Finite banana width effect on NTM threshold physics

K. Imada[1], J.W. Connor[2], H.R. Wilson[1]

[1] *York Plasma Institute, Dept. of Physics, University of York, Heslington, YO10 5DD, U.K.*

[2] *Culham Centre for Fusion Energy, Abingdon, Oxfordshire, OX14 3DB, U.K.*

Introduction

Neoclassical tearing modes (NTMs) are resistive MHD instabilities characterised by the formation and evolution of magnetic islands. If allowed to grow, they can limit tokamak performance by reducing the core pressure. Controlling NTMs in future devices, such as ITER, is therefore crucial for successful operations. Experimentally, it is observed that there exists a threshold to the island width, whereby a sufficiently small seed magnetic island heals itself and shrinks away [1]. Theoretically, it has been suggested that the finite banana width of trapped particle orbits may play a role in this threshold effect [2]. Since the observed threshold width is typically comparable to the banana width of trapped particles, it is important to determine the importance of this finite banana width effect, in order to develop an effective NTM control system, which would shrink seed islands to below the threshold width.

In toroidal geometry, the finite banana width effect gives rise to the neoclassical polarisation current, induced when the magnetic island moves relative to the plasma [2]. This in turn generates a parallel current perturbation, which may or may not stabilise the island depending on the conditions. The finite banana width effect also accounts for the radial transport of particles and heat, which tends to partially restore the flattened pressure profile near the island separatrix [3]. For a sufficiently small island, this reduces the perturbation in the pressure gradient-dependent bootstrap current which in turn reduces the drive for the NTM growth. Our goal is to investigate the full effect of the finite banana width using drift kinetic theory. We have developed a numerical code with a novel iteration scheme to calculate the electrostatic potential perturbation using quasineutrality as well as the momentum conservation term in a model collision operator. An accurate determination of the electrostatic potential is vital for calculating the neoclassical polarisation current, while the momentum conservation term is needed for an accurate calculation of the bootstrap current perturbation. In our calculation, we have relaxed the small ion banana width assumption, compared to the island width (i.e. $\rho_{bi} \sim w$). This enables us to explore the full particle orbit physics and its consequences for the island evolution. In this paper, we examine the ion response to the magnetic island perturbation using the drift kinetic equation, expanding in the small ratio of the island width to the tokamak minor radius: $\Delta = w/r \ll 1$, while retaining the ordering $\rho_{bi} = \varepsilon^{1/2}\rho_\theta \sim w$. We also demonstrate that our numerical code can produce a solution that is a function of a modified "flux" function, Ω_s, which is shifted from the perturbed flux surfaces by a distance $O(\rho_{bi})$, as indicated by analytic theory.

Drift Kinetic Equation

We work in the standard magnetic island coordinate system, (ψ, θ, ξ), where ψ, the poloidal magnetic flux, serves as the radial coordinate, θ is the poloidal angle that measures the distance along the helical equilibrium magnetic field lines, and ξ is the helical angle that measures along the length of the island (Fig.1). For a single helicity magnetic island perturbation, the perturbed magnetic flux is described by $\Omega = 2(\psi - \psi_s)^2/w_\psi^2 - \cos\xi$, where ψ_s is the flux at the rational surface where the island is located, $w_\psi^2 = 4\tilde{\psi} q_s/q_s'$, $\tilde{\psi}$ describes the perturbation amplitude, q_s is the safety factor and $q_s' = \mathrm{d}q/\mathrm{d}\psi|_s$. We solve the drift kinetic equation by expanding it in terms of $\Delta = w/r$. We seek a local solution around the rational surface, assuming the following form:

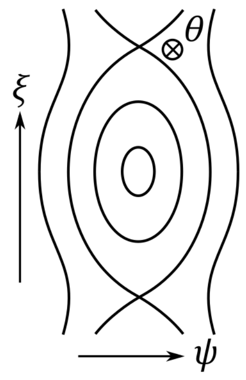

Figure 1: *Magnetic island geometry, with contours of constant* Ω.

