Low-threshold parametric excitation of electron Bernstein wave trapped in a filament in O-mode ECRH experiment in ITER

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1. Introduction.

High power ECRH is widely used nowadays for heating of electron component, current drive, control of neoclassical tearing modes and the first harmonic O-mode ECRH is considered for application for this purpose in ITER [1]. During the last decade it was shown both experimentally [2-4] and theoretically [5-6] that nonlinear effects can occur in the second harmonic ECRH experiments. Proposed theoretical explanation is based on a possibility of low-threshold parametric excitation of decay waves trapped in plasma due to non-monotonic behaviour of plasma density in radial direction [5, 6]. This mechanism is not specific for the second harmonic ECRH. As it was shown recently, a similar mechanism raises a possibility of low-threshold parametric excitation also by the O-mode pump [7]. In this case the upper hybrid waves localised in two dimensions in the axially symmetric plasma filament elongated in the magnetic field direction were excited parametrically. The threshold of the corresponding convective instability was determined there by energy loss in the axial direction.

In the present work we consider the convective parametric decay instability (PDI) of the pump ordinary wave leading to excitation of a high-frequency upper hybrid (UH) wave trapped in a vicinity of the local maximum of the density profile associated with the magnetic island. We analyse it in the conditions of the first harmonic O-mode ECRH experiments in ITER. The corresponding PDI threshold and gain coefficient are evaluated.

2. Basic equations

We use the most simple but nevertheless relevant to the experiment Cartesian coordinate system (x, y, z) where the $x-$ axis along the flux coordinate and the y, z imitate the coordinates perpendicular to and along to the magnetic field line on the magnetic surface, respectively. Neglecting a weak dependence of the high frequency wave number k_x on the coordinates but the flux coordinate x we introduce the pump ordinary wave in a form

$$\mathbf{E} = \frac{\mathbf{e}}{2} \frac{a_0(y,z)}{\sqrt{k_x(x)c/\omega_0}} \exp\left(i\int^x k_x(x')dx' + ik_z z - i\omega_0 t \right) + c.c. \quad (1)$$

with $k_x >> k_z$ and $e_z >> |e_y|$. The representation (1) describes a wide microwave beam propagating from the launching antenna inwards plasma with an amplitude $a_0 = \sqrt{8P_0/\left(w^2 c\right)}\exp\left[-(y^2+z^2)/(2w^2)\right]$, where P_0 is the pump wave power and w is the beam waist. The x – component of the dimensionless wavevector $\mathbf{n} = \mathbf{k}c/\omega_0$ is a solution of the local dispersion relation

$$D = D_X D_O - n_x^2 n_z^2\left(n_x^2 - \varepsilon\right) \qquad (2)$$

with $D_X = \varepsilon^2 - g^2 - \varepsilon n_x^2$ and $D_X = n_x^2 - \eta$ being the dispersion relation of the extraordinary and ordinary waves, respectively, at $n_z = 0$, and ε, g, η standing for the components of the "cold" dielectric tensor. Assuming $n_z << 1$, one can derive n_x explicitly $n_x \approx n_{x0} + n_{x1}$, $n_{x0} = \sqrt{\eta}$, $n_{x1} = n_{x0}^2 n_z^2\left(n_{x0}^2 - \varepsilon\right)/\left[2n_{x0}D_X\left(n_{x0}\right)\right] \propto O\left(n_z^2\right)$. The components of the polarization vector can be found out from Maxwell's equations as follows $\left|e_x\right|^2 \approx \omega_{ce}^2\left|e_y\right|^2/\omega_0^2$, $\left|e_y\right|^2 \approx \left[n_{x0}n_z/D_X\left(n_{x0}\right)\right]^2\left[\omega_{ce}/\omega_0\right]^2$, $\left|e_z\right|^2 \approx 1$. A basic set of the integral-differential equations describing the decay of an ordinary pumping wave (1) into a low frequency $\Omega << \omega_0$ short-wave oscillations $\mathbf{E}_I = -\nabla\phi_I\exp\left(-i\Omega t\right)$ and an UH wave $\mathbf{E}_E = -\nabla\phi_E\exp\left(i(\omega_0 - \Omega)t\right)$ is given as follows

$$\hat{D}_{\omega_0 - \Omega}\left\{\phi_E\left(\vec{r}'\right)\right\} = 4\pi\rho_E\left(\vec{r}\right),\ \hat{D}_\Omega\left\{\phi_I\left(\vec{r}'\right)\right\} = 4\pi\rho_I\left(\vec{r}\right). \qquad (3)$$

