An important electron acceleration process, producing energetic electrons in both space and laboratory plasmas, is the runaway mechanism. In the presence of an electric field which exceeds the minimum to overcome collisional friction, a fraction of the charged particles can be accelerated to relativistic energies, where radiative losses become important. Previous studies of laboratory plasmas [3] and lightning discharges [4] have shown that the energy lost by bremsstrahlung is important in limiting the energy of runaway electrons. However, only the average bremsstrahlung friction force on test particles has been considered in these studies. In this contribution, we present the first quantitative kinetic study of how bremsstrahlung emission affects the runaway-electron distribution function.

Starting from the Boltzmann electron transport equation, we derive a collision operator representing bremsstrahlung radiation reaction, fully accounting for the finite energies and emission angles of the emitted photons [5]. We find significant differences in the distribution function when bremsstrahlung losses are modeled with a Boltzmann equation (referred to as the "Boltzmann" or "full" bremsstrahlung model), compared to the model where only the average friction force is accounted for (the "mean-force" model). In the former model, the maximum energy reached by the energetic electrons is significantly higher than is predicted by the latter. In previous treatments which considered isotropic plasmas [2] or average energy loss [3, 4], the emission of soft (low-energy) photons did not influence the electron motion. We show that in the general case, emission of soft photons contributes significantly to angular deflection of the electron trajectories.

Formulation We treat bremsstrahlung as a binary interaction ("collision") between two charged particles, resulting in the emission of a photon. The effect of such a process on the electron distribution function f_e, due to interactions with another species b, is described by a Boltzmann collision operator $C^{\mathrm{B}}_{eb}\{f_e, f_b\} = (\mathrm{d}n_e)_{c,eb}/\mathrm{d}t\mathrm{d}\mathbf{p}$, where the differential change $(\mathrm{d}n_e)_{c,eb}$ in the phase-space density due to collisions in a time interval $\mathrm{d}t$ is given by $\big(\mathrm{d}n_e\big)_{c,eb} = f_a(\mathbf{p}_1)f_b(\mathbf{p}_2)\bar{g}_{\o}\mathrm{d}\bar{\sigma}_{eb}\mathrm{d}\mathbf{p}_1\mathrm{d}\mathbf{p}_2\mathrm{d}t - f_a(\mathbf{p})f_b(\mathbf{p}')g_{\o}\mathrm{d}\sigma_{eb}\mathrm{d}\mathbf{p}\mathrm{d}\mathbf{p}'\mathrm{d}t$. Here, $\mathrm{d}\sigma_{eb} = \mathrm{d}\sigma_{eb}(\mathbf{p}_1, \mathbf{p}_2, \mathbf{k}; \mathbf{p}, \mathbf{p}')$ is the differential cross-section for a particle a of momentum $\mathbf{p}$

and a particle b of momentum $\mathbf{p}'$ to be taken to momentum $\mathbf{p}_1$ and $\mathbf{p}_2$, respectively, while emitting a photon of momentum $\mathbf{k}/c$. We have also introduced the Møller relative speed $g_ø = \sqrt{(\mathbf{v}-\mathbf{v}')^2 - (\mathbf{v}\times\mathbf{v}')^2/c^2}$. The barred quantities $\mathrm{d}\bar{\sigma}$ and $\bar{g}_ø$ are defined likewise, but with $(\mathbf{p}, \mathbf{p}')$ and $(\mathbf{p}_1, \mathbf{p}_2)$ exchanged. Here, we consider only the contribution from spontaneous emission, however the contribution from absorption and stimulated emission [6] can be important in large systems where a significant background radiation is present.

The bremsstrahlung collision operator takes a particularly simple form when considering a cylindrically symmetric distribution function interacting with stationary targets. This is well motivated for runaway electrons in a magnetized plasma, which interact primarily with the bulk plasma which is near local thermal equilibrium. In terms of a decomposition in Legendre polynomials P_L, writing $A(p,\xi) = \sum_L A_L(p)P_L(\xi)$ for any function A of momentum coordinates $\mathbf{p} = (p, \xi, \varphi)$ in a spherical coordinate system where ξ is the pitch-angle cosine, the bremsstrahlung collision operator takes the form

