Gravitational waves generated by laser–plasma interaction

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We analyze theoretical models of gravitational wave generation in the interaction of high power lasers with matter in linear approximation of gravitational theory up to the quadrupole moment in moment expansion. The main purpose of our investigation is to analyze the three different models of high frequency gravitational waves (HFGW) in the interaction of high power laser pulse with a medium. These models were suggested in [1, 2], from which we have chosen to investigate the shock wave model, shock wave model with ablation [3] and the piston model [4]. The light sail model together with piston model was investigated in [8].

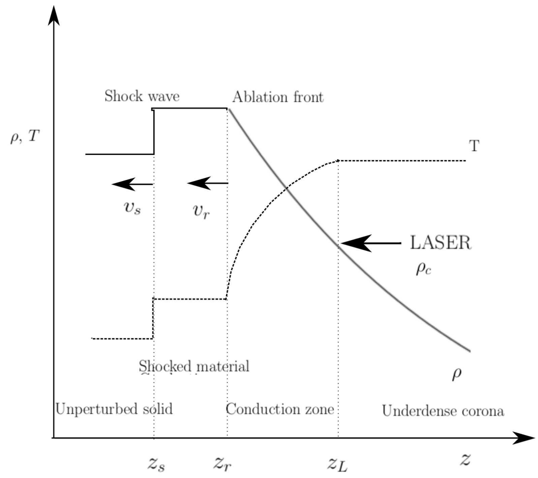

Figure 1: *The density and temperature profiles for shock wave and ablation models*

We derive the analytical formulas and estimates for the metric perturbations and the radiated power of the generated gravitational waves. The important input into calculations are the density profiles which can be seen in Fig. 1. Namely, the resulting formulae for the perturbation h_{ij}^{TT} and the luminosity $\mathscr{L}_{GW}$ for the shock wave model are:

$$\mathscr{L}_{\text{quad}} = \frac{24G}{5c^5}\frac{R_t}{\rho_0}P_L^2, \quad h_{zz} = \frac{8G}{rc^4}\left(\left(\frac{R_t}{\rho_0}\right)^{1/2} E_L - S f_1 R_t^{1/3} I_L^{2/3}\right), \tag{1}$$

where c is the speed of light and G is the gravitational constant, R_t is the target density $R_t = \frac{1}{2}\frac{A}{Z}m_p n_c$, $P_L = SI_L$ is the laser power and $E_L = SI_L t$ is the energy of the laser. For experimental values we will evaluate our expressions

$$P_L = 0.5\,\text{PW}, \quad \rho_0 = 30\,\text{mg/cm}^3, \quad E_L = 0.5\,\text{MJ}, \quad \tau = 1\,\text{ns}, \quad \lambda_L = 300\,\mu\text{m}$$

and the detection distance is $R = 10$ m or equivalently $f_1 = f = 10$ m, parameters a, b of the target foil are $a = b = 1\,\mu\text{m} = 10^{-4}\,\text{cm}$ and therefore $I_L = 0.5\times 10^8\,[\text{PW/cm}^2]$. The outgoing gravitational radiation has frequency $\nu_g = 1\,\text{GHz}$ and wave length $\lambda_g = 0.3$ m. The final estimations for our expressions of the luminosity and the perturbation are:

$$\mathscr{L}_{GW} \simeq 1.69172\times 10^{-23}[\text{erg/s}], \quad h_{zz}^{GW} \simeq 2.373\times 10^{-39}. \tag{2}$$

The resulting formulae for the ablation model are the following:

$$h_{zz} = \frac{64G}{rc^4}\left(\frac{1}{6}\left(\frac{R_t}{\rho_0}\right)^{1/2} E_L(\frac{2}{3}+e^{-b_I}) - \frac{SR_t^{1/3} I_L^{2/3}}{4}\left[f_2(\frac{2}{3}+e^{-b_I}) - \frac{3}{2}z_L e^{-b_I}\right]\right), \quad (3)$$

$$\mathscr{L}_{\text{quad}} = \frac{9G}{10c^5}\frac{R_t P_L^2}{\rho_0^2}[-\frac{2}{3}+e^{-b_I}(b_I^2+1)]^2, \quad (4)$$

and the estimations for the same experimental values as for the shock wave models (with $a = b = 1\,\text{mm} = 0.1\,\text{cm}$ for ablation) are

$$\mathscr{L}_{GW} \simeq 3.61\times 10^{-20}[\text{erg/s}], \quad h_{zz}^{GW} \simeq 4.7\times 10^{-39}. \quad (5)$$

