Modeling of seed magnetic island formation

I.G. Miron

National Institute for Lasers, Plasma and Radiation Physics, Magurele-Bucharest, Romania

Introduction

The dynamics of the neoclassical tearing modes and magnetic island is shown based on a quasi-analytic model that calculates the 3D perturbations spectrum inside and outside the magnetic island. The calculations are performed for the case of an ASDEX-Upgrade plasma surrounded by an inhomogeneously resistive wall. A spectrum of magnetic perturbations (MP) generated by a set of in-vessel saddle coils (B-coils) is considered[1].

The matching condition and the outer solutions

The 3D model considers a low plasma inverse aspect ratio approximation, a thin surrounding resistive wall and the assumption that the wall and the feedback coils lie on magnetic surfaces. The used geometry involves flux coordinates of Hamada type, (r, θ, φ), i.e. the "radial" flux coordinate, the poloidal and the toroidal angles, respectively. A constant local plasma toroidal rotation is kept in order to preserve the validity of the perturbed model, i.e. the small perturbations scale of variation from a static equilibrium state. According to [2], for a perturbed magnetic parametrization of the form $\mathbf{b} = \nabla\varphi \times \nabla\psi$, the tearing stability index measuring the jump of the perturbation across the magnetic island at the (m,n) magnetic surface is

$$\Delta_s'(t) = -(2m/r_s)\left[1 - \psi_{s,ext}^{mn}(t)/\psi_s^{mn}(t)\right] \tag{1}$$

$\psi_s^{mn}(t)$ is the NTM perturbation magnetic flux calculated inside the island (Fourier decomposition term) from the solving of the magnetic island perturbed resistive equations. r_s is the radial flux coordinate of the magnetic surface where the island develops. $\psi_{s,ext}^{mn}(t)$ is the perturbation calculated outside the magnetic island. Our calculations of the outer perturbations rely on the parametrization of the perturbed magnetic field in terms of ϕ, where $-\partial\phi/\partial t$ is the perturbed scalar electric potential, and of the perturbed plasma velocity $\mathbf{v}$, for an equilibrium magnetic field $\mathbf{B}$: $\partial\mathbf{b}/\partial t = \nabla\times(\mathbf{v}\times\mathbf{B})$, $\mathbf{v} = (1/B)\nabla(\partial\phi/\partial t)\times\mathbf{n} + (\mathbf{v}\cdot\mathbf{n})\mathbf{n}$, $\mathbf{n} = \mathbf{B}/B$. By comparing both parametrizations we get within the first order of the low inverse aspect ratio approximation that $\psi_{s,ext}^{mn}(t) = \mathrm{i}(n - m/q_s)\phi_s^{mn}(t)$, our calculated outer solutions being

$$\phi_s^{mn}(t) = A_s^{mn} + B_s^{mn}\exp(-\mathrm{i}n\Omega_{MP}t) + \sum_{p=1}^{6L} C_{ps}^{mn}\exp(\tau_p t) \tag{2}$$

q_s is the safety factor at r_s. Ω_{MP} is the toroidal rotation angular velocity of the rotating magnetic perturbations spectrum generated by the B-coils. τ_p are the roots of the determinant of the linearized system of the Laplace transformed perturbed equations: $\Delta_s(\tau_p) = 0,\ p = 1,..,6L$. $L = (m_2 - m_1 + 1)(n_2 - n_1 + 1)$, where $m_1 \le m \le m_2$ and $n_1 \le n \le n_2$ (see [3]). The calculated coefficients in (2) are

$$A_s^{mn} = \left.\frac{\Delta_s^l}{(\tau + \mathrm{i}n\Omega_{MP})\Delta_s}\right|_{\tau=0}, \quad B_s^{mn} = \left.\frac{\Delta_s^l}{\tau\Delta_s}\right|_{\tau=-\mathrm{i}n\Omega_{MP}}, \quad C_{ps}^{mn} = \left.\frac{(\tau - \tau_p)\Delta_s^l}{\tau(\tau + \mathrm{i}n\Omega_{MP})\Delta_s}\right|_{\tau=\tau_p} \tag{3}$$