$$C^{\mathrm{B}}_{eb,L}(p) = n_b \int \mathrm{d}p_1 \left[p_1^2 v_1 f_L(p_1)\, 2\pi \int_{-1}^{1} \mathrm{d}\cos\theta_s\, P_L(\cos\theta_s) \frac{\partial \bar{\sigma}_{eb}}{\partial \mathbf{p}} \right] - n_b v f_L(p) \sigma_{eb}(p). \qquad (1)$$

where $\sigma_{eb} = \int \mathrm{d}\mathbf{p}_1\, (\partial\sigma_{eb}/\partial\mathbf{p}_1)$ is the total cross-section. The integration limit in p_1 is determined by the conservation of energy in the collision, which gives $m_e c\sqrt{(\gamma + k_0/m_e c^2)^2 - 1} < p_1 < \infty$, where $\gamma = \sqrt{1 + p^2/m_e^2 c^2}$ is the Lorentz factor, and k is the photon energy.

The contribution from photon energies $k < k_0$, for some cut-off $k_0 \ll m_e c^2(\gamma - 1)$, needs to be treated with extra care, as the collision integral is logarithmically divergent: For small k, $\partial\bar{\sigma}_{eb}/\partial\mathbf{p} \propto 1/k$. The energy carried away by the photon energy can be neglected in this contribution to the collision operator. This yields a simpler, elastic, collision operator which describes those interaction, taking the form

$$C^{\text{small-}k}_{eb,L} = -n_b v f_L(p) \int_{-1}^{1} \mathrm{d}\cos\theta_s \left[1 - P_L(\cos\theta_s)\right] \int_{k_c}^{k_0} \mathrm{d}k \, \frac{\partial \sigma_{eb}}{\partial k \partial \cos\theta_s}. \qquad (2)$$

This operator needs to be cut off at a photon energy $k_c < k_0$, which due to dielectric suppression takes the value $k_c \sim \hbar\omega_p$ [7]. This term describes bremsstrahlung-induced pitch-angle scattering which, due to the logarithmic divergence in k, is enhanced relative to the $k > k_0$ part of the bremsstrahlung operator by a "bremsstrahlung logarithm" $\ln\Lambda_{\mathrm{B}} = \ln k_0/k_c$.

In our calculations, we will use the differential cross-section $\partial\sigma/\partial\mathbf{p}$ in the first Born approximation, first derived by Racah [8]. The rate of pitch-angle scattering from (2) can then be compared to the contribution from elastic Coulomb scattering, and the ratio between the $L = 1$ terms (the transport cross-section) of the two is analytically given by