The results for piston models read as:

$$\mathscr{L}_{\text{quad}} = \frac{8}{15}\frac{G}{c^5}\frac{1}{S\rho_0}\left(\frac{P_L}{c}\right)^3, \quad h_{zz} = \frac{8G}{rc^4}\frac{1}{\sqrt{S\rho_0}}\left(\frac{P_L}{c}\right)^{3/2} t. \quad (6)$$

When we substitute achievable laser parameters into expressions for luminosity and the perturbation we will get the estimations for the experiment:

$$P_L = 7\,\text{PW},\, \rho_0 = 1\,\text{g/cm}^3,\, \Phi = 30\,\mu\text{m},\, \tau = 1\,\text{ps}, \quad (7)$$

and the detection distance is again $R = 10$ m and $S = \Phi^2\pi/4$ where Φ is diameter of the target. The detection distance is $R = 10$ m or equivalently $f_2 = f = 10$ m, $z_L = 12$ m, parameters a, b of the target foil are $a = b = 1\,\mu\text{m} = 1\times 10^{-6}\,\text{m}$ and therefore $I_L = 7\times 10^8\,[\text{PW/cm}^2]$ and the velocity $v_r = 153008\,[\text{km/s}]$.

The wavelenght of the gravitational wave is $\lambda_g = 300\,\mu\text{m}$ and the frequency is $\nu_g = 1$ THz.

The final estimations for the luminosity and the perturbations are:

$$\mathscr{L}_{GW} \simeq 2.704\times 10^{-18}[\text{erg/s}], h_{zz}^{GW} \simeq 3\times 10^{-43}. \quad (8)$$

Therefore we claim that the piston model has the lowest luminosity and the lowest perturbations with quite low luminosity is the ablation model. This suggests that the ablation model is the best model for the experiment, but it is coupled with the shock wave model when realistically performing the experiment.

The radiative characteristic represents amount of energy going through various directions in the case of large distances r for quadrupole radiation is defines as

$$-t_{0r}^{GW} \equiv \mathscr{S} = \frac{c^3}{72G\pi r^2}\langle \dddot{I}_{ij}\dddot{I}_{ij} - 2\dddot{I}_{is}\dddot{I}_{sj}n_i n_j + \frac{1}{2}\dddot{I}_{ij}\dddot{I}_{rs}n_i n_j n_r n_s\rangle, \quad (9)$$

where $n = (\sin\theta\sin\phi, \sin\theta\cos\phi, \cos\theta)$ is the general vector. For the shock wave and piston models, the radiation in the general

$$\mathscr{S}_n = \frac{S^2 c^3\rho_0^2 v_s^6}{36G\pi r^2}\left[12 - 4(\sin^2\theta + 4\cos^2\theta) + (2\cos^2\theta - \sin^2\theta)^2\right]. \quad (10)$$

The directional part of the radiation is plotted in the Fig. (2) for ablation model and (3) for shock wave and piston models.

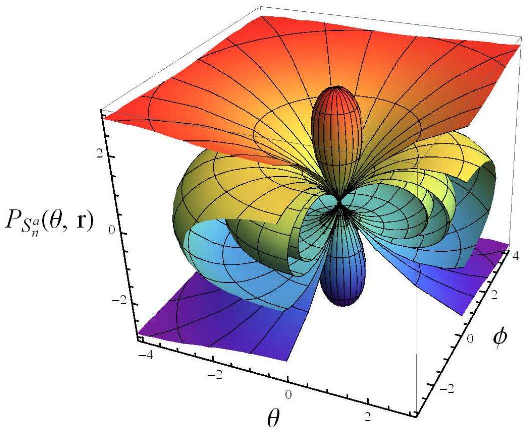

Figure 2: *The radiative structure for the ablation model.*

The structure has toroidal structure with the symetry center around the $z = 0$, which shows that the radiation in the direction of z axes vanishes. The radiation in Fig. (2) shows presents of such radiation in the direction of propagation of the laser which should not be there while working in the linearized gravitation. The present radiation is caused by the fact that the mass conservation is not valid for the ablation model and we integrate the density to some fixed value instead of the ∞.