Δ_s^l is Δ_s with $l = m - m_1 + 1 + (n - n_1)(n_2 - n_1 + 1)$ column replaced by right hand term vector of the following outer system of Laplace transformed equations (bar indicates the Laplace transform $\bar{\phi}(\tau) = \mathscr{L}(\phi(t))$)

$$\begin{aligned} \sum_{j,k}\sum_{\alpha=0}^{4} \tau^\alpha \left(P_{mn}^{jk\alpha}\bar{\phi}_s^{jk} + \tilde{P}_{mn}^{jk\alpha}\bar{\phi}_s'^{jk}\right) &= \sum_{j,k}\left(\sum_{\alpha=0}^{3} \tau^\alpha R_{mn}^{jk\alpha} - \frac{1}{\tau + \mathrm{i}k\Omega_{MP}}\sum_{\alpha=0}^{3} S_{mn}^{jk\alpha}\right) \\ \sum_{j,k}\sum_{\alpha=0}^{2} \tau^\alpha \left(W_{mn}^{jk\alpha}\bar{\phi}_s^{jk} + \tilde{W}_{mn}^{jk\alpha}\bar{\phi}_s'^{jk}\right) &= \sum_{j,k}\left(\sum_{\alpha=0}^{1} \tau^\alpha \tilde{R}_{mn}^{jk\alpha} - \frac{1}{\tau + \mathrm{i}k\Omega_{MP}}\sum_{\alpha=0}^{3} \tilde{S}_{mn}^{jk\alpha}\right) \end{aligned} \tag{4}$$

$\mathbf{P}^\alpha$, $\tilde{\mathbf{P}}^\alpha$ and $\mathbf{W}^\alpha$, $\tilde{\mathbf{W}}^\alpha$ are the plasma parameters and the wall and feedback parameters matrices, respectively. $\mathbf{R}^\alpha$, $\tilde{\mathbf{R}}^\alpha$ and $\mathbf{S}^\alpha$, $\tilde{\mathbf{S}}^\alpha$ are the initial perturbations and the rotating MP spectrum matrices having the toroidal angular velocity Ω_{MP}.

Inner solutions

Following the method from [2], but using our time-dependent solution derived outside the magnetic island, we get at early times

$$\begin{aligned} \psi_s^{mn}(t) = \frac{\mathrm{i}m(n - m/q_s)}{\pi t_R t_A} &\left\{A_s^{mn}t^2 + \frac{2B_s^{mn}}{n^2\Omega_{MP}^2}\left[1 - \mathrm{i}n\Omega_{MP}t - \exp(-\mathrm{i}n\Omega_{MP}t)\right]\right\} \\ &- \mathrm{i}(n - m/q_s)\sum_{p=1}^{6L}\frac{C_{ps}^{mn}}{\tau_p^2}\left[1 + \tau_p t - \exp(\tau_p t)\right] \end{aligned} \tag{5}$$

t_R and t_A are the resistive and the Alfven times, respectively. At later times (FKR and Rutherford regimes), a more complicated time dependent solution is obtained

$$\begin{aligned} \psi_s^{mn}(t) = \frac{\mathrm{i}(n - m/q_s)}{t_{FKR}^{5/4}} &\left\{\frac{A_s^{mn}t^{5/4}}{\Gamma(9/4)} - \frac{\mathrm{i}B_s^{mn}}{n\Omega_{MP}}\left[\frac{t^{1/4}}{\Gamma(1/4)}\left(4 + \exp(-\mathrm{i}n\Omega_{MP}t)E_{3/4}(-\mathrm{i}n\Omega_{MP}t)\right)\right.\right. \\ &\left.\left. - \frac{\exp(-\mathrm{i}n\Omega_{MP}t)}{(-\mathrm{i}n\Omega_{MP}t)^{1/4}}\right] - \sum_{p=1}^{6L}\frac{C_{ps}^{mn}}{\tau_p}\left[\frac{t^{1/4}}{\Gamma(1/4)}\left(4 + \exp(\tau_p t)E_{3/4}(\tau_p t)\right) - \frac{\exp(\tau_p t)}{\tau_p^{1/4}}\right]\right\} \end{aligned} \tag{6}$$

Γ and E_ν are gamma and generalized exponential integral function, respectively. t_{FKR} is the linear tearing mode diffusion time $t_{FKR} = (t_R^{3/5}t_A^{2/5}/m^{6/5})[\pi\Gamma(3/4)/\Gamma(1/4)]^{4/5}$.

