Kinetic analysis of the interaction between particles and magnetic islands

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The equilibrium configuration in a tokamak consists of axi-symmetric nested toroidal flux surfaces, called magnetic surfaces. This favorable configuration, however, can be perturbed by a radial magnetic perturbation that arises on a rational magnetic surface of safety factor $q = m/n$ where integers m and n are, respectively, the poloidal and toroidal mode numbers of the perturbation. On these flux surfaces the parallel current can exhibit a current singularity layer. The natural response of the system is to regularize the singularity by non-ideal effects [1]. This leads to the breaking and reconnecting of the magnetic field-lines, resulting in the modification of the topology of the field by the destruction of resonant surfaces and the formation of magnetic islands [2]. This electromagnetic instability is called the *tearing* instability. In tokamak plasma, the presence of magnetic islands can have a severe impact on the confinement. The parallel streaming of particles following the perturbed field lines across the island results in enhanced cross-field transport leading to the flattening of the radial profiles. This can significantly reduce core confinement and eventually lead to a disruption [3]. Hence the importance of the control of this resistive instability. Furthermore, in a tokamak, a large fraction of plasma particles can have large gyro-orbits and banana orbits due to their large kinetic energy. These high energy populations arise from various heating mechanisms like ion cyclotron radio frequency heating (ICRF). The ensuing non-Maxwellian distribution function can strongly affect the equilibrium, stability, and transport in the plasma.

Classical tearing modes are known to be driven by parallel current density gradient and their stability is characterized by the parameter Δ' [4]. In the absence of fast particles and neglecting ion contribution, the growth rate of the mode scales linearly with Δ' [5]. It was shown in ref. [6] that the effects of energetic ions on tearing modes mainly come from the ideal region of the

instability due to the large drift orbit width of energetic ions. Furthermore, C. Hegna showed in [8] that a population of energetic ions can suppress the nonlinear island growth when its density profile peeks just outside of the rational surface. Energetic particles can provide an additional source or sink of free energy affecting the tearing modes by changing the value of Δ' which is determined by equilibrium quantities. In the presence of energetic particles the background distribution function is no longer a Maxwellian. It was shown in ref. [7] that in this case, the energetic particles can affect the perturbed parallel current response due to their resulting magnetic drift.

In the present work we investigate the stabilization mechanisms of magnetic islands by studying their interaction with particles, both thermal and energetic. The gyrokinetic theory is used for this purpose as it is a theoretical framework that allows the analysis of the impact of wave-particle resonances on plasma instabilities. The mode frequency is analytically calculated in two cases, first in the presence of an energetic electron population taking into account the Landau resonance only and second in the absence of fast particles taking into account both the Landau and curvature drift resonances. The considered geometry is slab and the equilibrium magnetic field is of the form $\mathbf{B_0} = B_0(\mathbf{e_z} + \mathbf{e_y} x/L_s)$, where $\mathbf{e_y}$ and $\mathbf{e_z}$ are unit vectors and L_s is the magnetic shear length. Closely following refs. [9, 10] the linear growth rate of a tearing mode is calculated starting from the linearized gyrokinetic collisionless Vlasov equation for each species,

$$\left[-i\omega + \mathbf{v} \cdot \nabla + \frac{q_j}{m_i} \frac{\mathbf{v} \times \mathbf{B_0}}{c} \cdot \frac{\partial}{\partial \mathbf{v}} \right] f_j = -\frac{q_j}{m_j} \left[\tilde{\mathbf{E}} + \frac{\mathbf{v} \times \tilde{\mathbf{B}}}{c} \right] \cdot \frac{\partial F_j}{\partial v} \tag{1}$$

coupled to parallel Ampère's law,

$$(\nabla \times \mathbf{B})_\parallel = \frac{4\pi}{c} \sum_{e,i} q_j \int \mathbf{d^3v} v_\parallel f_j \tag{2}$$

where ω is the frequency of the mode, f_j and F_j are the perturbed and equilibrium distribution functions, $\tilde{\mathbf{E}}$ and $\tilde{\mathbf{B}}$ are the perturbed fields represented as a typical perturbation of the form $Q(\mathbf{x}, t) = \tilde{Q}(\mathbf{x}) \exp(-i\omega t)$. The dispersion relation for the parallel vector potential is then obtained.

In the following, the first case (energetic electrons, Landau resonance) is presented. The equilibrium distribution function can be written as $F = F_M + F_{EP}$, where the energetic particles distribution is a shifted Maxwellian. The dispersion relation for the mode is derived taking into account only the resonance in parallel velocity. For simplicity the energetic particles population is taken to be of the electrons species. The frequency of the mode is found to have a real (ω) and

