Resonance Line Radiation Flux in a Plasma Slab: Self-Similar Radiative Transfer Model

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1. Introduction. The representation of kinetic equations in the self-similar variables allows one to obtain analytic solutions, which may be very helpful for testing the respective blocks of numerical codes for transport phenomena. The examples include the steady-state collisional kinetic equations for electrons [1] and neutral atoms [2] in a strongly inhomogeneous plasma. The self-similarity appears to be applicable to the cases of nonlocal (non-diffusive) correlations of the distribution function like the cases of superthermal electrons [1] and fast neutrals produced by the charge exchange [2]. Another type of self-similarity was found [3] for the non-steady-state Biberman-Holstein (B-H) equation for radiative transfer in resonance atomic/ionic lines. Here again the self-similarity is expressed in terms of characteristics of nonlocality of the B-H radiative transfer. The approach [1, 2] was extended to the case of the steady-state B-H equation in an inhomogeneous plasma slab. It was shown that for some types of similarity of spatial profiles of three characteristics, namely, background plasma density, line shape width and non-radiation source of atomic excitation, the profile of excited atoms density may be described analytically in terms of the similarity of the above-mentioned profiles. The revealed cases of analytical solutions for the 1D transfer were suggested for testing the radiative transfer codes in edge plasmas. Here we extend the model [4] to the case of 3D radiative transfer in resonance lines in a plasma slab. The model gives transparent description of the nonlocal effects in the resonance-line radiative transfer and gives reliable benchmarks for complicated numerical modeling of superdiffusive transport.

2. Three-dimensional radiative transfer in an inhomogeneous plasma slab. We will consider three-dimensional transfer in a slab of thickness of L. As a linear coordinate across the slab we choose a variable z. The Biberman-Holstein (B-H) equation for radiative transfer in resonance atomic/ionic lines in an inhomogeneous media in a three-dimensional steady-state case in the case of isotropic radiators when probabilities of emission of a photon and the corresponding cross-section of absorption in the rest frame of the atom doesn't depend on the direction of a photon, has the following form (cf. [5, 6]):

$$\int_0^\infty d\omega\, k(\omega,z)\int_0^L dz_1 \frac{n(z_1)P(\omega,z)}{4\pi\tau}\int_0^\infty \frac{2\pi\rho_1 d\rho_1}{(z-z_1)^2+\rho_1^2}\exp\left(-\sqrt{1+\frac{\rho_1^2}{(z-z_1)^2}}\left|\int_z^{z_1} k(\omega,z_2)\,dz_2\right|\right)= \\ =\left(\frac{1}{\tau}+\sigma_{\text{quench}}(\mathrm{z})\right)n(z)-q(z). \tag{1}$$

where $n(x)$ is the density of excited atoms, $P(\omega, x)$ is the (normalized over frequency ω) line shape of the photon emission by an excited atom at the point x, $k(\omega, x)$ is the coefficient of absorption of a photon by the atom (i.e. inverse free path of the photon) at the point x; σ_{quench} is the excited atom's inverse lifetime with respect to quenching; τ is the mean lifetime of the atom's excited state in the case of no quenching (so called, lifetime with respect to spontaneous radiative decay), $q(x)$ is the source of excitation of atoms by all processes, except absorption of photons. Integral terms consider the emission of a photon at a distant point z_1 and the absorption at the current point z. taking into account the possibility of absorption by the other atoms on this way.

In some cases, the transfer equations allow substantial simplification. In [1, 2] it was shown that, under condition λ/S = const, where λ is the mean free path, S is the characteristic scale length of variation of the distribution function (see [1, 2] for λ and S definitions), it is possible to introduce the self-similar variables allowing to find an exact analytical solutions of the kinetic equations. Here the condition λ_{ph}/S = const can be rewritten as

$$\left(\omega_T(z)/n_0(z)\right)\left(d\ln\omega_T(z)/dz\right)\equiv\gamma=\text{const}. \tag{2}$$

where $n_0(x)$ is the density of atoms in the ground state, $\omega_T(x)$ is the line shape width.

