# Weakly nonlinear dust acoustic shock waves in a charge varying electronegative magnetized dusty plasmas with nonisothermal trapped electrons: Application to the Halley Comet.

M. Bacha, and M. Tribeche

*USTHB, Faculty of Physics, PB 32, El Alia, 16111 Bab-Azzouar, Algiers*

**Introduction**

In a dusty plasma, the dust charge may fluctuate and then becomes a new dynamical variable, and the electron behavior can be strongly modified by the nonlinear potential of the localized dust acoustic (DA) structure by generating a population of non- Maxwellian trapped particles. A large number of contributions have focused on the effects of particle trapping on different types of linear and nonlinear collective processes in varying charge dusty plasmas [1]-[9]. It has been demonstrated that the presence of such non-isothermal particles can significantly modify the wave propagation characteristics in collisionless varying charge dusty plasmas. On another side, electronegative dusty plasmas which are highly chemically reactive have attracted a great deal of interest [10]-[14] because of their wide technological applications (semiconductor materials processing [15], synthesis of nanomaterials [16], . . . etc.) and vital role in astrophysical plasmas (D- region of the ionosphere, the mesosphere, the solar photosphere, the cometary comae . . . etc). In most of these astrophysical and laboratory plasmas, the electron number density decreases because of the electron attachment on the dust grain surface. Hence, in this situation the negative ions may play a crucial role. Therefore, it may be of practical interest to examine the combined effects of an oblique magnetic field and electron trapping on the DAW in varying charge electronegative dusty plasmas. To this end, we go parallel to what has been done in the work of Ghosh et al. [17] and we use the correct trapped electrons charging current based on the orbit-motion limited approach [9] with application to the Halley Comet.

**Basic equations**

Let us consider a collisionless, electronegative magnetized dusty plasma having trapped electrons (e), Maxwellian positive (+) and negative ions (-), and mobile charge fluctuating negatively charged dust grains (d). Thus, at equilibrium the charge neutrality condition reads as $n_{e0} + Z_{d0} + n_{-0} = n_{+0}$, where the subscript “0” and $Z_{d0}$ stand for the unperturbed quantities and dust grain charge number, respectively. The constant external magnetic field $\mathbf{B}$ lies in the $(x-z)$ plane making an angle $\theta$ with the x-axis, $\mathbf{B} = B_0 \cos\theta\, \mathbf{e_x} + B_0 \sin\theta\, \mathbf{e_z}$. The wave propagation vector $\mathbf{k}$ lies along the x-axis. The positive and negative ions and electrons densities are given by

$$n_+ = n_{+0}\exp(-e\phi/k_BT_+),\; n_- = n_{-0}\exp(e\phi/k_BT_-)\;, \quad (1)$$

$$n_e(\phi) = n_{e0}\left[\exp\left(\frac{e\phi}{T_{ef}}\right)erfc\left(\sqrt{\frac{e\phi}{T_{ef}}}\right) + \frac{1}{\sqrt{\beta}}\exp\left(\beta\frac{e\phi}{T_{ef}}\right)erfc\left(\sqrt{\beta\frac{e\phi}{T_{ef}}}\right)\right] \text{ for } \beta > 0,$$
$$n_e(\phi) = n_{e0}\left[\exp\left(\frac{e\phi}{T_{ef}}\right)erfc\left(\sqrt{\frac{e\phi}{T_{ef}}}\right) + \frac{2}{\sqrt{\pi}}\frac{1}{\sqrt{-\beta}}W\left(\sqrt{-\beta\frac{e\phi}{T_{ef}}}\right)\right] \text{ for } \beta < 0. \quad (2)$$

where $erfc(x) = 1 - erf(x) = 1 - \frac{2}{\sqrt{\pi}}\int_0^x \exp(-t^2)dt$ and $W(x) = \exp(-x^2)\int_0^x \exp(t^2)dt$ is the Dawson integral.

