

# Influence of tangential drifts on neoclassical transport in optimized stellarators

Iván Calvo<sup>1</sup>, Félix I. Parra<sup>2,3</sup>, José Luis Velasco<sup>1</sup> and J. Arturo Alonso<sup>1</sup>

<sup>1</sup>*Laboratorio Nacional de Fusión, CIEMAT, 28040 Madrid, Spain*

<sup>2</sup>*Rudolf Peierls Centre for Theoretical Physics, University of Oxford, Oxford, OX1 3NP, UK*

<sup>3</sup>*Culham Centre for Fusion Energy, Abingdon, OX14 3DB, UK*

## Introduction

In general, the orbit-averaged radial magnetic drift is non-zero for trapped particles in stellarators. Stellarators in which the orbit-averaged radial magnetic drift vanishes are called omnigeneous. Although exactly omnigeneous configurations are not mathematically forbidden, achieving perfect omnigeneity in practice requires such an accuracy in the design and placement of the coils that, even in future devices, deviations from omnigeneity are unlikely to be negligible. These deviations are more deleterious at small collisionalities. The  $1/v$  regime has been recently treated in stellarators close to quasisymmetry [1], which are particular cases of stellarators close to omnigeneity. Here, the techniques learnt in [1] are generalized to stellarators close to omnigeneity and applied to collisionality values below the  $1/v$  regime.

## Omnigeneous stellarators and stellarators close to omnigeneity

We use spatial coordinates  $\{\psi, \alpha, l\}$ , where  $\psi$  determines the flux surface,  $\alpha$  is a poloidal angle (this is simply to fix ideas;  $\alpha$  might have a different helicity and the treatment would be analogous) that labels magnetic field lines once  $\psi$  has been fixed, and  $l$  is the arc length over the magnetic field line selected by fixing  $\psi$  and  $\alpha$ .

Passing particles always have vanishing average radial magnetic drift. A stellarator is called omnigeneous if the orbit-averaged radial magnetic drift is zero for all trapped particles [2]. This is equivalent to saying that  $\partial_\alpha J = 0$ , where

$$J = 2 \int_{l_{b1}}^{l_{b2}} |v_{||}| dl \quad (1)$$

is the second adiabatic invariant, and  $l_{b1}$  and  $l_{b2}$  are the bounce points. Hence, a stellarator is omnigeneous if and only if  $J$  is a flux function.

Below, we write magnetic fields for stellarators close to omnigeneity as

$$\mathbf{B} = \mathbf{B}_0 + \delta \mathbf{B}_1, \quad (2)$$

where  $\mathbf{B}_0$  is omnigeneous and  $\delta\mathbf{B}_1$  is a perturbation with  $0 \leq \delta \ll 1$  and  $\mathbf{B}_1 \sim \mathbf{B}_0$ . We assume that  $|\nabla \ln B_0|^{-1} \sim |\nabla \ln B_1|^{-1} \sim L_0$ .

### Low-collisionality drift-kinetic equation: $\rho_{i*} \ll v_{i*} \ll 1$

Let us use coordinates  $\{v, \lambda, \sigma\}$  in velocity space, where  $v$  is the magnitude of the velocity,  $\lambda = v_\perp^2/(v^2 B)$  is the pitch angle coordinate and  $\sigma$  is the sign of the parallel velocity. In the standard drift-kinetic expansion, the distribution function  $F_i(\psi, \alpha, l, v, \lambda, \sigma)$  is expanded as  $F_i = F_{i0} + F_{i1} + O(\rho_{i*}^2 F_{i0})$ , where  $F_{i1} \sim \rho_{i*} F_{i0}$ ,  $F_{i0}$  is a Maxwellian distribution with density  $n_i$  and temperature  $T_i$  constant on flux surfaces and  $\rho_{i*} \ll 1$  is the ion gyroradius over  $L_0$ . The electrostatic potential is expanded as  $\varphi(\psi, \alpha, l) = \varphi_0(\psi) + \varphi_1(\psi, \alpha, l)$ , with  $\varphi_0(\psi) \sim T_i/(Z_i e)$  and  $\varphi_1/\varphi_0 \sim \rho_{i*}$ . Here,  $Z_i e$  is the charge of the ions and  $e$  is the proton charge.

