Estimation of anomalous viscosity based on modeling of experimentally observed plasma rotation braking induced by applied resonant magnetic perturbations

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Introduction

A resonant magnetic perturbation (RMP) of poloidal harmonics $m = 1$ causes braking and locking of core-resonant, naturally rotating $m = 1$ tearing modes in the Madison symmetric torus (MST) *reversed-field pinch* (RFP). This has allowed a test of time-dependent mode locking theory, including an RMP with multiple toroidal harmonics [1]. The kinematic viscosity ($\nu_\perp$) is the single free parameter in the model, adjusted such that the modeled tearing mode (TM) velocity matches the experimentally measured time evolution. The estimated viscosity [1] is consistent with a previous measurement with a biased probe [2], inserted to temporarily spin up the plasma. The model-required viscosity is about 100 times larger than the classical prediction and is linked to magnetic stochasticity. Viscosity is a key parameter in visco-resistive MHD codes like NIMROD [3], which are being used, e.g., to model tokamak disruptions, which can also involve stochasticity. But these codes require validation, which requires measurement of the viscosity over a broad parameter range. This is not possible with a biased probe, which is limited to low-energy-density plasmas. The RMP technique has no such limit and is thus ideal for validation. In the present work, the standard deviation of the $\nu_\perp$ estimate is calculated by variation of the model input-parameters within their experimental uncertainty. Furthermore, the $\nu_\perp$ is estimated in different MST plasmas and its relation to the other plasma parameters is investigated.

Experimental apparatus – application of a resonant magnetic perturbation in the MST

The presented tearing mode data were measured in MST [4]. The RMP was applied during the plasma current flat top and in-between the so called sawtooth crashes. The coils used to produce the perturbation consist of a poloidal array of 38 saddle coils, which are located at a vertical cut in MST's thick conducting shell. The radial field at the cut is measured by a set of 32 sense coils [5]. The coils are programmed to produce a $m = 1$ magnetic perturbation (MP) (figure 1(a)). However, due to the limited extent in toroidal angle a large number of toroidal n are produced. The n spectrum produced by a 165 G $m = 1$ MP have been measured in vacuum conditions (figure 1(b)). The poloidal (b_θ) and toroidal (b_ϕ) components are measured by a toroidal array of 32 and 64 coils, respectively, located at the inner surface of the shell. The relative amplitude of the radial component (b_r/b) is calculated from the Newcomb equation, which combined with the measured b_θ and b_ϕ amplitudes result in an estimation of the b_r amplitude (green circles in figure 1(b)) [1]. The relative amplitude of each n harmonics to the total applied $m = 1$ radial field at the cut, $b_r^{m=1,n}/b_r^{m=1}$, is used as scaling in the modeling to estimate the applied $b_r^{m=1,n}$ at the plasma edge in the analyzed RMP–plasma experiments.