$$f_i = \left(1 - \frac{e\Phi}{T_i(0)}\right) F_{\mathrm{M}i}(0) + G_i, \qquad F_{\mathrm{M}i}(0) = \frac{n_i(0)}{\pi^{3/2} v_{\mathrm{th}i}^3(0)} \exp\left[-\frac{v^2}{v_{\mathrm{th}i}^2(0)}\right], \tag{1}$$

where (0) indicates quantities evaluated at the rational surface, Φ is the perturbed electrostatic potential, and $T_i = m_i v_{\mathrm{th}i}^2$. As in previous analytic works [2, 4], the perturbation can take the form $G_i = F_{\mathrm{M}i}'(0) p_\phi + g_i$, where $F_{\mathrm{M}i}'(0) = F_{\mathrm{M}i}(0)\left[1 + (v^2/v_{\mathrm{th}i}^2 - 3/2)\eta_i\right]/L_i$, $L_i = (\mathrm{d}n_i/\mathrm{d}r)/n_i(0)$ and $\eta_i = L_{Ti}/L_{ni}$. $g_i = \sum_k \Delta^k g_k$ is the non-adiabatic response to the island perturbation, $p_\phi = \psi - Iv_\parallel/\omega_c$ is the toroidal canonical momentum, which is a conserved quantity on particle orbits, $v_\parallel = \sigma v\sqrt{1-\lambda B}$, $\sigma = v_\parallel/|v_\parallel|$, λ is the pitch angle and $I(\psi) = RB_T$. We work in the island rest frame such that there exists an equilibrium radial electric field, so that $\Phi = \Phi_{eq}'\psi + \varphi$, where φ is the potential perturbation. Using Eq.(1), we find that the leading order contribution to the drift kinetic equation comes from the parallel streaming and magnetic drift response:

$$\frac{v_\parallel}{Rq}\left[\left.\frac{\partial g_0}{\partial\theta}\right|_\psi + I\frac{\partial}{\partial\theta}\left(\frac{v_\parallel}{\omega_c}\right)\frac{\partial g_0}{\partial\psi}\right] = 0, \tag{2}$$

so that $g_0 = \bar{g}_0(p_\phi, \xi, v, \lambda)$.

The solution for g_0 is obtained from the $O(\Delta)$ contribution to the drift kinetic equation, which includes the island physics, $\mathbf{E}\times\mathbf{B}$ response and collisional effects. The higher order term in g_1 (i.e. $\left.\frac{\partial g_1}{\partial\theta}\right|_{p_\phi}$) can be annihilated by making use of the periodicity in θ at fixed p_ϕ (hence varying ψ) for the passing particles, and the particle conservation at bounce points for trapped particles.

The result is an orbit-averaged equation for g_0:

$$\left[p_\phi \Theta(\lambda_c-\lambda)+\omega_D-\omega_{E,\xi}\right] \frac{\partial g_0}{\partial \xi}\bigg|_{p_\phi} - \left[m\tilde{\psi}\cos\xi\,\Theta(\lambda_c-\lambda)-\omega_{E,\psi}\right] \frac{\partial g_0}{\partial p_\phi}\bigg|_{\xi} - \left\langle \frac{Rq}{v_\parallel} C(g_0+F'_{\mathrm{M}i}p_\phi)\right\rangle_\theta = F'_{\mathrm{M}i}\left[m\tilde{\psi}\cos\xi\,\Theta(\lambda_c-\lambda)-\omega_{E,\psi}\right], \tag{3}$$

$$\omega_{E,\psi} = \left\langle \frac{\rho_\theta}{v_\parallel}\frac{\partial \Phi}{\partial \xi}\right\rangle_\theta,\ \omega_{E,\xi} = \left\langle \frac{\rho_\theta}{v_\parallel}\frac{\partial \Phi}{\partial \psi}\right\rangle_\theta,\ \omega_D = \langle \rho_\theta v_\parallel\rangle_\theta + \frac{q_s'}{q_s}\left\langle \frac{\partial}{\partial\psi}\left(\frac{\rho_\theta v_\parallel}{\omega_c}\right)\right\rangle_\theta, \tag{4}$$

