Reduced Order Modelling of Resistive Wall Modes in EXTRAP T2R Reversed-Field Pinch

A.C. Setiadi[1], F. Villone[2], P.R. Brunsell[1], A. Polsinelli[2], S. Mastrostefano[2], and L. Frassinetti[1]

[1] *Department of Fusion Plasma Physics, School of Electrical Engineering, KTH Royal Institute of Technology, SE-10044 Stockholm, Sweden*

[2] *DIEI, Università di Cassino e del Lazio Meridionale, Italy*

Introduction

The Resistive Wall Modes (RWM) is a kink-like instability that limits the performance of toroidal fusion devices such as the tokamak and the reversed-field pinch (RFP). The RWM in RFP appears in multiple toroidal Fourier mode number. The RFP device EXTRAP T2R is equipped with an array of 128 control coils distributed at 32 toroidal positions. EXTRAP T2R can, with suitable modification of the power supply, also be used for studying feedback stabilization in a tokamak plasma equilibrium, and the large coil array will then enable the investigation of simultaneous stabilization of multiple toroidal modes. To this end, this study investigates the development of reduced order models for control of RWMs in the EXTRAP T2R device, including models for the n=0 mode stabilization in tokamak equilibrium.

Stabilizing the RWM is a challenging task from control engineering perspective. The use of a sophisticated model-based control algorithm might be necessary to stabilize the RWM in a future fusion device [1][2]. Many of the available control algorithms depend on the availability of an accurate model of the system to be controlled. One way of obtaining an accurate model is to perform an electromagnetic modeling of the fusion device that takes into account the plasma coupling with the complex structure of the conducting structures, which is essentially the approach taken when using the CarMa code [3]. However, such approach yields a complex model with a high number of state variables, which in turn might lead to a complex feedback controller. There is a concern regarding the computational cost since the execution of the feedback controller needs to be sufficiently fast to stabilize the multiple RWMs, which have growth rates in the order of millisecond.

To this end, it is important to find a way to obtained and an accurate and simple model of the RWM in view of successful implementation of the feedback controller.

Electromagnetic modelling of EXTRAP T2R

The EXTRAP T2R is a RFP with an aspect ratio of R/a=1.24/0.183 m. The vacuum chamber is made of stainless steel which is closely fitted by two layers of copper shells. In a typical experiment of EXTRAP T2R without RWM feedback, the plasma current is Ip=80 kA and the pulse length is 15-23 ms. The RWM diagnostics include a large number of passive and active saddle coils. The coils are distributed in 4 poloidal positions and 32 toroidal positions. Furthermore, the signals from the top coils are subtracted with the signals from the bottom coils, similarly, the signal from the outward coils are subtracted with the signal from the inward coils. For the present configuration, the Fourier mode numbers that can be resolved are m=1 and -16<=n<=15.

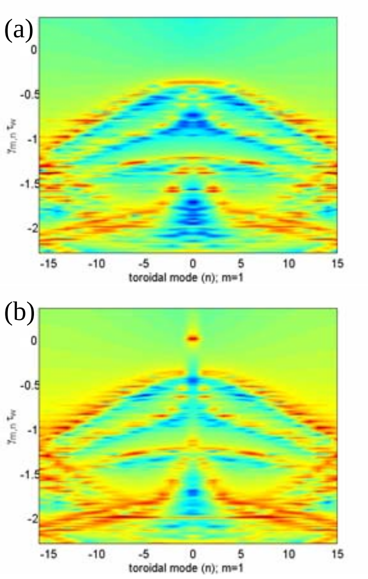

Figure 1 Spectrum of the mode-dependent growth/decay for the full order model obtained from CARIDDI code; (a) is for vacuum case and (b) tokamak case

To give a correct description of the 3D conducting structure that surrounds the plasma, the magneto-quasi-static code CARIDDI [4] is used. CARIDDI is a finite elements code based on 3D integral formulation of the eddy current equations and can be coupled with plasma equilibrium response matrices with different toroidal mode numbers (e.g. computed by CREATE_L [5] for n=0) to model the RWM, giving rise to the CarMa code [3,6]. The code has been validated experimentally on other devices, for instance in [3] the RWM in RFX-mod is modeled, while in [7] the EAST tokamak is considered. The key assumption in the modeling process is that the plasma is massless thus, it reacts instantaneously to external excitation. The resulting model can be cast into the state space form [3,6]:

$$\frac{dx}{dt} = Ax(t) + Bu(t) \; ; \; y(t) = Cx(t) + Du(t)$$

Here the state variables x(t) are associated with the current in the conducting structure, u(t) is the vector of 64 input currents of the active coils, and y(t) is the vector of 64 magnetic field as measured in the centre of the sensor coils. The complexity of the model is determined by the number of state variables, which in EXTRAP T2R case is up to 4285.