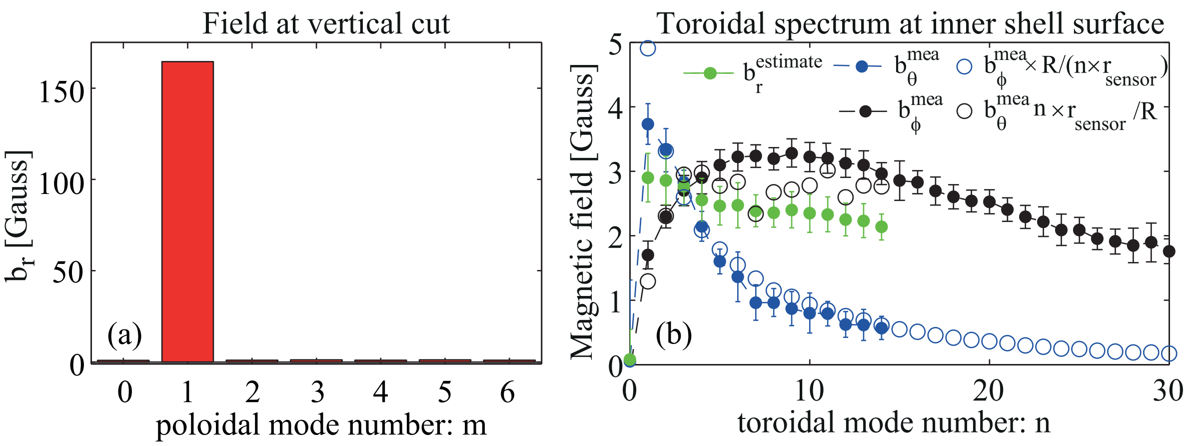

Figure 1: **(a)** The applied MP at the vertical cut. **(b)** The spectrum of toroidal mode numbers (n) in vacuum when an $m = 1$ MP of 165 G is applied at the cut.

Model - tearing mode dynamics

The dynamics of the tearing modes are described by the three coupled equations [6]: (I) the equation of fluid motion, (II) the no-slip condition and (III) the modified Rutherford equation. The equation of fluid motion describes the momentum balance in the plasma. The poloidal component can be neglected in the MST core [1] and the present model includes only the toroidal component

$$\rho(r)\frac{\partial \Delta\Omega_\phi}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\rho\nu_\perp(r)\frac{\partial \Delta\Omega_\phi}{\partial r}\right) + \sum_n \frac{T_{EM,\phi}^{m,n}(t)}{4\pi^2 r R^3}\delta(r - r_s^{m,n}), \tag{1}$$

where r is the minor radius, R is the major radius, $\rho(r)$ is the plasma density radial profile, $\Delta\Omega_\phi(r,t)$ is the toroidal component of the perturbed angular plasma velocity, $T_{EM,\phi}^{m,n}(t)$ is the electromagnetic torque caused by the interaction between the tearing mode and external resonant fields (caused, e.g., by induced currents in the wall or by the RMP-coils). The $\delta(r - r_s^{m,n})$ indicates that the $T_{EM,\phi}^{m,n}$ acts locally at the resonant surface $r_s^{m,n}$. The single free parameter is $\nu_\perp$ and it is estimated by matching the experimental TM velocity evolution. The magnitude of the electromagnetic torque caused by the RMP–TM interaction is proportional to the amplitude of both the TM ($|b_{TM}|$) and the RMP ($|b_{RMP}|$).

The model [1] can include the dynamics of several resonant tearing modes. However, the central mode ($n = 6$) has the largest impact on the fluid motion and the inclusion of two modes ($n = 6$ and $n = 7$) is sufficient to estimate the kinematic viscosity [1]. Higher mode numbers ($n > 9$) are very weakly connected to the $n = 6$ mode via the viscosity. Consequently, the model is run including the two dominant modes.

Results

The model-required kinematic viscosity is well constrained assuming that the other model input is well known, as shown in figure 2. Figure 3(a) shows another match to the experimental tearing mode velocity. The input parameters are varied within the experimental 1σ standard error and thus provide an error estimation of the estimated kinematic viscosity (figure 3(b)). The largest uncertainty is caused by the fluctuations in the measured tearing mode amplitude and velocity, which are present also before the RMP application. The estimated viscosity for a set of discharges is plotted against various plasma parameters in figure 4. There are weak trends against the density and the temperature: $\nu_\perp \propto 1/<n_e>$ and $\nu_\perp \propto T_e$. That the two parameters give the inverse relations is expected since they are roughly related as $T_e \propto 1/<n_e>$. The range of the plasma current in the present data set is too narrow to draw any conclusions. The $\beta_{\theta e}$ is calculated from measurements of the electron temperature and density, but the total beta is likely to follow the same trend [7]. The beta value has the best correlation with $\nu_\perp$, figure 4(d), where the trend is $\nu_\perp \propto 1/\beta_{\theta e}$. A possibility is that the energy confinement time τ_E increases with higher beta. The τ_E is expected to be inversely proportional to the viscous diffusion, i.e., $\tau_E \propto 1/\nu_\perp$. However, τ_E depends on several other parameters and has to be carefully calculated.

