Enhancements to the ELITE code and application to QH-mode in DIII-D

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The quiescent H-mode (QH) is a high performance edge localised mode (ELM) free mode of operation which was discovered on DIII-D [1]. There are now two types of known QH-modes [2]. In the standard QH-mode [1], the pedestal is limited by an MHD edge harmonic oscillation (EHO) with a low toroidal mode number (n), typically $n \sim 1 - 3$. This EHO has been found to be a saturated kink-type mode, which is destabilised by flow shear, and provides density control [3, 4]. The newly discovered wide-pedestal QH-mode has increased pedestal width, height and thermal confinement. It generally has no EHO and is characterised by broad band MHD which provides the density control [2]. ELITE [5, 6] is an edge ideal MHD stability code that is optimised for the study of intermediate-high n modes associated with ELMs, using peeling-ballooning theory. The code solves a simplified set of equations for the eigenfunction based on a rigorous expansion in inverse toroidal mode number, n^{-1}. These equations are derived from δW, the change in energy associated with a radial perturbation of a plasma fluid element X [5, 6]. In intermediate-high n ELITE the δW ideal MHD energy principal can be written in terms of a single radial perturbation, X, [5, 7] which leads to the peeling-ballooning equation. The associated Euler equation is then solved using Fourier decomposition of X in straight field line poloidal angle, ω:

$$X = \sum_m u_m(\psi) e^{-im\omega} \tag{1}$$

where ψ is the poloidal flux. This leads to a set of coupled differential equations in the form:

$$A''_{km} \frac{d^2 u_m}{d\psi^2} + A'_{km} \frac{d u_m}{d\psi} + A_{km} u_m = 0 \tag{2}$$

ELITE has been extended to arbitrary n for the study of pedestals dominated by low n activity. Extending the formalism requires that the ideal MHD energy principle be taken in the incompressible limit with two perturbation displacements in the direction perpendicular to the field lines, U and X. We write an expansion for U as the leading order terms in X and group all remaining terms into W:

$$U = \frac{i}{n} \frac{\partial X}{\partial \psi} + \frac{ip'}{nB^2} X + \frac{iW}{n} \tag{3}$$

This leads to a modified δW equation with two perturbation displacements. The resulting Euler equation can be solved using Fourier expansions for both X and W, which produces two sets of coupled equations for $u_m(\psi)$ and $w_m(\psi)$ (the Fourier coefficients of W):

$$A''_{km} \frac{d^2 u_m}{d\psi^2} + A'_{km} \frac{du_m}{d\psi} + A_{km} u_m + B'_{km} \frac{dw_m}{d\psi} + B_{km} w_m = 0$$

$$C_{km} w_m + D'_{km} \frac{du_m}{d\psi} + D_{km} u_m = 0$$

(4)

As there are no W derivatives in the second equation it is readily inverted and inserted into the first equation to yield the same ELITE-like form as equation 2, but with modified coefficients.

This allows the equations to be solved using the same framework as the original ELITE code. We have performed various successful benchmarks for different types and shapes of equilibria to verify the results produced by the new low n ELITE. Specifically we have compared with the original ELITE code, as well as the codes GATO and MARG2D at low n. Agreement has been obtained in both the growth rates and eigenfunctions.

Figure 1: *D_α trace for shot 163520, with colours showing the times where the shot was analysed. 2200ms (green), 2650ms (red), 3000ms (orange), 3500ms (purple) and 3985ms (blue). The transition from EHO to wide -pedestal QH-mode is just after 2400ms*

We have used the new arbitrary n version of ELITE to study low n modes in DIII-D QH-mode shot 163520. This shot has both wide-pedestal and EHO QH-modes. The transition from standard to wide-pedestal QH mode, which appears at low input torque in near double null discharges, is seen as a sudden increase in electron pressure pedestal width coupled with the appearance of broad band magnetic fluctuations. We have analysed the stability at 5 times in the shot, which are marked on the D_α trace in figure 1. The stability boundary has been taken as $\gamma/\omega_A = 0.02$.