The drift-kinetic equation for the non-adiabatic component of the distribution function  $G_{i1} = F_{i1} + (Z_i e \varphi_1 / T_i) F_{i0}$  is<sup>1</sup>

$$v_{||} \hat{\mathbf{b}} \cdot \nabla G_{i1} + \Upsilon_i \mathbf{v}_{M,i} \cdot \nabla \psi F_{i0} = C_{ii}^\ell [G_{i1}], \quad (3)$$

where  $\mathbf{v}_{M,i}$  is the ion magnetic drift,

$$\Upsilon_i = \frac{n'_i}{n_i} + \left( \frac{m_i v^2}{2T_i} - \frac{3}{2} \right) \frac{T'_i}{T_i} + \frac{Z_i e \varphi'_0}{T_i}, \quad (4)$$

primes stand for differentiation with respect to  $\psi$  and  $C_{ii}^\ell$  is the linearized ion-ion collision operator. In this paper we focus on ion transport and assume that a mass ratio expansion  $\sqrt{m_e/m_i} \ll 1$  has been taken, so that the ion-electron collision term has been dropped. Here,  $m_e$  is the electron mass and  $m_i$  is the ion mass.

Define the ion collisionality as  $\nu_{i*} = \nu_{ii} L_0 / v_{ti}$ , where  $\nu_{ii}$  is the ion-ion collision frequency,  $v_{ti} = \sqrt{T_i/m_i}$  is the ion thermal speed. If  $\nu_{i*} \ll 1$ , one can perform an expansion in powers of the collisionality. To lowest order one finds that  $G_{i1}$  is homogeneous along the coordinate  $l$ . The function  $G_{i1}$  is found from averages of the drift-kinetic equation to next order in  $\nu_{i*}$ . For trapped particles we average over the orbit,

$$\int_{l_{b1}}^{l_{b2}} |v_{||}|^{-1} C_{ii}^\ell [G_{i1}] dl = \left( \int_{l_{b1}}^{l_{b2}} |v_{||}|^{-1} \mathbf{v}_{M,i} \cdot \nabla \psi dl \right) \Upsilon_i F_{i0}. \quad (5)$$

For passing particles we take the flux surface average, that we denote by  $\langle \cdot \rangle_\psi$ . That is,

$$\left\langle B v_{||}^{-1} C_{ii}^\ell [G_{i1}] \right\rangle_\psi = 0. \quad (6)$$

From the last two equations, it is clear that  $G_{i1} \sim \nu_{i*}^{-1} \rho_{i*} F_{i0}$ . This defines the  $1/v$  regime.

<sup>1</sup>The ambipolarity condition and the quasineutrality equation that allow to solve for  $\varphi_0$  and  $\varphi_1$  are discussed in [3].

### Drift-kinetic equation for collisionalities below the $1/v$ regime: $v_{i*} \ll \rho_{i*}$

In general, if  $v_{i*} \lesssim \rho_{i*}$  the drift-kinetic expansion breaks down because  $G_{i1}$  becomes as large as  $F_{i0}$ . In addition, the drift-kinetic equation becomes radially non-local because there is no reason, in principle, to neglect terms like  $\mathbf{v}_{M,i} \cdot \nabla \psi \partial_\psi G_{i1}$  (in this paper we do not discuss large aspect ratio effects). However, if  $\delta \ll 1$  there is a rigorous way to carry out the expansion and to derive a radially local drift-kinetic equation [3]. If  $\delta \ll 1$ , it is possible to show that  $G_{i1} = g_i(\psi, \alpha, v, \lambda)$  in the trapped region and that it vanishes in the passing region. The correct ansatz to deal with the regime  $v_{i*} \ll \rho_{i*}$  is  $g_i = \delta g_i^{(1)} + \dots$ , where  $g_i^{(1)} \sim F_{i0}$ . Analogously, we take  $\varphi_1 = \delta \varphi_1^{(1)} + \dots$ , where  $\varphi_1^{(1)} \sim \varphi_0$ .

Expanding in  $\delta$ , we find a radially-local drift-kinetic equation valid<sup>2</sup> for  $v_{i*} \ll \rho_{i*}$ . Namely,

$$-\partial_\psi J^{(0)} \partial_\alpha g_i^{(1)} + \partial_\alpha J^{(1)} \Upsilon_i F_{i0} = \sum_{\sigma} \frac{Z_i e \Psi'_t}{m_i c} \int_{l_{b10}}^{l_{b20}} \frac{dl}{|v_{||}^{(0)}|} C_{ii}^{\ell(0)} [g_i^{(1)}],$$

where  $\Psi_t$  is the toroidal magnetic flux over  $2\pi$ ,  $l_{b10}$  and  $l_{b20}$  are the orbit bounce points calculated using  $B_0$ ,  $c$  is the speed of light,

$$\partial_\psi J^{(0)} = - \int_{l_{b10}}^{l_{b20}} \frac{\lambda v \partial_\psi B_0 + 2Z_i e / (m_i v) \partial_\psi \varphi_0}{\sqrt{1 - \lambda B_0}} dl$$

and

$$\partial_\alpha J^{(1)} = - \int_{l_{b10}}^{l_{b20}} \frac{\lambda v \partial_\alpha B_1 + 2Z_i e / (m_i v) \partial_\alpha \varphi_1^{(1)}}{\sqrt{1 - \lambda B_0}} dl.$$

We have employed a superindex (0) for quantities corresponding to  $\mathbf{B}_0$  and a superindex (1) for perturbed quantities.