where λ_c corresponds to the trapped/passing boundary in the pitch angle space and $C(g)$ is a momentum-conserving model collision operator. Eq.(3) can be written more compactly by introducing the modified flux function, $\Omega_S = p_\phi^2/2w_\psi^2 - \cos\xi$. Since $\frac{\partial F'_{\mathrm{M}i}p_\phi}{\partial \xi}\Big|_{p_\phi} = 0$, we obtain:

$$\boxed{p_\phi\Theta(\lambda_c-\lambda)\frac{\partial G_0}{\partial \xi}\bigg|_{\Omega_s} + (\omega_D-\omega_{E,\xi})\frac{\partial G_0}{\partial \xi}\bigg|_{p_\phi} + \omega_{E,\psi}\frac{\partial G_0}{\partial p_\phi}\bigg|_{\xi} = \left\langle \frac{Rq}{v_\parallel}C(G_0)\right\rangle_\theta .} \tag{5}$$

Analytic Limit

We now demonstrate that the orbit averaged equation (5) produces the same solutions in the limit of small ion banana width as the ones in [2] . We make a secondary expansion in terms of $\delta_i = \rho_{bi}/w \ll 1$, and consider the leading order solution to Eq.(5), $G_{0,0}$. In this limit, we may assume that the electrostatic potential is a function of the perturbed flux function to leading order, i.e. $\Phi = \Phi_0(\Omega)+\Phi_1$, $\Phi_1/\Phi_0 \sim O(\delta_i)$, where Φ_1 describes the correction to the potential originating from the particle orbit effects. Then, writing $\omega_{E,\psi}$ and $\omega_{E,\xi}$ in terms of Φ_0 and Φ_1:

$$\left[\Theta(\lambda_c-\lambda)-\left\langle \frac{\rho_\theta}{v_\parallel}\frac{\partial\Phi_0}{\partial\Omega}\right\rangle_\theta\right] p_\phi \frac{\partial G_0}{\partial \xi}\bigg|_{\Omega_s} + (\omega_D-\delta\omega_{E,\xi})\frac{\partial G_0}{\partial \xi}\bigg|_{p_\phi} + \delta\omega_{E,\psi}\frac{\partial G_0}{\partial p_\phi}\bigg|_{\xi} = \left\langle \frac{Rq}{v_\parallel}C(G_0)\right\rangle_\theta, \quad \delta\omega_{E,\xi} = \left\langle \frac{\rho_\theta}{v_\parallel}\frac{\partial\Phi_1}{\partial\psi}\bigg|_\xi\right\rangle_\theta, \quad \delta\omega_{E,\psi} = \left\langle \frac{\rho_\theta}{v_\parallel}\frac{\partial\Phi_1}{\partial\xi}\bigg|_\psi\right\rangle_\theta. \tag{6}$$

Then, $O(\delta_i^0)$ terms in Eq.(6) yield:

$$\left[\Theta(\lambda_c-\lambda)-\left\langle \frac{\rho_\theta}{v_\parallel}\frac{\partial\Phi_0}{\partial\Omega}\right\rangle_\theta\right] p_\phi \frac{\partial G_{0,0}}{\partial \xi}\bigg|_{\Omega_s} = \left\langle \frac{Rq}{v_\parallel}C(G_{0,0})\right\rangle_\theta . \tag{7}$$

