

Flux conservation and inductive effects in the disruption modelling

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1. Introduction. Disruptions in tokamaks induce large currents in the conducting structures around the plasma. This results in huge forces on the vacuum vessel (simply called ‘wall’), as observed in JET [1, 2]. The level of danger increases for tokamaks with stronger magnetic fields and plasma currents [2–4] so that even the vessel integrity becomes an issue [3, 4]. For future devices like ITER and IGNITOR the problem is aggravated by a wide scatter in theoretical estimates because of uncertainties in the underlying disruption physics basis [3]. Additionally, the plasma-wall electromagnetic interaction is usually, with rare exceptions, treated within simplified models. For example, in several famous codes used in fusion, the wall is replaced by a set of axisymmetric toroidal filaments, see [4–6] and references therein. Then the poloidal current in the wall is technically forbidden. The filamentation has been introduced long ago ‘for convenience’, but without detailed physics justification and fixing of the applicability limits. Its consistency with flux conservation constraints is analyzed here.

Here, the disruption-induced poloidal current in the tokamak wall is evaluated within the large-aspect-ratio flux-conserving model. The derived formulas describe the consequences of both thermal quench (TQ) and current quench (CQ). They give explicit dependence of the current on the plasma parameters. The estimates are compared with numerical results of [4].

2. Formulation of the problem. Disruptions are the events with the whole plasma undergoing gross rapid changes. In the ITER tokamak, the thermal quench duration is expected to be ~ 1 ms, while the vacuum vessel time constant for the $m/n = 1/1$ mode is 0.25 s [3] and the plasma resistive time is even larger. Then, at the early stage of disruptions, both plasma and wall should react on magnetic perturbations as ideal conductors. In particular, the toroidal fluxes

$$\Phi_\alpha \equiv \int_a \mathbf{B} \cdot d\mathbf{S}_\alpha = \frac{1}{2\pi} \int_\alpha \mathbf{B} \cdot \nabla \zeta d\tau \quad (1)$$

must remain constant during the plasma evolution. Here \mathbf{B} is the magnetic induction, α denotes, respectively, the plasma (pl) and plasma-wall gap (g), and $d\tau$ is the volume element. For the toroidal plasma, the relevant mathematics is briefly described in [7]. Here, for estimates, we use a simpler ‘quasi-cylindrical’ approach and prescribe the fluxes by

$$\Phi_{pl} = \int_{pl} B_z dS_{pl} \equiv \bar{B}_z S_{pl}, \quad (2)$$

$$\Phi_g = B_g S_g, \quad (3)$$

where S_{pl} and S_g are, respectively, the plasma and gap areas in the toroidal cross-section, B_z is the toroidal field in the plasma, B_g is B_z at its boundary, the bar is the averaging over S_{pl} :

$$\overline{XS}_{pl} \equiv \int XS_{pl}. \quad (4)$$

The plasma is assumed axially symmetric when it evolves subject to equilibrium force balance

$$\nabla p = \mathbf{j} \times \mathbf{B} \quad (5)$$

with p the plasma pressure, $\mathbf{j} = \nabla \times \mathbf{B} / \mu_0$ the current density, μ_0 the vacuum permeability.

3. Variations at flux conservation. It follows from (2) and (3) that

$$\overline{B_z} - B_g = \frac{\Phi_{pl}}{S_{pl}} - \frac{\Phi_g}{S_g}. \quad (6)$$

Therefore, when the plasma state is changed,

$$d(\overline{B_z} - B_g) = \left(\frac{\Phi_{pl}}{S_{pl}^2} + \frac{\Phi_g}{S_g^2} \right) dS_g + \frac{d\Phi_{pl}}{S_{pl}} - \frac{d\Phi_g}{S_g}, \quad (7)$$

where a natural relation

$$S_{pl} + S_g \equiv S_w = \text{const} \quad (8)$$

has been used. With

$$B_g dS_g = d\Phi_g - S_g dB_g, \quad (9)$$

which is a consequence of (3), equation (7) yields

$$S_{pl} d(\overline{B_z} - B_g) = d\Phi_{pl} + \frac{\overline{B_z}}{B_g} d\Phi_g - \left(S_{pl} + S_g \frac{\overline{B_z}}{B_g} \right) dB_g. \quad (10)$$

If the fluxes Φ_{pl} and Φ_g (or, at least, their sum) remain constant, this gives us

$$dB_g = \frac{S_{pl}}{S_{pl} + S_g} d(B_g - \overline{B_z}), \quad (11)$$

where a small correction of the order of $(1 - \overline{B_z}/B_g)$ is disregarded. Here we treat B_g as a constant, but by replacing it by the gap-averaged field we make (11) applicable for a torus [7].