Where $\phi$ is the electrostatic potential, $n_j$ represents the number density of particles of species $j$. $T_j$ is the temperatures, and $m_j$ is the masses. $\beta$ is a parameter determining the number of trapped electrons and its magnitude is defined as the ratio of the free hot electron temperature $T_{ef}$ to the hot trapped electron temperature $T_{et}$, i.e. $|\beta| = T_{ef}/T_{et}$ . The dynamics of low frequency dust- acoustic oscillations are studied using the reductive perturbation technique of Shamel [18] and construct a weakly nonlinear theory [19] for the DAWs by introducing the stretched coordinates $\xi = \varepsilon^{1/4}(X - U_{ph}T), \tau = \varepsilon^{3/4}T,$ where $\varepsilon$ is a small parameter characterizing the strength of the nonlinearity and $U_{ph}$ is the normalized phase velocity of the line DAW. The dependent variables are expanded in power series of $\varepsilon$. The fellowing modified Korteweg-de vries Burger (mKdVB) equation is obtained

$$\frac{\partial N_d^{(1)}}{\partial\tau} + A\sqrt{N_d^{(1)}}\frac{\partial N_d^{(1)}}{\partial\xi} + B\frac{\partial^3 N_d^{(1)}}{\partial\xi^3} = C\frac{\partial^2 N_d^{(1)}}{\partial\xi^2}. \quad (3)$$

The coefficients of nonlinearity A, dispersion B, and the Burger term C are given by

$$A = \frac{1}{(1-\alpha_d\beta_{ch})^2}\frac{\delta_+(1-\beta)}{\sqrt{\pi}U_{ph}\gamma\sec^2(\theta)}, B = \frac{U^3{}_{ph}\tan^2(\theta)}{2\omega^2{}_{cd}} + \frac{\cos^2(\theta)}{2U_{ph}}\left(\frac{1}{1+\alpha_d\beta_{ch}}\right)^2,$$
$$C = \frac{\alpha_d\omega_{ch}(1+\sigma_+ + \gamma_2)/(c_1\sigma_+)}{2\sec^2(\theta)(1+\alpha_d\beta_{ch})^2}, \text{with } c_1 = Z\left[\frac{1}{Z+\sigma_+} + \frac{A_-}{\sigma_-} + A_+(\beta - \beta^2 Z) + A_+\left(1 - \frac{1}{\beta}\right)\right],$$
$$U_{ph} = \cos(\theta)\left(\frac{\gamma_d\sigma_d}{\alpha_d} + \frac{1}{1+\alpha_d\beta_{ch}}\right)^{1/2}.$$

Now transforming to the wave frame, $\eta = \xi + V_f\tau$, the mKdVB with $\psi = N_d{}^{(1)}$ reduces to

$$\frac{d^2\psi}{d\eta^2} = \left(\frac{V_f}{B}\right)\psi - \left(\frac{2A}{3B}\right)\psi^{3/2} - \left(\frac{C}{B}\right)\frac{d\psi}{d\eta}. \quad (4)$$

**Numerical results**

Before proceeding further, we first give the Halley Comet ($\sim 10^4$ km from the nucleus) approximated physical parameters we used in our numerical computation. These parameters are [20]-[24], $n_{+0}$~$2\times10^8$ $m^{-3}$,$n_{d0}$~1 $m^{-3}$,$T_e$~100 eV, $m_+=m_-$~$1.6756\times10^{-27}$ kg, $B_0$~$7.5\times10^{-3}$ T, $r_d$~5 μm and $\gamma_d$~5/3. Our results show that the only admissible positive trapping parameter β-values which satisfy the condition $\delta_-<1[=n_{-0}/n_{+0}=1-(n_{e0}/n_{+0}+Z_{d0}n_{d0}/n_{+0})<1]$ are β>0.75 for $\sigma_-=1$ and β>0.73 for $\sigma_-=0.01$. Similarly only admissible negative trapping parameter β-values which satisfy the condition $\delta_-<1$ are β<-0.884 for $\sigma_-=1$ and 0.01. The compressive shock waves may develop and set in our plasma model, i. e., the dust charge variation induced nonlinear wave damping leads to the development of collisionless DA shock waves (Fig.**1**). we note that the effect of separation of charges, which is manifested by the appearance of some oscillations in the shock wave profile. This effect becomes less pronounced as the value of the trapping parameter β decreases. It is turns out that the anomalous dissipation effects may prevail over that dispersion as the electrons evolve far away from their Maxwellian equilibrium. The collisionless shock wave is compressive in nature and therefore may lead to dust density enhancement. This latter, which is important in the astrophysical context, is more pronounced as the electrons evolve towards their thermodynamic equilibrium. As |β| increases, the strength of the shock wave increases, and the number of oscillations appearing in the shock weve profile decreases. The effects of the obliqueness (Fig.**2**) and magnitude (Fig.**3**) of the magnetic field on the DA shock wave profile is investigated. As may be expected, the obliqueness and magnitude of the magnetic field are found to modify the dispersive properties of the DA shock wave. It can be seen that an increase of θ or decrease of $B_0$ (via $\omega_{cd}$) render the shock structure more dispersive. We obtain qualitatively the same results for a negative values of β.