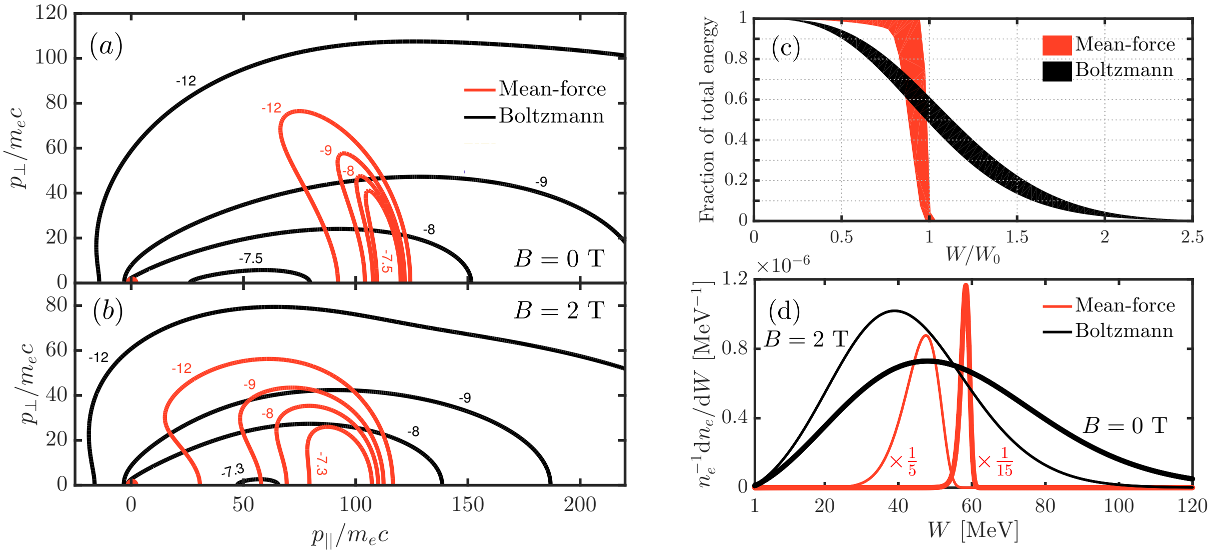

Figure 1: Comparison of the Boltzmann and mean-force bremsstrahlung models. (a), (b) 2D distribution functions (contours of $\log_{10}[f(m_e c)^3/n_e]$); (c) fraction of plasma kinetic energy carried by electrons more energetic than $W = m_e c^2(\gamma - 1)$; (d) angle-averaged distribution functions.

$C_1^{\text{small-k}}/C_1^{\text{Coulomb}} = \alpha(2/\pi)(\ln\Lambda_{\text{B}}/\ln\Lambda)\{[\ln(2p/m_e c) - 1]^2 + 1\}$, where $\alpha \approx 1/137$ is the fine-structure constant. The bremsstrahlung pitch-angle scattering thus increases in importance with electron energy: at the 10 MeV scale it typically represents a 10% modification, while at 10 GeV energies it equals the elastic pitch-angle scattering rate.

Results We shall determine the effect of bremsstrahlung losses on the steady-state runaway distribution. The situation of primary interest is when the electric field is stronger than (but comparable to) the critical field, $E > E_c = m_e c/e\tau_c$, and we consider the quasi steady-state distribution which is set up over times longer than the relativistic collision time, $t \gg \tau_c = 4\pi\varepsilon_0^2 m_e^2 c^3/(n_e \ln\Lambda e^4)$, where $\tau_c \sim 20\,\text{ms}$ at $n_e = 10^{21}\,\text{m}^{-3}$. We use the Fokker-Planck solver CODE [9, 10] to solve the kinetic equation, accounting for acceleration by an electric field, elastic Coulomb collisions and radiation losses. We will compare the Boltzmann model with the mean-force model, where bremsstrahlung losses are modeled as a continuous slowing-down force, given by the stopping-power formula $\mathbf{F}_{\text{B}} = -\hat{\mathbf{p}}\sum_b n_b \int_0^{m_e c^2(\gamma-1)} \mathrm{d}k\, k\partial\sigma_{eb}/\partial k$ [1], the sum taken over all particle species in the plasma.

The results are summarized in Fig. 1. We use parameters representing a tokamak disruption scenario with massive gas injection, with electron density $n_e = 3\cdot 10^{21}\,\text{m}^{-3}$, plasma effective charge $Z_{\text{eff}} = 10$ and electric field $E = 2E_c$. The distribution functions are shown in (a), (b) and (d), demonstrating large qualitative differences between the mean-force and Boltzmann models. Both models show that radiation losses will limit the maximum electron energy, but the Boltzmann model predicts a significantly wider distribution of energies. Synchrotron radiation reaction losses, associated with the gyromotion of electrons in a straight magnetic field, has also been shown to be an important energy-loss mechanism [11–14]. Figure 1(b) shows that, in conjunction with bremsstrahlung losses, synchrotron losses (modeled as in Ref. [13]) shifts the distribution towards lower energies but does not change its qualitative features. In addition, the low-energy photon emissions described by Eq. (2) cause a wider pitch-angle distribution, which can be seen in (a) and (b) where the black contours extend further in the perpendicular direction. Figure 1(c) shows the fraction of plasma kinetic energy carried by electrons more energetic than $W = m_e c^2(\gamma - 1)$, normalized to the kinetic energy W_0 which solves the force-balance equation $F_{\rm B}(W) = eE - eE_c$. The filled regions represent the values spanned as $Z_{\rm eff}$ is varied between 1 and 35, and $(E/E_c - 1)/(1 + Z_{\rm eff})$ between 0.05 and 0.25. From the figure we see that a significant fraction of energy, of order 5%, is carried by electrons with kinetic energy more than twice that which the mean-force model predicts. This result is very robust, with the same behavior being observed for a large range of electric fields and effective charges. In terms of the normalized units shown in figure 1(c), the behavior is independent of plasma density, except for a weak logarithmic dependence in the Coulomb logarithm. The behavior we have observed here may therefore apply to a wide range of plasmas where bremsstrahlung losses are important.

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