In weakly inhomogeneous plasma the operators $\hat{D}_\omega\left(\mathbf{r},\mathbf{r}'\right) = \hat{D}_\omega\left(\mathbf{r}-\mathbf{r}',\left(\mathbf{r}+\mathbf{r}'\right)/2\right)$, $\omega = \left[\omega_0 - \Omega, \Omega\right]$ in these integral equations exhibiting much stronger dependence on the first argument $\mathbf{r}-\mathbf{r}'$ than on the second - $\left(\mathbf{r}+\mathbf{r}'\right)/2$, associated with the plasma inhomogeneity, can be represented as $\hat{D}_\omega\left[\mathbf{r}-\mathbf{r}',(\mathbf{r}+\mathbf{r}')/2\right] = (2\pi)^{-3}\int D\left[\omega,\mathbf{q},(\mathbf{r}+\mathbf{r}')/2\right]\exp[i\mathbf{q}(\mathbf{r}-\mathbf{r}')]d\mathbf{q}$ where $D_\omega\left(\mathbf{q},\mathbf{r}\right) = q^2 + \chi_e\left(\omega,\mathbf{q},\mathbf{r}\right) + \chi_i\left(\omega,\mathbf{q},\mathbf{r}\right)$ with χ_e and χ_i being defined at a fixed coordinate $\mathbf{r}$ and consisting of the real and imaginary part are familiar expressions for electron and ion susceptibilities in homogeneous plasmas. We have also introduced in (3) the nonlinear charge densities ρ_I and ρ_E at frequencies Ω and $\omega_0 - \Omega$ correspondingly generated by the ponderomotive force and being responsible for coupling of low and high frequency waves:

$$\rho_I \square\ \chi_i\left(\Omega\right)\frac{a_B^*}{8\pi}\frac{\chi_e\left(\omega_0-\Omega\right)\phi_E}{q_E^2}, \rho_E \square\ \chi_e\left(\omega_0-\Omega\right)\frac{a_B}{8\pi}\frac{\chi_i\left(\Omega\right)\phi_I}{q_I^2}, \left|a_B\right| \approx \frac{2a_0}{H}\frac{cq_E}{\omega_0}\frac{\omega_{ce}^2}{\omega_{pe}^2}|e_y|. \qquad (4)$$

3. Perturbation theory approach and the convective instability threshold.

As is known the PDI threshold decreases substantially when the high frequency UH daughter wave is trapped at least in x - direction that is possible if the turning point of the UH wave dispersion curve and the local maximum of the non-monotonous density profile are close one to another. Seeking a solution of the system $D'\left(\omega_0-\Omega,\mathbf{q},x\right)|_{\vec{q}_E,\omega_E,x_E}\equiv D'_E=0$, $\partial D'\left(\omega_0-\Omega,\mathbf{q},x\right)/\partial q_x\,|_E=0$, $\partial D'\left(\omega_0-\Omega,\mathbf{q},x\right)/\partial x\big|_E=0$ where the first equation is the dispersion relation of the UH wave, the second one is a condition of the turning point of its dispersion curve and the latter is the extremum condition for the dispersion function $D'\left(\omega_0-\Omega,\mathbf{q},x\right)|_E$ over variable x, we can get frequency $\omega_0-\Omega=\omega_E$, wave number $\mathbf{q}=\mathbf{q}_E=\left(q_E,0,0\right)$ and coordinate $x=x_E$ which, while additional constraints $\partial^2 D'/\partial q_x^2\,|_E>0$ и $\partial^2 D'/\partial x^2\,|_E>0$ hold, provide a global minimum of the dispersion function $D'_E\left(\omega_E,q_E+q_x,x_E+x\right)$ over two variables (q_x,x) and thus guarantee the UH wave trapping between two nearby turning points of the UH wave dispersion curve leading to full suppression of the corresponding convective loss from the decay region. Assuming the nonlinear pump and the convective energy loss of the UH wave along the magnetic field weak, in the first step we analyze the equation $\left[\partial^2 D'/2\partial x^2\,|_E\cdot x^2-\partial^2 D'/2\partial q_x^2\big|_E\cdot\partial^2/\partial x^2\right]\phi_E=0$ which can be solved by separation of variables $\phi_E\left(x,z\right)=b\left(z\right)\varphi_n\left(x\right)\exp\left(-iq_E x+i\omega_E t\right)$, $\varphi_n\left(x\right)=H_n\left(x/\delta_x\right)\exp\left[-x^2/2\delta_x^2\right]$ with H_n standing for the Hermitian polynomial and δ_x being the size of the UH mode localization region. In the next step of the perturbation theory procedure we take into account the UH wave PDI pumping and the effect of its energy loss from a spot of the pump beam along z direction. We multiply both sides of the first equation in (3) by $\varphi_n^*\left(x\right)$ and integrate it that leads to