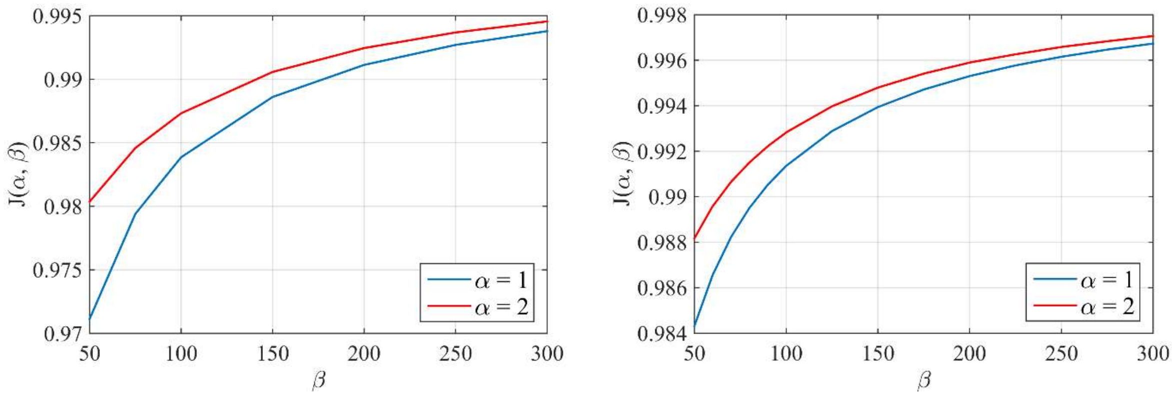
Equation (1) differs from the equation in [4] only in the fact that there will be an exponential integral $E_1(x)$ in it instead of ordinary exponent. It means that Eq. (1) may be transformed into the form

$$\left(1+\tau\sigma_{\text{quench}}(z)\right)=J(\alpha,\beta)+q(z)\cdot\left(\tau/n_0(z)\right)\cdot\left(\omega_T(z)/\tilde{\omega}_T\right)^\alpha, \tag{3}$$

where $J_{\text{slab}}^{(3\text{D})}(\alpha,\beta)$ function is defined as

$$J_{\text{slab}}^{(3\text{D})}(\alpha,\beta)=\frac{\beta}{2}\int_{-\infty}^{\infty}d\xi\int_{\frac{\omega_T(z)}{\omega_T(L)}\xi}^{\frac{\omega_T(z)}{\omega_T(0)}\xi}d\eta\frac{1}{\eta}\left(\frac{\eta}{\xi}\right)^\alpha a(\eta)a(\xi)E_1\left(-\beta\left|\int_\xi^\eta\frac{1}{\sigma}a(\sigma)\,d\sigma\right|\right). \tag{4}$$

Here we introduce the dimensionless parameter $\beta \equiv C_{01}/\gamma$ related to optical depth, $C_{01} = B_{01}\,\hbar\omega_0 / 4\pi$, B_{01} is the Einstein coefficient (for absorption). Function $a(x)$ characterizes emission $P(\omega, x)$ and the coefficient of absorption $k(\omega, x)$ line shapes. The function $J(\alpha, \beta)$ is shown in Fig. 1. That fact that function $J(\alpha, \beta)$ tends to a constant equal 1 with $\beta \to \infty$ (large optical depth) can be proved in general case. It corresponds to a full compensation of absorption and emission of photons at a given point in the infinite media.



(a) (b)

Fig. 1. Function $J(\alpha, \beta)$ in (a) one-dimensional and (b) three-dimensional cases.

Following [4] we obtain the excited atoms density profile from Eq. (3):

$$n(z) = \frac{n_0(z)\left(\tilde{\omega}_T / \omega_T(z)\right)^{\alpha}}{\left(1+\tau\sigma_{\text{quench}}\right) - J(\alpha,\beta)}. \tag{5}$$

