Inverse Problem for Fast Nonlocal Heat Transport Events in Magnetic Fusion Plasmas

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1. Introduction. One of the most surprising examples of the fast nonlocal heat transport (FNHT) exhibits unexpected direction of heat flux, namely, the immediate, in the diffusion time scale, rise of electron temperature T_e in the central plasma in the case of the fast cooling of the edge plasma (the so-called "cold pulse" events in tokamaks and stellarators, see refs. in [1, 2]). Several approaches based on the superdiffusion formalisms (i.e. on an integral equation with nonlocal, longer than diffusive, spatial correlations and, respectively, the respective dominance of long mean-free-path energy carriers) have been suggested to describe anomalous heat transport in magnetized fusion plasmas (see, e.g., refs. in [2]). Among these, the approach [3] suggested an integral equation for T_e evolution at the initial stage of the cold-pulse experiments. The formalism [3] was extended in [4] to a much broader class of the kernels of integral equation with a pronounced nonlocality provided by hypothetical energy carriers (presumably, of electromagnetic wave origin). Also, an inverse problem was formulated for recovery of the kernel from the observed space-time dependence of perturbations of plasma temperature and density. The following explanation was suggested: the abrupt cooling of the edge plasma leads to an abrupt decrease of the absorption in the edge that, in turn, allows more transparent circulation of the energy carriers and, under condition of almost full reflection from the edge, an increase of the intensity/flux of the carriers. The latter leads to a growth of temperature in the central plasma at initial stage of the event.

Here we extend the model [4] in the following directions: (i) inclusion of conventional mechanism of anomalous diffusion of heat transport and, respectively, an extension of applicability of the model to a longer time period, (ii) allowance for an internal transport barrier (ITB) for hypothetical long-free-path (i.e. superdiffusive) carriers while for anomalous diffusion there is no ITB (i.e. no jump of diffusion coefficient). The results of interpreting the data from stellarator LHD [5] and tokamak TFTR [1, 6, 7] are presented.

2. An extension of FNHT model [4] to the case of an ITB. Application of the FNHT model [4] to tokamak TFTR data suggested that, contrary to stellarator LHD data, the inverse problem [4] has no acceptable solutions. On the other hand, in TFTR the peak of temperature rise in the core, just after fast cooling of the edge, takes place near magnetic surface with rational-number safety factor q (e.g., q=2 for TFTR shots 31880-31891 [6]). Also, the extension of applicability of the model to a longer time period requested the account of heat diffusion. The latter is especially important for the case of ITB. This gives the following extension of transport equation suggested in [4]:

$$\left[k\big(T(\rho,t)\big)\, I_\beta(\{T(\rho,t)\}) - k\big(T_0(\rho)\big)\, I_\beta(\{T_0(\rho)\})\right] + \hat{L}_{diff}[T(\rho,t) - T_0(\rho)] = \Pi(\rho,t),$$

$$0 < \rho < \rho_{max}; \quad \beta = inner,\ outer; \qquad (1)$$

$$I_{inner}(\{T_\alpha\}) = \frac{1}{D}\left(\int_0^1 n_{e\alpha}(\rho_1,t)\, Q\big(T_\alpha(\rho_1,t)\big)\rho_1 d\rho_1 + \boldsymbol{E}\int_{\rho_{ITB}}^1 n_{e\alpha}(\rho_2,t)\, k(T_\alpha(\rho_2,t))\, \rho_2 d\rho_2 \int_0^{\rho_{ITB}} n_{e\alpha}(\rho_1,t)\, Q\big(T_\alpha(\rho_1,t)\big)\rho_1 d\rho_1\right)$$

$$D = \int_0^1 n_{e\alpha}(\rho_1,t)\, k(T_\alpha(\rho_1,t))\rho_1 d\rho_1 + \boldsymbol{E}\int_0^{\rho_{ITB}} n_{e\alpha}(\rho_1,t)\, k(T_\alpha(\rho_1,t))\, \rho_1 d\rho_1 \int_{\rho_{ITB}}^1 n_{e\alpha}(\rho_2,t)\, k\big(T_\alpha(\rho_2,t)\big)\, \rho_2 d\rho_2$$

where ρ is the normalized minor radius coordinate; β=*inner* for $0 < \rho < \rho_{ITB}$, β=*outer* for $\rho_{ITB} < \rho < \rho_{max}$; α=0 before perturbation, and α=1 after it ($T_1(\rho,t) \equiv T(\rho,t)$). Expression for the intensity of carriers in the outer plasma, I_{outer}, may be obtained from that for I_{inner} by replacing, in the second term, k with Q and Q with k. Also

$$\hat{L}_{diff}[T] = \chi_0\, \chi_{rel} \frac{\partial}{\rho\partial\rho}\left(\rho \frac{\partial T}{\partial \rho}\right), \quad \chi_0 = 10^4 \frac{cm^2}{s}, \ \Pi(\rho,t) = A\frac{\partial}{\partial t}\,[T(\rho,t) - T_0(\rho)].$$