The first time is 2200ms shown in figure 1 as green. This is during the coherent EHO phase, at high torque, before the wide-pedestal transition. Here we studied modes from $n = 2 - 10$, finding that this pulse lies on the kink/peeling boundary. From figure 2 it can be seen that all the low-intermediate n modes have non-zero growth rate
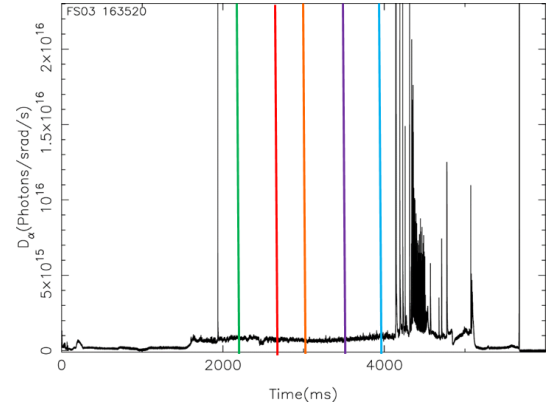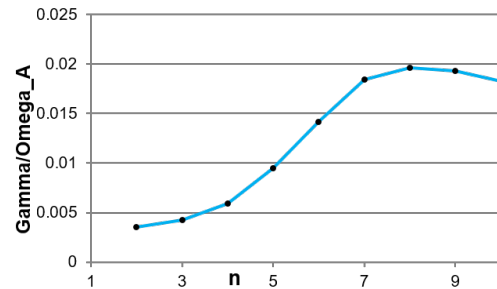

Figure 2: *growth rate (γ/ω_A) vs. n for time=2200ms*

and at $n = 8$, the growth rate peaks very close to the stability boundary. The mode structures for low n are highly radially localised and kink/peeling-like in structure, consistent with the observation of coherent EHO. Additional analysis was performed using a new diagnostic in ELITE which analyses the individual contributions to δW. Analysing $n = 5 - 15$, shown in figure 3, confirms the drive is predominately kink/peeling. The next 3 times analysed are the first and two middle wide pedestal phases; shown in figure 1 and given by: 2650ms (red), 3000ms (orange), and 3500ms (purple).

Time 2650ms is just after the transition to the wide-pedestal QH-mode phase, so the pedestal width is still relatively small and torque is still significant. Time 3000ms is in the middle of the wide-pedestal phase just after reaching zero torque, before the power and density ramp occurred in the shot. Time 3500ms is in the middle of the wide-pedestal phase just after reaching zero torque, but during the power and density ramp phase.

Figure 3: *Amplitude of drive terms for $n = 5 - 15$ at time=2200ms. Positive magnitude is stabilising, negative is destabilising*