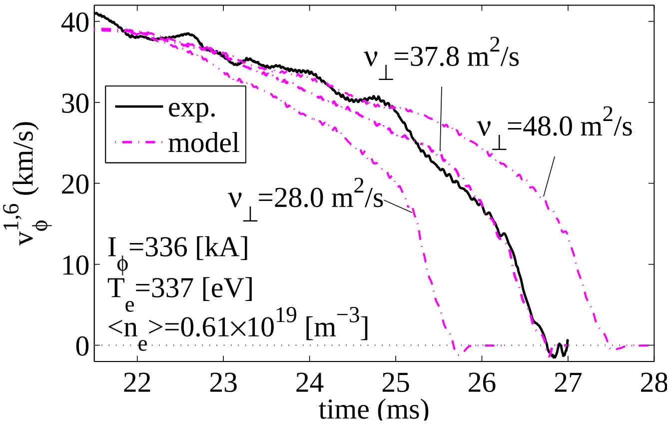

Figure 2: Time evolution of the experimental $n = 6$ TM velocity (black line) and the modeled velocity for various kinematic viscosities. The signals are smoothed in a 0.5 ms time window (see unsmoothed signals in Ref. [1]). The applied $m = 1$ RMP has the form of a step function and maximum $|b_r| = 130$ G. The best match (minimization of chi-square) is for $\nu_\perp = 37.8$ m^2/s. The experimental MST data is from shot 1150304025.

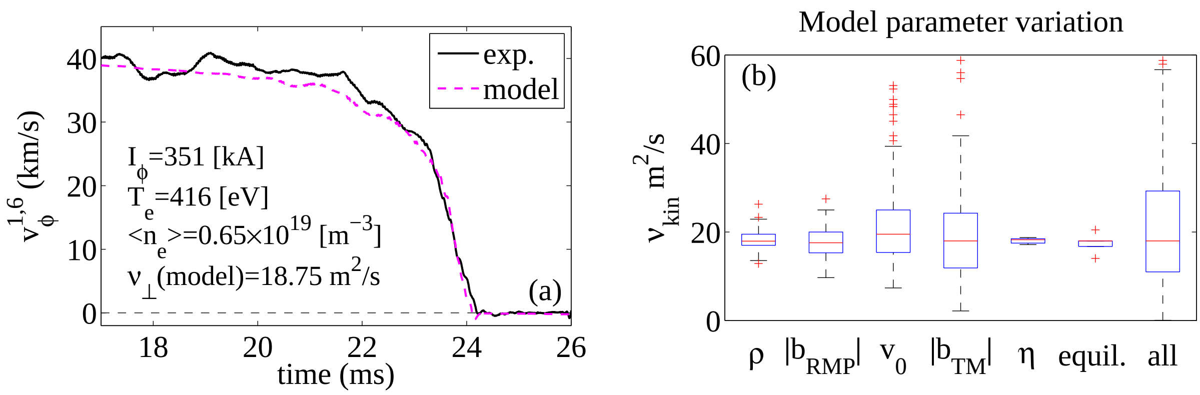

Figure 3: **(a)** Time evolution of the experimental and modeled $n = 6$ TM velocity. The applied $m = 1$ RMP has the form of a ramp function with maximum $|b_r| = 139$ G. **(b)** The $\nu_\perp$ resulting from a model parameter variation of the shot in frame (a). The box-and-whisker shows the median (red line), the 25/75 percentiles blue box, the vertical black dashed line are considered to be within the distribution and the red crosses are outliers. From left to right are the single input-parameter variation of the ρ, $|b_{RMP}|$, v_0 (initial TM velocity), $|b_{TM}|$ (initial TM amplitude), η (resistivity) and the plasma equilibrium profiles. Finally all the input-parameters are varied together. The experimental MST data is from shot 1150310065.