Here, k stands for temperature dependence of absorption coefficient and Q, the source of hypothetical superdiffusive carriers (electron density is taken as a separate factor in the source and sink functions). $\Pi(\rho)$ comes from time derivative of plasma energy density; A=3 under conditions of T_e=T_i, Z_{eff}=1, and slow relaxation of density in the core plasma. Similarly to [4], Eq. (1) assumes a monochromatic (“one-energy-group”) transport by superdiffusive carriers, and the relation $Q(T) = const\ T\ \kappa(T)$ assumes the Kirchhoff law (in the Rayleigh-Jeans limit). The constant E depends on the parameters of carriers (group velocity, etc.) and inversely proportional to $(1 - R_{ITB})\rho_{ITB}$, where R_{ITB} is reflectivity of ITB for the carriers.

3. Inverse problem solutions. The inverse problem is formulated as a minimization of the deviation from the equality in Eq. (1), where Q(T) and χ_{rel} and E are the sought-for values while T_0(ρ), T_1(ρ, t), Π(ρ, t) are the input data. The extension of time interval as compared with interpretation of LHD data in [4] has shown that only the introduction of the freedom of input data in a certain interval around original input data enables one to obtain stable solutions of inverse problem. In fact, the inverse problem is extended to a search of small deviations of the input data from original ones that allows attaining a very small defect of equality in Eq. (1). The solver combines the spline approximation of experimental data and the minimization of error of satisfying the Eq. (1).

Some results of solving the inverse problem are shown in Fig. 1.

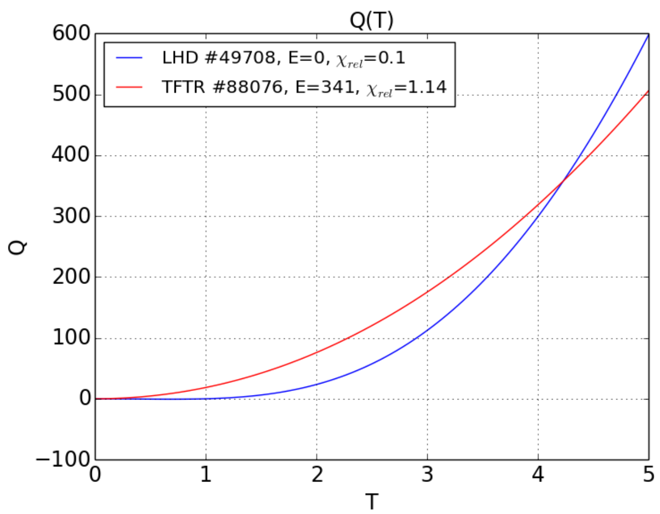

Figure 1. Temperature dependence of the source function obtained by solving the inverse problem based on the transport equation (1), for experimental data shown in the legend. The values of complementary sought-for parameters, E and χ_{rel}, are also indicated.

It appears that in the frame of the formalism for the case of ITB it is possible, for TFTR shot #88076 [1], to find a stable solution. Some details of the inverse problem solution are shown: modification of the input data for temperature (Fig. 2) and the radial profiles of separate terms in Eq. (1) for the obtained solution of inverse problem (Fig. 3).