$$\frac{\partial^2}{\partial z^2}b_E\left(z\right)=\frac{\chi_i\left(\Omega\right)\chi_e\left(\omega_0-\Omega\right)}{2q_E^2}\frac{\int_{-\infty}^{\infty}\frac{dx}{\sqrt{k_x c/\omega_0}}\exp\left[i\int^x\left(k_x\left(x'\right)-q_E\right)dx'+ik_z z\right]a_B\phi_I\varphi_n^*}{\eta\left(\omega_0-\Omega\right)\int_{-\infty}^{\infty}dx\left|\varphi_n\left(x\right)\right|^2}\tag{5}$$

Due to a finite value of the pump wavenumber along the magnetic field direction usual for the ECRH experiments the nonlinear current excites a heavily damped low frequency short-wave length oscillations for which the energy convection out of the excitation region is not important. Thus, seeking the potential of these induced oscillations at frequency Ω with the radial component of the wave vector $q_E - k_0\left(x\right)$ as follows $\phi_I = b_I \exp\left(-i\int^x \left(q_E - k_x\left(x'\right)\right)dx' - ik_z z\right)$, we get the amplitude of theirs b_I by means of WKB approximation $b_I\left(x,z\right) = \chi_i\left(\Omega\right)\chi_e\left(\omega_0-\Omega\right)/\left(2q_I^2 D_I\right)a_B^*\left(x,z\right)b_E\left(z\right)\varphi_n\left(x\right)$ where $D_I\left(q_E - k_0\left(x\right)\right) = D_I'\left(q_E - k_0\left(x\right)\right) - iD''\left(q_E - k_0\left(x\right)\right)$ and $D' = \left(q_E - k_0\left(x\right)\right)^2 + \chi_e' + \chi_i'$ corresponds to the residual part of the dispersion relation of the induced oscillations, D'' describes their strong absorption by the ions. Substituting ϕ_I into the RHS of (5), solving it, we can get the power threshold from the condition

$$\Gamma = 2wq''\left(P_0^{th}\right) = \sqrt{2}w\frac{\chi_i\chi_e}{q_E^2}|e_y|\left.\frac{q_E c}{\omega_{pe}}\right|_{x_E}\frac{\left|a_0\left(z\right)\right|}{H}\sqrt{\frac{\int_{-\infty}^{\infty}\frac{D''\left(\Omega,x\right)}{\left|D'\left(\Omega,x\right)\right|^2+\left|D''\left(\Omega,x\right)\right|^2}\frac{\left|\varphi_n\left(x\right)\right|^2}{k_x c/\omega_0}dx}{\int_{-\infty}^{\infty}\left|\varphi_n\left(x\right)\right|^2 dx}} = 1$$

as $P_0^{th} \approx 40/n_z^2[kW]$. Here we supposed the deuterium plasma parameters in the filament to be in the ITER range (*H = 48 kGs;* T_e*= 6 keV;* T_i*= 6 keV;* n_e*= 0.8* $10^{14}cm^{-3}$). We assume the magnetic island width to be *w=1cm* and excess of density in it equal to *6 %.*

4. Conclusions

The obtained threshold could be overcome for the microwave beam produced by a single 1 MW gyrotron only for n_z>0.2. It is important that the power threshold of the instability is not dependent on the ECRH beam width so that the beam focusing is possible without the decay instability enhancement. At small n_z< 0.2 the instability could be dangerous in the case of coupling of the power output of several generators into a single beam. However the instability threshold could be further decreased in this case for the UH wave trapped in the island also in the magnetic field direction which is possible close to the equatorial plane. This case will be considered in a separate paper.

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