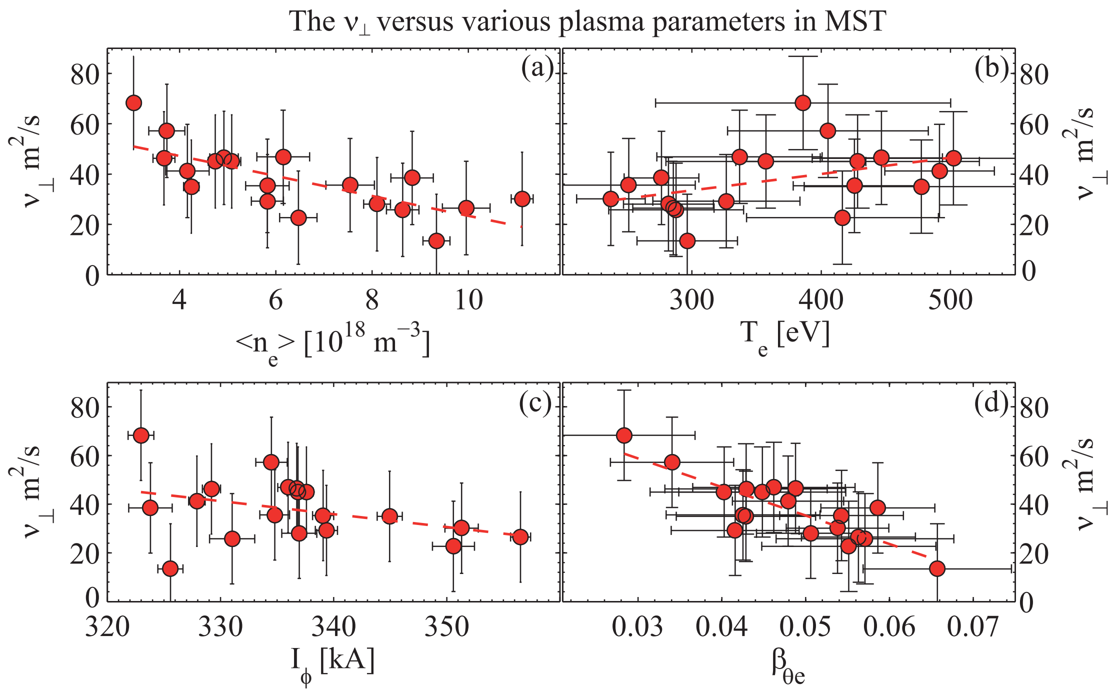

Figure 4: The $\nu_{\perp} \pm \sigma$ is estimated for a set of MST shots and plotted against the experimental **(a)** central line-averaged electron density, **(b)** the electron temperature (measured with the Thomson scattering chord closest to the $n = 6$ TM resonant surface), **(c)** the plasma current and **(d)** the poloidal beta, $\beta_{\theta e} = 2\mu_0 < n_e > T_e/B_\theta^2(a)$. The red lines are linear fits to the data and the error bars are 1σ.

Conclusion and future work

In the main part of the plasma discharges the estimated viscosity is within the range 20-50 m^2/s, which is similar to measurements made with other methods in MST [2, 8]. The standard error of the estimated $\nu_\perp$ is large (about $\sigma \approx 20$ m^2/s), but might be reduced by including knowledge of the pre-RMP fluctuations in the TM signals, which is the largest source of uncertainty in the estimated $\nu_\perp$. In another RFP, EXTRAP T2R, the core–viscosity has been estimated to $\nu_\perp \approx$ 4–40 m^2/s [9], which is also anomalous and similar to the MST value. Estimation of $\nu_\perp$ with the RMP–technique in different MST operations (confinement regimes), and comparison with energy confinement time and the $\nu_\perp$ measured using the biased–probe–technique [2] should be interesting future work.

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