While the population as a function of z is obtained, one can estimate the radiative energy flux density in one direction (to the right through the surface at the point z):

$$\frac{dE}{dSdt} = \frac{\hbar\omega_0}{\tau}\int_0^z n(z_1)\,dz_1 \int_0^1 \frac{d\mu}{2}\int_0^{\infty} P(\omega, z_1)\exp\left[-\frac{1}{\mu}\int_{z_1}^{z} k(\omega, z_2)\,dz_2\right]d\omega, \tag{6}$$

where the excited atoms density is given by (5) with function $J(\alpha, \beta)$ defined by (4). The absorption coefficient (without stimulated emission) can be written as $k(\omega, z) = C_{01} n_0(z)\varphi(\omega, z)$, where $\varphi(\omega, z)$ is the is the (normalized over frequency) line shape of the photon absorption. Assuming the line shape of the following form $P(\omega, z) = \varphi(\omega, z) = (1/\omega_T(z))\, a(\omega/\omega_T(z))$ one can transform integral in (6) to integration over $\omega_T(z)$ and to introduce dimensionless variables

$$\xi(\omega) = \omega / \omega_T(z), \quad \xi'(\omega_T(z_1)) = \omega / \omega_T(z_1), \quad \xi''(\omega_T(z_2)) = \omega / \omega_T(z_2). \tag{7}$$

Thus Eq. (6) takes form

$$\frac{dE}{dS\,dt} = \frac{\hbar\omega_0}{\tau}\frac{(\tilde{\omega}_T)^{\alpha}}{B}\frac{1}{(\omega_T(z))^{\alpha-1}}\frac{1}{C_{01}}\frac{\beta}{2}\int_{-\infty}^{\infty} d\xi \int_0^1 d\mu \int_{\xi}^{\frac{\omega_T(z)}{\omega_T(0)}\cdot\xi} d\xi' \,\frac{1}{\xi'}\left(\frac{\xi'}{\xi}\right)^{\alpha} a(\xi') \times$$
$$\times \exp\left[-\frac{\beta}{\mu}\int_{\xi}^{\xi'} d\xi'' \frac{a(\xi'')}{\xi''}\right], \quad B \equiv \left(1+\tau\sigma_{\text{quench}}\right) - J(\alpha,\beta). \tag{8}$$

Let us denote the integral in the right hand side of Eq. (8) as $I(\alpha, \beta)$. For example, $I(\alpha, \beta)$ for Doppler line shape in case of $\alpha = 1$ is shown in Fig. 2.

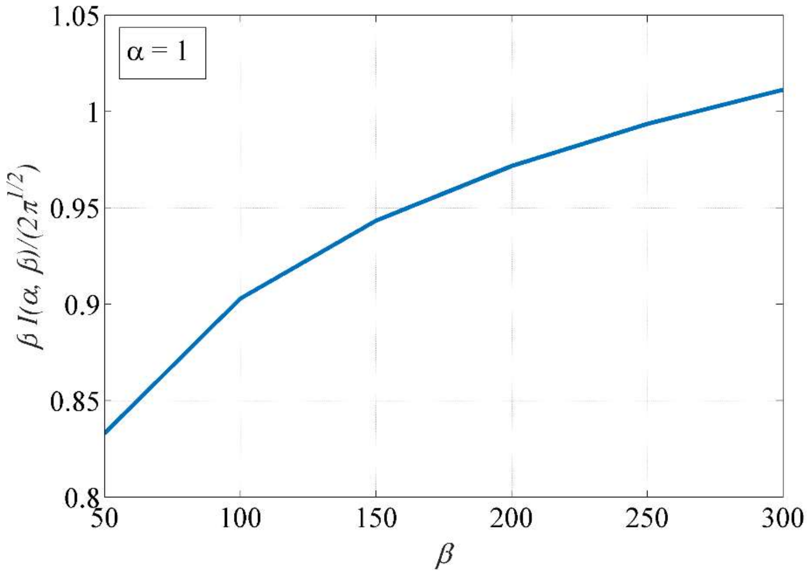


Fig.2. $I(\alpha, \beta)$ for Doppler line shape in case of $\alpha = 1$.

3. Conclusions. The model [4] is extended to the case of 3D radiative transfer in resonance lines in a plasma slab. Semi-analytic expression for the radiation flux is derived, which may be applied for a wide range of radiative transfer problems in fusion and astrophysical plasmas, including the edge plasmas in fusion facilities.

Acknowledgements. The part of work concerning the analysis of superdiffusive transport is supported by the Russian Foundation for Basic Research (project RFBR 15-07-07850-a). This work is supported by US DOE grants DE SC000199, DESC0008660, and DE FG02 04ER54739 at the UCSD and grant 14.Y26.31.008 of the MES of the Russian Federation at MEPhI.

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