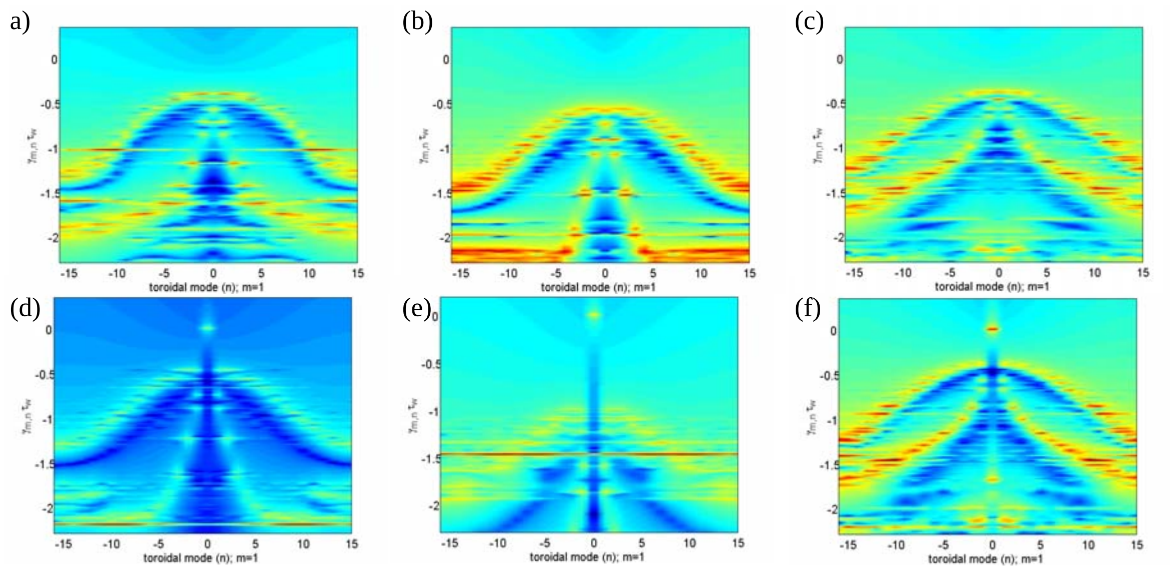

Figure 2 Spectrum of the mode-dependent growth/decay rates for the reduced order model, (a)-(c) are for the vacuum case, (d)-(f) are with the tokamak configuration with only n=0 plasma; (a) and (d) are obtained by balanced singular perturbation with order of 118, (b) and (e) by Hankel norm approximation with order of 118, finally (c) and (f) by Arnoldi algorithm with order of 128.

A reduced order model needs to be obtained without sacrificing much of the accuracy of the model. The first class of model order reduction methods is based on removing the eigenmodes of a system that has the least “importance” to the system. The notion of “importance” can be quantified by borrowing the definition of controllability and observability from control theory [8]. In the context of EXTRAP T2R model, there are 4285 eigenmodes and each of them represents a particular current distribution in the conducting structures. Some of this eigenmodes might correspond to highly localized and quickly decaying current distributions, which do not give a significant contribution to the global dynamics of the system. Two methods that belong to this class are the so-called balanced singular perturbation and the optimal Hankel norm approximation [8]. The second class of methods is based on using low order Lauren series to approximate the complex frequency response of a system such as the Arnoldi algorithm [8]. Both classes of the method have been implemented in EXTRAP T2R.

Result and Discussion

We apply the model order reduction techniques for two cases. The first is the vacuum case. The second is the case is the plasma case, where EXTRAP T2R is run as a tokamak. At the moment the plasma response is only included for the n=0 mode, for the other n-modes, the plasma response is assumed to be equal to the vacuum case. We investigate the mode-dependent

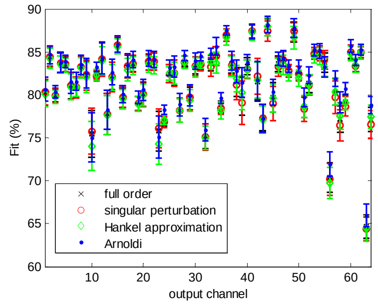

Figure 3 the average fitness value of resulting vacuum model of EXTRAP T2R with respect to the experimental data

growth rates $\gamma_{m,n}$ of EXTRAP T2R. To obtain this information, a bivariate function is introduced as in reference [2] to obtain the correlation between the eigenvalue of the system with the input-output helicity of the system. The results for the full order model can be seen in figure 1, where the red color indicates the mode-dependent growth rates normalized with the long wall time of 11.3 ms. The values of the modeled growth rates are consistent with past experiments [2]. There are two sets of contours in figure 1, which can be explained by the fact that each toroidal mode has several eigenvalues corresponding to different m-modes, and the eigenvalues appear in pairs, corresponding to two different field directions. The shell has a horizontal cut in the equatorial midplane. Due to the cut, the field diffuses faster in the horizontal direction. Similar information is obtained for the reduced order case, as can be seen in figure 2. The model reduction technique removed up to 97% of the state variables. In figure 2, only one set of contours appears that shows different growth rates for vertical and horizontal direction. The resulting model and the reduced order models are validated with 29 sets of experimental data, where the coils currents are randomized for the vacuum case. The results can be seen in figure 3. The x-axis is the individual output channel and the y-axis is the fitness rating with respect to the experimental data quantified by variance-accounted [2]. It can be seen that overall the model has good agreement with the experimental data, furthermore, the obtained reduced order models do not show significant difference with respect to the full order model while managed to reduce up to 97% of the state variables.

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