All three phases were shown in previous ELITE analysis to $n \geq 5$ to be below the kink/peeling stability boundary. Using the new low n ELITE stability was found down to $n = 2$, consistent with other previous findings in [2]. To study the proximity to marginal stability, the current density was steadily increased from the experimental point, towards the stability boundary. It was found in all three phases that the first mode to be destabilised was $n = 4$. The final time in the shot to be analysed is at 3985ms, shown in figure 1 as blue. This is at the end of the power and density ramp, just before the ELMs return. This was previously shown in ELITE analysis to be below the kink/peeling boundary for $n \geq 5$. Using the new low n ELITE, the growth rates for this phase with experimental pedestal and edge current density were found to
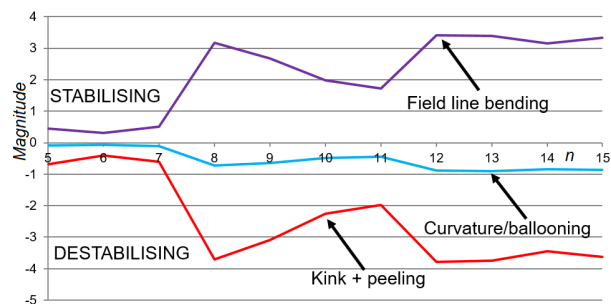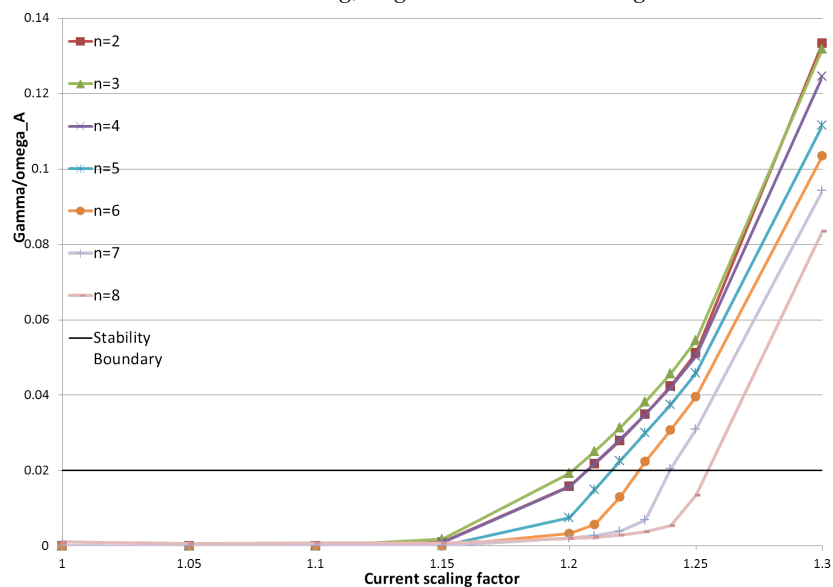

Figure 4: *Time 3985 growth rate (γ/ω_A) vs. current scaling factor, where 1=100% and 1.1=110%, for $n = 2 - 8$. Stability boundary=0.02*

be exclusively stable. However, a low n mode of $n = 1$ or $n = 2$ was seen on the magnetics in this pulse just before the crash. Note that this is lower than the typical $n = 4 - 6$ mode seen as ELM precursors. Therefore another scan of increasing current density was performed to explore the sensitivity. The results can be seen in figure 4. These results show that low n modes of $2 < n < 4$ are destabilised first, for a 20% increase in current density. The low n mode structures are very different to those seen in the EHO QH-mode phase: they are much broader global modes. Note there is no $n = 1$ analysis in these results as the $n = 1$ mode structure extends to the magnetic axis and we have not yet implemented an appropriate boundary condition for such situations.

To conclude, ELITE has been extended to arbitrary n to allow the study of low n dominated phenomena. Successful benchmarks have been performed against the original ELITE formalism, as well as against the codes MARG2D and GATO. The new δW diagnostic provides improved physics insight into the peeling-ballooning drive. An analysis of DIII-D shot 163520 has also been performed, which allows comparison between the standard EHO and wide-pedestal regimes. The QH-mode with EHO has a low n peeling mode present which is localised in the pedestal region. During most of the wide-pedestal phases the plasma is predicted to be stable to $1 < n < 10$ modes and sits below the peeling boundary. This is consistent with observations in [2]. When the calculated current density is increased by 20% an $n = 4$ mode is seen. In the final wide-pedestal phase increasing the current density by 20% destabilises low n modes between $2 < n < 4$. These modes are much more global than seen in the standard EHO phase. Additionally including these low n modes in the ELITE analysis does not significantly change the stability boundary. Further work to be done is to prepare the low n code for release to ELITE users. Also to be done is to include an on-axis boundary condition and wall physics for wall and global modes. Additional work will seek to incorporate flow shear into arbitrary n ELITE.

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