Let us define the poloidal frequency  $\omega_\alpha = m_i c \partial_\psi J^{(0)} / (Z_i e \Psi'_t \tau_b^{(0)})$ , where  $\tau_b^{(0)}$  is the orbit time in the magnetic field  $\mathbf{B}_0$ . Typically,  $\omega_\alpha \sim \rho_{i*} v_{ti} / L_0$  and the drift-kinetic equation is solved by expanding in  $v_{ii} / \omega_\alpha \sim v_{i*} / \rho_{i*} \ll 1$ . To lowest order in the  $v_{ii} / \omega_\alpha$  expansion one obtains  $g_i^{(1)} = g_0 + \dots$ , with

$$g_0 = \frac{1}{\partial_\psi J^{(0)}} \left( J^{(1)} - \frac{1}{2\pi} \int_0^{2\pi} J^{(1)} d\alpha \right) \Upsilon_i F_{i0}.$$

### Energy flux when $v_{i*} \ll \rho_{i*}$

It is easy to realize that  $g_0$  does not contribute to the energy flux,  $Q_i$ . The physical effect that explains neoclassical transport when  $v_{i*} \ll \rho_{i*}$  depends on certain properties of  $\omega_\alpha$ .

Customarily, for non-zero  $\varphi_0'$  there exists a minimum value of  $v$  for which  $\omega_\alpha = 0$  for some value of  $\lambda$  (the value of  $\lambda$  for each  $v$  is usually unique). We denote this value of  $v$  by  $v_{\min}$ . When

<sup>2</sup>As explained in [3], it is expected that this drift-kinetic equation also ceases to be valid for sufficiently low collisionality.

$v \geq v_{\min}$ , we define  $\lambda_r(\psi, v)$  as the value of  $\lambda$  such that  $\omega_\alpha(\psi, v, \lambda_r) = 0$ . Of course,  $\lambda_r$  is a function of  $\psi$  and  $v$ ,  $\lambda_r \equiv \lambda_r(\psi, v)$ .

(i)  $\sqrt{v}$  regime

If  $v_{\min} \gg v_{ti}$ , transport is dominated by the discontinuity of  $g_0$  at the trapped/passing boundary. This discontinuity originates a collisional layer of size  $\Delta\lambda \sim B_0^{-1}(v_i/\omega_\alpha)^{1/2}$ , and the energy flux can be shown to scale as

$$Q_{i,\sqrt{v}} \sim \delta^2 \frac{v_{ii}^{1/2}}{\omega_{\alpha,c}^{3/2}} \rho_{i*}^2 n_i m_i v_{ti}^4 L_0^{-1} S_\psi,$$

where  $S_\psi$  is the area of the flux surface. This is the  $\sqrt{v}$  regime.

(ii) *Superbanana-plateau regime*

If  $v_{\min} \lesssim v_{ti}$ , transport is dominated by the divergence of  $g_0$  at the resonant values of the pitch-angle coordinate,  $\lambda_r(\psi, v)$ . In this case, the energy flux turns out to be independent of the collisionality,

$$Q_{i,sb-p} \sim \delta^2 \rho_{i*} n_i m_i v_{ti}^3 S_\psi.$$

### Additive formula for the energy flux

Since the layers corresponding to the  $\sqrt{v}$  and to the superbanana-plateau regimes are small and located at different regions of phase space, their contributions to transport are additive. This means that we can write, for  $v_{i*} \ll \rho_{i*}$ ,

$$Q_i = Q_{i,\sqrt{v}} + Q_{i,sb-p}.$$

An explicit expression for this formula is provided in [3].

### Acknowledgments

This work has been carried out within the framework of the EUROfusion Consortium and has received funding from the Euratom research and training programme 2014-2018 under grant agreement No 633053. The views and opinions expressed herein do not necessarily reflect those of the European Commission.

### References

- [1] I. Calvo et al., *Plasma Phys. Control. Fusion* **56**, 094003 (2014).
- [2] J. R. Cary et al., *Phys. Plasmas* **4**, 3323 (1997).
- [3] I. Calvo F. I. Parra, J. L. Velasco and J. A. Alonso, "The effect of tangential drifts on neoclassical transport in stellarators close to omnigeneity", in preparation.