

On growth rates and scalings for slow and fast resistive wall modes

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A theory-based scaling for resistive wall modes (RWMs) growth rate is proposed and tested in numerical calculations. Similar to the earlier result for ITER [1], it can be cast in a simple form $\tau_D \gamma = (\beta - \beta_{no}) / (\beta_{id} - \beta)$, where β_{id} and β_{no} are the beta limits with and without wall stabilization respectively, τ_D is an effective resistive wall time. The latter depends on the wall proximity to plasma and the poloidal harmonic structure of the mode at the plasma boundary. In the thin-wall regime, the scaling closely follows the variational Haney-Freidberg estimate [2] (H-F) for slow RWMs in terms of ideal MHD beta limits set by external kink modes. The results of the RWM growth rate calculations with the KINX-RWM code [3] are presented to show that the RWM growth rate scaling is typically valid with τ_D values close to the cylindrical approximation. Application of the thin-wall RWM growth rate scaling predicts the transition to the thick-wall regime at $\beta = \beta_{id} - (\beta_{id} - \beta_{no}) / (1 + \tau_D / \tau_{sk})$, which must be rather close to $n=1$ limit β_{id} for ITER parameters because of high values of the ratio τ_D / τ_{sk} , where $\tau_{sk} = \tau_w d_w / r_w$ and $\tau_w = \mu_0 \sigma d_w r_w$ are the skin time and resistive time of the wall of thickness d_w , minor radius r_w and conductivity σ . Calculations with a thick wall should be used for $\tau_{sk} \gamma > 1$ and the fast RWM growth rate estimate proposed in [4] can be utilized for that.

1. H-F estimate accuracy The Haney-Freidberg theory [2] gives the following expression for the real growth rate γ of magnetic field perturbations \mathbf{b} that varies in time as $e^{\gamma t}$:

$$\gamma \tau_D = -W_{no} / W_{id}. \quad (1)$$

Here W_{no} and W_{id} are the perturbed energies without and with an ideal wall and τ_D is the “resistive wall diffusion time” defined by Eq. (66) in Ref. [2]. This dispersion relation is derived in the thin-wall approximation. In fact τ_D is a functional depending on the trial perturbed field used for the estimate (1):

$$\tau_D = (W_{id} - W_{no}) / D_w, \quad (2)$$

where the quadratic form D_w is related to dissipated power in the resistive wall (see for example [5]). To estimate the accuracy of approximation (1) for the RWM growth rate, calculations of $n=1$ RWMs were performed with the KINX-RWM code for the standard sequence of ITER scenario 4 equilibria with different values of β_N [6]. Only the first resistive wall was taken into account. Two estimates for RWM growth rates γ (Figure 1a) and τ_D (Figure 1b) obtained with different normal magnetic field perturbation at the plasma boundary – exact RWM solution and no wall ideal kink mode – are compared.

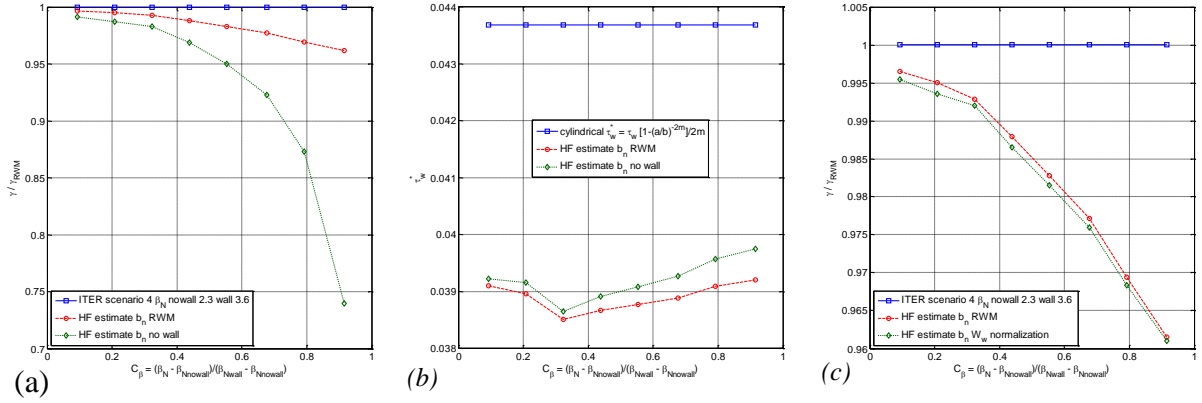


Figure 1. Comparison of H-F estimates and τ_D for different trial perturbed fields: (a) the growth rate estimate normalized by exact RWM growth rate in inertia-less thin wall approximation γ_{RWM} ; (b) resistive wall time τ_D and cylindrical estimate; (c) comparison of the H-F for the exact RWM mode and for the mode structure from minimization of $\delta W_{no} / \delta W_{id}$.