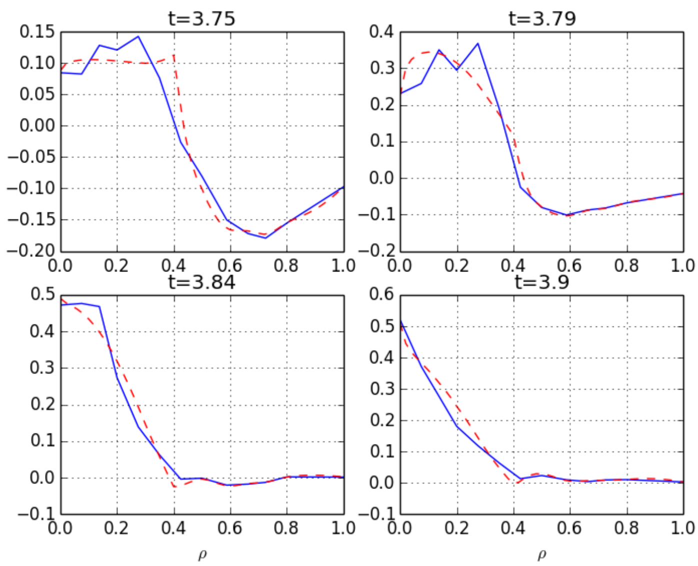

Figure 2. Time dependence of radial profile of the difference of electron temperatures after and before the perturbation. The original TFTR #88076 data (blue solid line) are compared with those used in solving the extended inverse problem (red dashed line, E=341) where the freedom of input data is allowed in a certain interval around original input data to obtain a stable solution of inverse problem.

The effect of the change of absorption of superdiffusive carriers after perturbation is shown in Figure 4 for TFTR data. Similar dependence takes place for LHD data. According to Eq. (1), the increase of the absorption in the core plasma is caused by the decrease of absorption in the edge plasma.

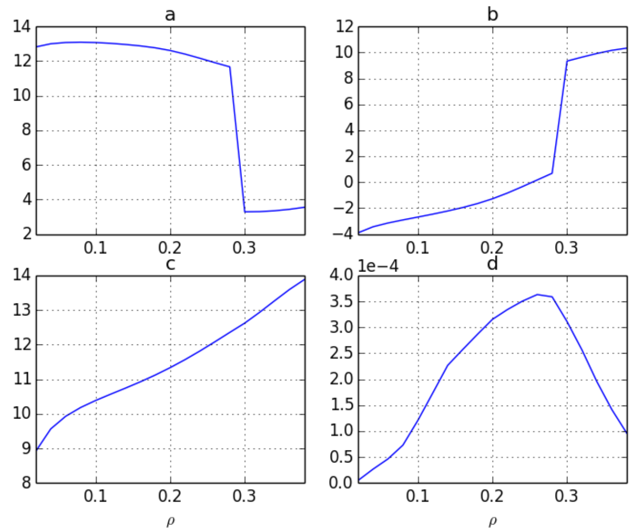

Figure 3. Radial profiles of the terms in Eq. (1) for the inverse problem solution of Figure 1 (E =341), at time 3.75 s: contribution to power density balance from (a) superdiffusive carriers, (b) diffusion, (c) time derivative of temperature. The defect of equality in Eq. (1) is shown in figure d. It is seen that the accuracy of satisfying the Eq. (1) is high enough.

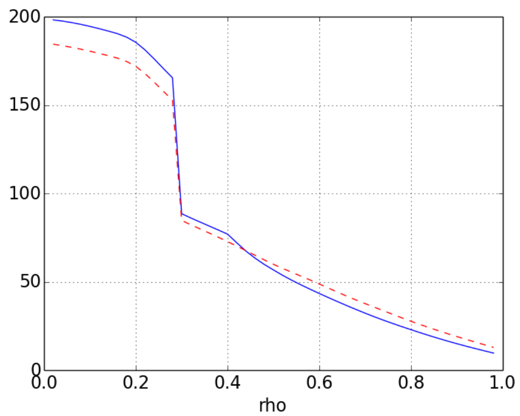

Figure 4. Radial profiles of absorbed power density before (red dashed line) and after (blue solid) perturbation, for TFTR shot #88076, at time 3.75 s (case E=341).

The case of E=341 actually corresponds to the limit of negligible transparency of the ITB to superdiffusive carriers. In this limit, penetration of the heat though the ITB is provided exclusively by the diffusion.

4. Conclusions. It is shown that introduction of the internal transport barrier (ITB) for hypothetical long-free-path (i.e. superdiffusive) carriers, with a free reflectivity of the ITB, and of the diffusion, with a free diffusivity in the range of anomalous diffusivity and with no ITB for this mechanism, enabled us to find stable solution of inverse problem aimed at the recovery of such a temperature dependence of the source function of these carriers, which may be rather close for stellarator LHD and tokamak TFTR.

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