Modeling of the island evolution

The above calculated solutions are used to analytically derive a time dependent formula of the tearing stability index (1). By solving the modified Rutherford equation the magnetic island width evolution is obtained. As a general observation, it should be noted that the model presented here is a perturbations theoretical model that is obviously valid as long as the plasma equilibrium is not changed. Therefore the model cannot describe the NTM saturation regime. An on-going confinement degradation invalidates the perturbed dynamic model. Time traces of the island width are shown in Fig. 1 with and without the bootstrap term. An expected destabilizing behavior is obtained. Due to the multimode approach the effect of the adjacent modes to the central NTM perturbation is found. This aspect is of a special interest when a spectrum of external MP (of error field type) is taken into account, such as the one generated by the ASDEX-Upgrade B-coils. Fig. 2 and 3 show the island evolution in the single mode case along with the cases when adjacent poloidal modes are considered. Whereas for the $(2,1)$ island the both more negative and more positive neighboring modes destabilize the central mode (Fig. 2), for the $(3,2)$ island the more negative neighboring mode has a more destabilizing effect (Fig. 3). A similar analysis performed in the toroidal case proves that the adjacent toroidal modes have a significantly lower influence on the central unstable mode compared to the poloidal neighboring modes.

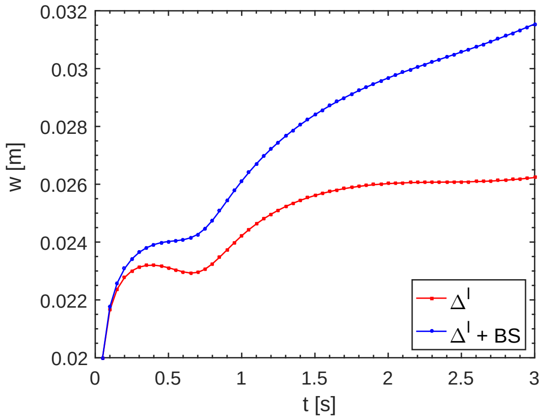

Figure 1: *Calculated island width in the absence vs. in the presence of the bootstrap term.* Δ' *is the calculated tearing stability index.*

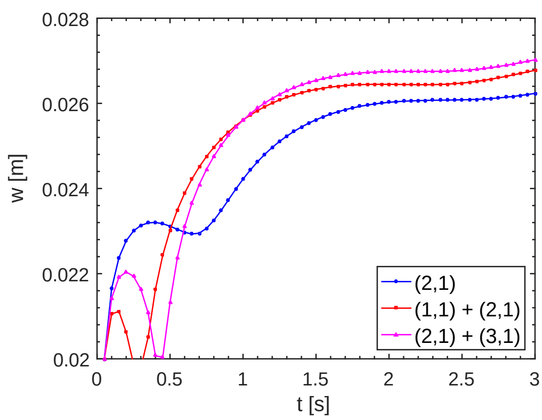

Figure 2: *The effect of the neighboring poloidal modes to the* $(2,1)$ *island width dynamics.*

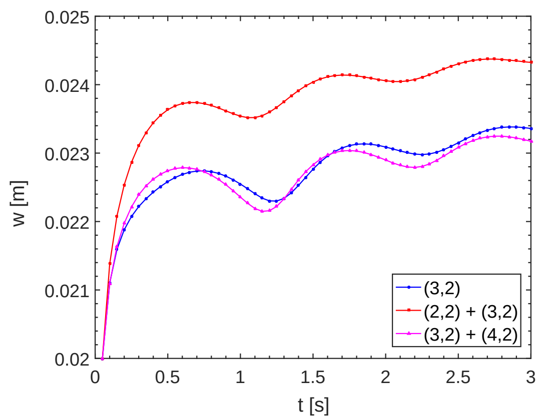

Figure 3: $(3,2)$ *island evolution in the presence of neighboring poloidal modes.*