The cylindrical value $\tau_{Dcyl} = \tau_w (1 - x_{pl}^{-2m}) / (2m)$ [5] is also shown for ITER model with ratio of the wall minor radius to the plasma radius $x_{pl} = 1.35$, $m = 2$. The deviation of the estimate (1) from the calculated growth rate γ_{RWM} is less than 5% for the normalized beta $C_\beta = (\beta - \beta_{no}) / (\beta_{id} - \beta_{no}) < 0.9$ in case of exact RWM mode. Small variation of the $\tau_D = (\delta W_{id} - \delta W_{no}) / D_w$ for different field perturbations suggests that minimizing the ratio $\delta W_{no} / \delta W_{id}$ over trial magnetic field perturbation at the plasma boundary would give better estimate. However, such a minimization gives the H-F result very close but inferior to the exact RWM trial perturbation (Figure 1c).

2. Scaling for slow RWM growth rates The H-F estimate suggests a simple scaling for the RWM growth rate in terms of the standard limits β_{id} and β_{no} , if we use the approximation

$$W_{no} \approx C_{no} (\beta - \beta_{no}) W_{id} \approx C_{id} (\beta - \beta_{id}) \text{ with } C_{no} \approx C_{id} :$$

$$\tau_D \gamma = (\beta - \beta_{no}) / (\beta_{id} - \beta). \quad (3)$$

To test its accuracy, we approximate the computed RWM growth rate by the function $\tau_{D_{cyl}}\gamma = C_{fit}(\beta - \beta_{no})/(\beta_{id} - \beta)$ with a free fitting parameter C_{fit} and the cylindrical estimate of the resistive wall time with $m=2n$, where n is toroidal mode number. For the case presented in Figure 1 the value of C_{fit} is 0.78.

The calculations performed with the KINX-RWM code for the ITER Scenario 4 equilibrium with separatrix at the boundary under variations of the pedestal height show that scaling (3) is applicable to medium- n ELM-RWM modes. The cylindrical estimate for the τ_D value is also quite accurate for these modes. In Figure 2 the computed RWM growth rates and the fits for them are presented for $n=1$, $n=5$ and $n=10$ modes. Note that the fitting parameter C_{fit} is typically less than 1 meaning that the effective resistive wall time in scaling (3) must exceed the cylindrical value $\tau_{D_{cyl}}(m=2n)$ and $C_{no} < C_{id}$. However, the opposite tendency is found for high $n=10$ and the wall very close to the plasma ($x_{pl}=1.05$) with lower effective resistive time due to $C_{fit}=1.7$. The latter suggests earlier transition to the fast RWM regime as explained in the next section.

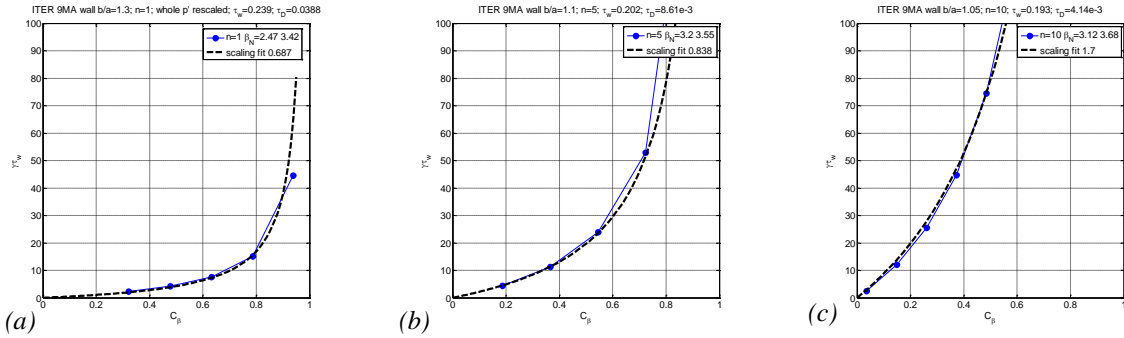


Figure 2. The RWM growth rate in thin wall inertia-less approximation versus normalized beta C_β for the ELM-RWM mode: (a) $n=1$, $x_{pl}=1.3$, $C_{fit}=0.69$; (b) $n=5$, $x_{pl}=1.1$, $C_{fit}=0.84$; (c) $n=10$, $x_{pl}=1.05$, $C_{fit}=1.7$