In the low collision frequency limit $\nu_{ii}/\varepsilon\omega \ll 1$, where ν_{ii} is the ion-ion collision frequency and ω is the island rotation frequency (in our island rest frame, this is expressed in terms of the equilibrium radial electric field), the solution is: $G_{0,0} = (\omega_*^T/\omega_* - e\Phi'_{eq}L_i/T_i)(F_{\mathrm{M}i}(0)/L_i)H(\Omega_s)$, where $\omega_*^T/\omega_* = 1+(v^2/v_{\mathrm{th}i}^2 - 3/2)\eta_i$ and $H(\Omega_s)$ is a free modified flux function, which is to be determined from the collisional constraints on $G_{0,0}$. Then, Taylor-expanding Ω_S about Ω in the limit of small ρ_{bi}, we find that the full ion distribution function as determined from Eq.(5) is:

$$\boxed{f_i = \left(1 - \frac{e\Phi}{T_i}\right) F_{\mathrm{M}i} + \left(\frac{\omega_*^T}{\omega_*} - \frac{e\Phi'_{eq} L_i}{T_i}\right) \frac{F_{\mathrm{M}i}}{L_i} \left[H(\Omega) - \rho_\theta v_\parallel \frac{\partial H}{\partial \psi}\right] + G_{1,0}.} \tag{8}$$

If we compare with the solution from [2]:

$$f_i = \left(1 - \frac{e\Phi}{T_i}\right) F_{\mathrm{M}i} + \left(\frac{\omega_*^T}{\omega_*} - \frac{e\Phi'_{eq} L_i}{T_i}\right) \frac{F_{\mathrm{M}i}}{L_i} \left[h(\Omega) - \rho_\theta v_\parallel \frac{\partial h}{\partial \psi}\right] + \rho_\theta v_\parallel \frac{e\Phi'_{eq}}{T_i} \frac{F_{\mathrm{M}i}}{L_i} + \bar{h}_i, \tag{9}$$

we see that the two are identical in form, if $H(\Omega) \to h(\Omega)$ and $G_{1,0} \to \bar{h}_i$ (the additional term in Eq.(9) comes from transforming the solution from the $\mathbf{E} \times \mathbf{B}$ rest frame to the island rest frame). Therefore, we have verified that our equation (5) can reproduce the correct solution in the same limit as previous analytic works.

Summary

We have demonstrated that our new drift kinetic equation for calculating the ion response to the island perturbation produces the same form of solution in the limit of small ion banana width as the previous analytic work. In order to explore the new physics, we must solve Eq.(5) in full, which requires a numerical treatment. This will provide a valuable benchmark for a numerical solution of the full Eq.(5). The code under development does produce a solution that is a function of the modified perturbed flux, i.e. $G = G(\Omega_s)$ (Fig.2). However, there are differences in the numerical and analytic limit results for the perturbed ion density and parallel flow profiles, which need to be understood. Once the benchmarking of our code is complete and its algorithm fully verified, we will be able to explore the new physics.

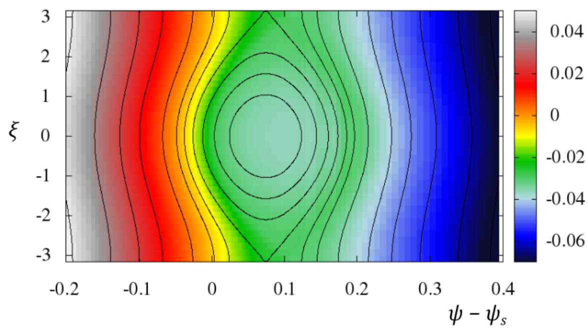

Figure 2: *Numerical solution for G_i on $\psi - \xi$ plane. Contours are of constant Ω_s Note that the colour contour aligns well with the contours of Ω_s, demonstrating that $G = G(\Omega_s)$.*

References

[1] Z.Chang et al, Physical Review Letters **74**, 4663 (1995)

[2] H.R. Wilson et al, Physics of Plasmas **3**, 248 (1996)

[3] R. Fitzpatrick, Physics of Plasmas **2**, 825 (1995)

[4] K. Imada, H.R. Wilson, Physics of Plasmas **19**, 032120 (2012)

Acknowledgment: This project is funded by EPSRC grant number EP/N009363/1. Computational time is provided by IFERC-CSC, and Plasma Physics HEC Consortia, grant number EP/L000237/1.