The right-hand side of (11) can be expressed through the plasma parameters by using the integral consequence of (5), as described in [7]. Below, the relations allowing easy estimates are derived in the standard cylindrical model where (5) reduces to

$$\rho \frac{d}{d\rho} (2\mu_0 p + \mathbf{B}^2) = -2B_\theta^2 \quad (12)$$

with ρ denoting the radius in the plasma perpendicular cross-section, B_θ the poloidal field, and θ the poloidal angle. The integral consequence of (12) is

$$2\mu_0\bar{p} = B_g^2 - \overline{B_z^2} + B_J^2 \quad (13)$$

with B_J being B_θ at the plasma boundary. By exploiting this relation and

$$B_g^2 - B_z^2 = 2B_g(B_g - B_z) - (B_g - B_z)^2, \quad (14)$$

where the last term is negligibly small and hence disregarded, we transform (11) into

$$2\frac{dB_g}{B_g} = \frac{S_{pl}}{S_{pl} + S_g} d\left(\beta - \frac{B_J^2}{B_g^2}\right) \quad (15)$$

($\beta \equiv 2\mu_0\bar{p}/B_g^2$ is the ratio of the plasma and magnetic pressures).

Note that equations (1)–(11) are valid at any plasma shape. A circular plasma appeared when equation (12) was introduced for finding the right-hand side of (11). For a noncircular plasma, one can use the approximate relation [8, 9]

$$\Phi_{pl} - B_g S_{pl} \approx \frac{2K}{K^2 + 1} \frac{(\mu_0 J)^2}{8\pi B_g} (1 - \beta_p), \quad (16)$$

where J is the plasma net toroidal current, $\beta_p \equiv 2\mu_0\bar{p}/B_{pb}^2$, $B_{pb} \equiv \mu_0 J/L_p$, K and L_p are, respectively, the vertical elongation and the poloidal circumference of the plasma surface. With (16) we obtain from (11) a generalization of (15):

$$\frac{dB_g}{B_g} = \frac{1}{S_{pl} + S_g} \frac{2K}{K^2 + 1} d\left[\frac{(\mu_0 J)^2}{8\pi B_g^2} (\beta_p - 1)\right]. \quad (17)$$

Equations (15) and (17) give dB_g , the change in the toroidal field in the plasma-wall vacuum gap under the condition that $\Phi_{pl} + \Phi_g = \text{const}$. Since

$$2\pi R_0 B_g = \mu_0 I_g, \quad (18)$$

where R_0 is the tokamak major radius and I_g is the net poloidal current external to the plasma,

$$dB_g / B_g = dI_g / I_g. \quad (19)$$

Combining this with (17), we have (at $K = \text{const}$)

$$dI_g = \frac{\mu_0}{4B_g} \frac{R_0}{S_{pl} + S_g} \frac{2K}{K^2 + 1} d[J^2(\beta_p - 1)]. \quad (20)$$

At fixed currents in the toroidal coils, this gives the poloidal current induced in the wall.

4. Estimates. This analytical prediction can be compared with the results of [4], where a detailed analysis of disruptions in the IGNITOR tokamak has been performed by using two

numerical codes, MAXFEA and CarMa0NL. With $R_0 = 1.32$ m, $S_{pl} = 1.27$ m², $S_g = 0$, $K = 1.83$, $B_g = 13$ T, $J = 11$ MA and $\beta_p = 0.2$, the same as parameters in optimized scenario described in [4], equation (20) gives us $dI_g \approx 2$ MA at the end of disruption ($\beta_p = J = 0$). In [4], the maximum total poloidal current in the wall was found to be of the order of 1 MA on the whole torus. The difference by factor of 2 may be attributed to inherent limitations of analytical modelling or the flux conservation constraint used here, but it also points out to necessity of further theoretical and numerical studies.

In ITER [3], $R_0 = 6.2$ m, $S_{pl} = 22$ m², $K = 1.7$, $B_g = 5.35$ T and $J = 15$ MA. Assuming $S_{pl} + S_g = 1.44S_{pl}$ and $\beta_p = 0.62$ before TQ, as in [6], at the end of TQ (when β_p drops to zero at the fixed current) we have from (20) $dI_g \approx -1.4$ MA. Similarly, we obtain $dI_g \approx 0.9$ MA at the end of CQ ($\beta_p = J = 0$). This quantifies the consequence of (15) and (17) that the TQ and CQ induce the wall current of opposite polarities.

5. Conclusions. It should be noted that the key element here is the small deformation of the plasma cross-section, $dS_{pl} = -dS_g \neq 0$ when the plasma is evolved in a flux-conserving manner. This is clearly seen in (9) that gives $dB_g/B_g = -dS_g/S_g$ when Φ_g is fixed. Accordingly, as implied by (15), the flux-conserving plasma must slightly expand at β drop and shrink at decrease of J . The relative change of the volume can hardly exceed 1%, but this is sufficient for inducing the poloidal current in the wall on the MA level in ITER-like or reactor tokamaks. Such current (not accounted for when the wall is represented by toroidal filaments) flowing perpendicular to the toroidal field must be a source of strong local forces on the wall. Our analysis confirms the importance of this effect recently found [4] in simulations for IGNITOR. The estimates show that this should be taken into account in the studies of disruptions and disruption mitigation in ITER.

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