3. Fast RWM growth rates The thin wall approximation breaks down at $\tau_{sk}\gamma > 1$ when the RWM growth rate is high enough. Then the skin effect in the wall along with plasma inertia effects come into play. Figure 3 presents the computed growth rates for the case illustrated by Figure 1 ($n=1$ RWM) and for the case represented by Figure 2c ($n=10$ ELM-RWM). In accordance with scaling (3) and using the corresponding effective resistive wall time we get $\tau_{sk}/\tau_D = 0.1$ and 2.2 for these two cases so that the expected transition to the fast RWM regime $C_\beta > 1/(1 + \tau_{sk}/\tau_D)$ must be at 0.9 and 0.3, respectively. Calculations confirm that the fast RWM growth rate significantly differ from the thin wall estimates beyond these thresholds. The formulation based on the ideal MHD potential and kinetic energy forms described by Eq. (61) in [4] provides a way to avoid calculations inside the thick wall when

the skin depth is small, $s \ll d_w$ or $\tau_{sk}\gamma \gg 1$. The applicability range of this formulation seems to be $C_\beta > 0.9$ for instabilities with $n=1$ and $C_\beta > 0.5$ for $n=10$ in the considered ITER configuration.

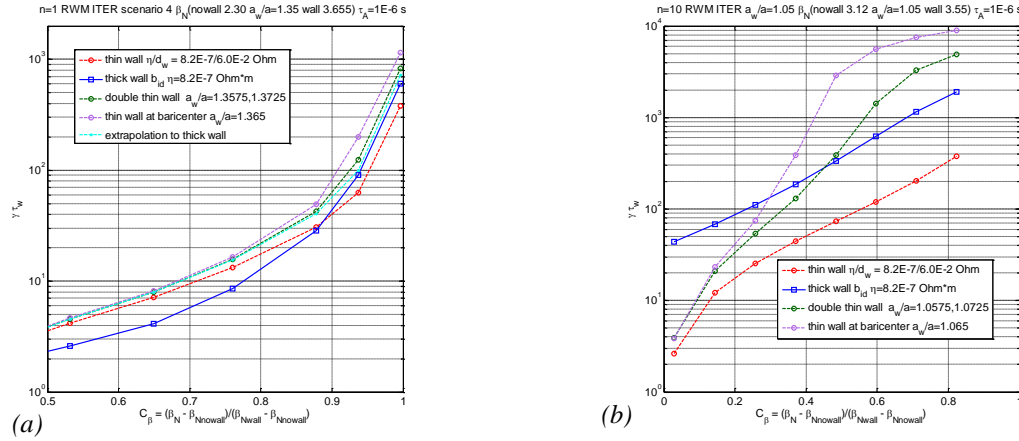


Figure 3. Comparison of fast growth rate estimates to the thin wall (red dash line) growth rates for (a) $n=1$ RWM case with ITER wall; (b) $n=10$ ELM-RWM with $x_{pl}=1.05$ wall position. The single and double wall approximations for the thick wall are presented together with the formulation based on the ideal MHD potential and kinetic energy forms.

Conclusions The scaling for slow RWM growth rates $\tau_D \gamma = (\beta - \beta_{no}) / (\beta_{id} - \beta)$ is proposed and validated in numerical calculations for the ITER Scenario 4 equilibria with different wall positions and toroidal mode numbers of the modes. The effective resistive wall time τ_D entering the H-F estimate (1) is reliably approximated by the analytic expression for cylindrical plasma $\tau_D = \tau_w (1 - x_{pl}^{2m}) / (2m)$, where the dominant poloidal number is selected to be $m = nq$, $q = 2$. Far from the no-wall limit the skin effect in the wall and plasma inertia affect the results, so that at $s < d_w$ the growth rates are described by the fast RWM scaling [4,5] and can be estimated based on the ideal MHD potential and kinetic energy forms. The value of normalized beta for transition to fast RWM regime decreases with increased mode number and is well described by the slow RWM scaling $C_\beta > 1 / (1 + \tau_{sk} / \tau_D)$.

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Acknowledgement The work was supported by the grant from the Russian Science Foundation №16-11-10278 and by the Competitiveness Programme of NRNU MEPhI.