We investigate the geometrical characteristics of polarization of the generated gravitational waves and the behavior of test particles in the gravitational field of the wave [6] which will be important for the detection. We have used the geodesics equation for the study.

The shock wave test particles change its shape from the circle to the ellipse which changes in time thanks to the time dependence of the amplitude A_+, see Fig. (4). The behaviour of test particles in $+$ mode of polarization is the same for piston model, the ablation model test particles differ because the amplitude has non–trivial time dependence and the circle of test particles is just larger in time.

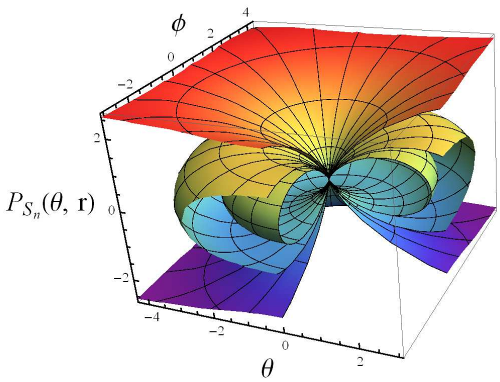

Figure 3: *The radiative structure for the shock wave and piston models.*

To summarize our work shortly, see [6] and [7], we have investigated generation of gravitational waves by laser–plasma interaction in three models, shock wave, ablation and piston models. We have investigated the perturbations and luminosities of such waves and other properties which relate to the polarization of the GW, such as type of polarization, the maplitudes in two modes of GW, the properties in spherical coordinates in linear gravity. Furthermore, we have analyzed the radiative characteristics of the radiation which shows to be toroidal character for $+$ mode and bulb for $\times$ mode. We analyzed the behaviour of test particles under influence of the GW.

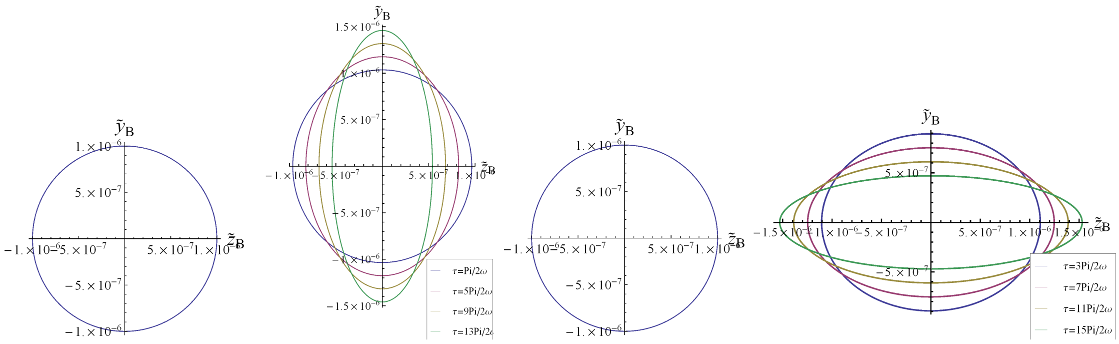

Figure 4: The diagrams depict the position of test particles as function of time under influence of GW wave with + polarization for the shock wave model.

The resulting perturbations of the waves are about 10^{-40} which is out of range of today's known detectors. The recent detection of gravitational waves on LIGO with magnitude 10^{-21} [5] would definitely open a new era of fundamental physics and thus our research becomes even more relevant in today's physics. The results are summarized in the papers [6](shock wave model) and [7] (ablation and piston models).

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