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an imaginary part (γ). In figure 1 ω (left) and γ (right) are compared to the growth rate γ_0 found
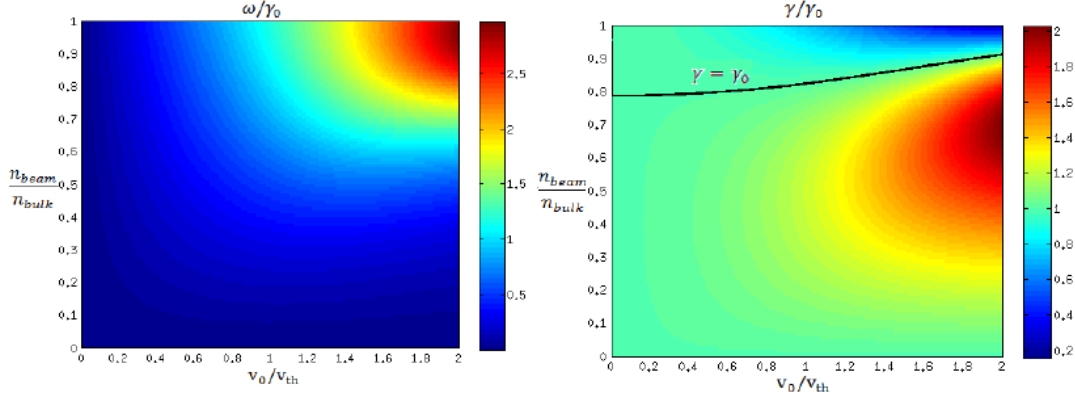

Figure 1: *Real (left) and imaginary (right) parts of the mode frequency in terms of the velocity and the density of the beam of fast ions compared to the growth rate γ_0 (solid black line) in the absence of energetic particles.*

in the absence of energetic particles. They are plotted as a function of n_{beam}/n_{bulk} (n_{beam} is the density of the energetic particle population and n_{bulk} is the plasma bulk density) and v_0/v_{th} (v_0 is the velocity of the beam and v_{th} is the thermal velocity). The result shows the appearance of a real component of the mode frequency (contrary to the case without energetic particles in which $\omega = 0$) for significant beam velocity and beam density. The mode exhibits a growth at significant values of the beam velocity but is seen to be stabilized by an increase in the density of energetic particles.

We are also interested in analyzing the effects of the drift motion of thermal particles on the tearing stability. We calculate the growth rate of the mode taking into account the electron ∇B drift resonance together with the Landau resonance. We are considering short wavelength variations across the magnetic field and long wavelength variations parallel to $\mathbf{B_0}$, so we make the approximation $\rho_i \nabla_\perp \sim 1$. The perturbations with such rapid spatial variations perpendicular to the magnetic field can be represented in the form of an eikonal function $S(x,y,z)$ [11] such that $\tilde{Q}(\mathbf{x}) = \hat{Q}(\mathbf{x}) \exp(iS)$, where $\hat{Q}(\mathbf{x})$ is the amplitude and S satisfies $\mathbf{b} \cdot \nabla S = 0$. For large parallel currents in the singular layer, the electrostatic field is shown to be small, so we will neglect its contributions [5]. The constraint boundary condition leading to the dispersion relation takes the form

$$\Delta' = \int dx \left[\frac{4\omega}{\sqrt{\pi}} \sum_{j=e,i} \frac{\omega_{pj}^2}{c^2} \int \frac{\exp\left(-v^2\right)}{\omega - k_\parallel v_\parallel - \omega_D} J_0^2 \left(\alpha_j v_\perp\right) v_\perp v_\parallel^2 dv_\perp dv_\parallel \right] \quad (3)$$

in which the Landau and the curvature drift resonances are apparent, ω_p is the plasma frequency, J_0 is the Bessel function of first kind, $\alpha_j = \nabla S/\Omega_j$ and $\omega_{Dj} = \mathbf{v_{Dj}} \cdot \nabla S$ where $\mathbf{v_{Dj}} =$

$\mathbf{b} \times \mu \nabla B_0 / (m_j \Omega_j)$. Integrating over $v_\parallel$ we get

$$\Delta' = \int dx \left[\frac{4\omega_{pe}^2}{c^2} \zeta_e^2 \int_{\mathbb{R}^+} \Lambda \left[1 + \Omega_e' Z \left(\Omega_e' \right) \right] v_\perp e^{-v_\perp^2} J_0^2(\alpha_e v_\perp) \, dv_\perp \right] \tag{4}$$

where $\Omega_e' = \zeta_e \Lambda = \zeta_e (1 - \frac{\omega_{Be}}{\omega} v_\perp^2)$ and $\zeta_e = \omega L_s / k_y v_e x$ and $\omega_{Be} = -v_e^2 / 2\Omega_e a L_s$. $Z(\Omega_e')$ is the plasma dispersion function. Integrating eq. (4) first over the radial coordinate x and then over $v_\perp$ taking the Bessel function to the first order in $\alpha_e v_\perp$, we find the growth rate

$$\gamma = \frac{c^2}{\omega_{pe}^2} \frac{k_y v_e}{\sqrt{\pi} L_s} \Delta' \tag{5}$$

where k_y is the wave number in the binormal direction and v_e is the electron thermal velocity. The growth rate is found to have the same scaling with Δ' as in the case where only the $k_\parallel v_\parallel$ resonance is considered but with double the coefficient. Consequently, using this approach we conclude that the drift resonance has the effect of doubling the growth rate of the tearing mode. However, one should be careful about the generality of the result since it is only valid for $\alpha_e v_\perp < 1$, because of the approximation made on the argument of the Bessel function $J_0(\alpha_e v_\perp)$, and for $\omega_{Be} v_\perp^2 / \omega x < 1$ because they are arguments of the plasma dispersion function that must tend to zero at the boundary limit of the integral.

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