Time traces of the normalized $(2,1)$ mode amplitude are shown in Fig. 4 for different phase shifts between the upper and lower B-coils rows $\Delta\phi$. The coils are switched on between 1.5 s and 2.5 s. The signal spectrum has a maximum current of $I_{max} = 1$ kA at $f = 0.5$ Hz toroidal frequency. The maximum resonance between the MP and the mode occurs for $\pi/2 < \Delta\phi < 3\pi/4$.

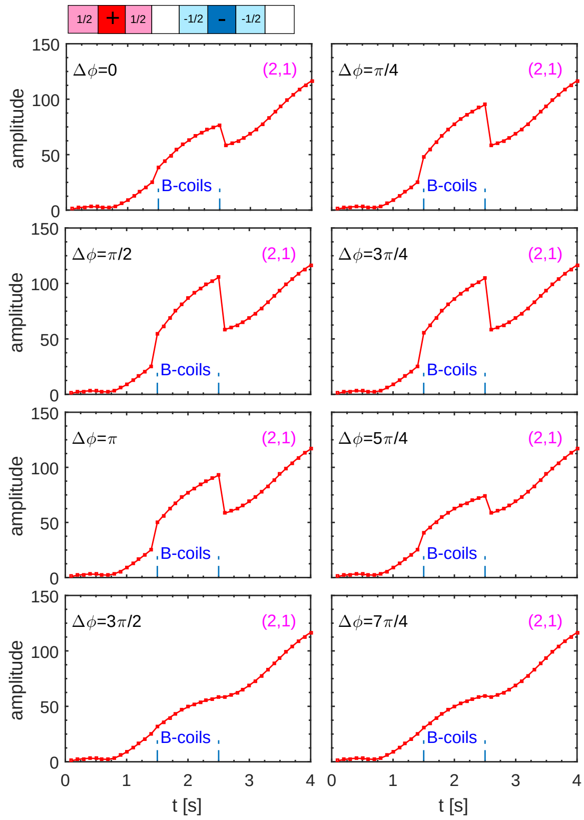

Figure 4: $(2,1)$ *NTM normalized amplitude for different toroidal phasing* $\Delta\phi$ *of the coil currents.* $I_{max} = 1$ *kA.*

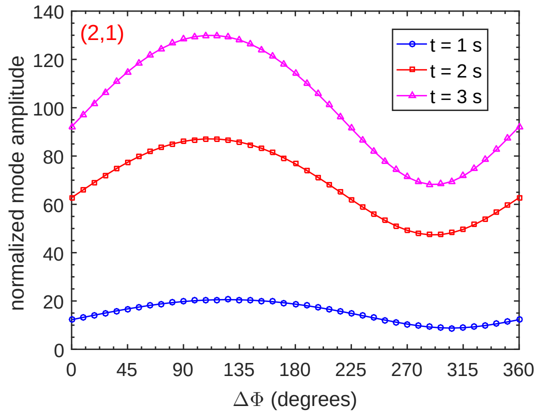

Figure 5: *Calculated normalized* $(2,1)$ *NTM amplitude versus the toroidal phasing* $\Delta\phi$ *of the coil currents at* $t = 1s$*,* $2s$ *and* $3s$*, respectively.*

More precisely, Fig. 5 drawn at different time points shows that $\Delta\phi \approx 110^o$ corresponds to the maximum resonance. Plasma response to applied perturbations is explicitly calculated. To conclude, the modeling of the island evolution is possible within the regimes of interest as long as the model validity requirements are fulfilled. The solutions derived here could be easily used to further calculate the MP induced braking torques that damp the plasma rotation and subsequently affect the island evolution.

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