**Conclusion**

To conclude, we have investigated the combined effects of an oblique magnetic field and electron trapping on weak DAWs in a charge varying electronegative dusty plasmas with application to the Halley Comet. A weakly nonlinear analysis has been carried out to derive a modified Korteweg- de Vries- Burger like equation. Making use of the equilibrium current balance equation, we have first constrained the physically admissible values of the electron trapping parameter. The influence of the electron trapping on the shock front height has been investigated in the case of oblique propagation. Moreover, increases of obliqueness of the magnetic field or a decrease of its magnitude render the shock structure more dispersive. We hope that results will aid to explain and interpret the nonlinear oscillations that may occur in the Halley Comet plasma.

**References**

[1] S. Ghosh, S. Sarkar, M. Khan, and M. R. Gupta. Phys. Plasmas **9**, 1150 (2002).
[2] S. K. El-Labany, W. M. Moslem, and A. E. Mowafy. Phys. Plasmas. **10**, 4217 (2003).

[3] S. K. El-Labanyand al. phys. Plasmas **11**, 926 (2004).
[4] E. F. El-Shamy, chaos, Solitons and Fractals **25**, 665 (2005).
[5] K. R. Chowdhury, A. P. Mishra, and A. R. chowdhury, chaos, Solitons and fractals **29**, 125 (2006).
[6] M. Tribeche, L. Ait Gouagam, and T. H. Zerguini, phys. Scr. **75**, 354 (2007).
[7] S. Younsi and M. Tribeche, Phys. Letr. A. **372**, 5181 (2008).
[8] M. Tribeche, S. Younsi, and T. H. Zerguini, Chaos, Solitons and Fractals 41, 1277 (2009).
[9] A. Berbri and M. Tribeche, J. Phys. Plasmas **10**,1017 (2009).
[10] R. L. Merlino and J. J. Loomis, Phys. Fluids B**2**, 2865 (1990).
[11] S. Ghosh, phys. Plasmas **12**, 094504 (2005).
[12] S. Ghosh, R. Bharuthram, M. Khan, and M. R. Gupta. Phys. Plasmas **13**, 112305 (2006).
[13] M. Rosenberg and R. L. Merlino, Planet. Space Sci. **55**, 1464 (2007).
[14] M. Tribeche and M. Benzekka, Phys. Plasmas **16**, 083702 (2009).
[15] H. Kokura and al., Jpn. Appl. Phys., Part. 1 **38**, 5256 (1999).
[16] L. Boufendi and A. Bouchoule, plasma Sources Sci. Technol. **11**, A211 (2002).
[17] S. Ghosh, Z. Ehsan, and G. Murtaza, Phys. Plasmas **15**, 023701 (2008).
[18] H. Schamel, J. Plasma Phys. **9**, 377 (1973).
[19] H. Washimi and T. Taniuti, Phys. Rev. Lett. **17**, 996(1966).
[20] A. Wekhof, Moon Planets **24**, 157 (1981).
[21] I. M. Podgorny, E. M. Dubinin, and P. L. Israelevich. Moon Planets **27**, 135 (1982).
[22] G. H. Schwehm and B. Kneissel, Proceedings of the Joint NASA/ESA Working Group Meeting, ESA SP-174 (1981), p.77.
[23] N. Divine, H. Fetching, T. I. Gombosi et al., Space Sci. Rev. **43**, 1 (1986).
[24] K. A. Anderson, C. W. Carlson, D. W. Curtis et al., Proceedings of the 20th ESLAB *Symposium on the Exploration of Halley's Comet*, Heidelberg, 27-31Octobre1986, EA SP-250, p.259.

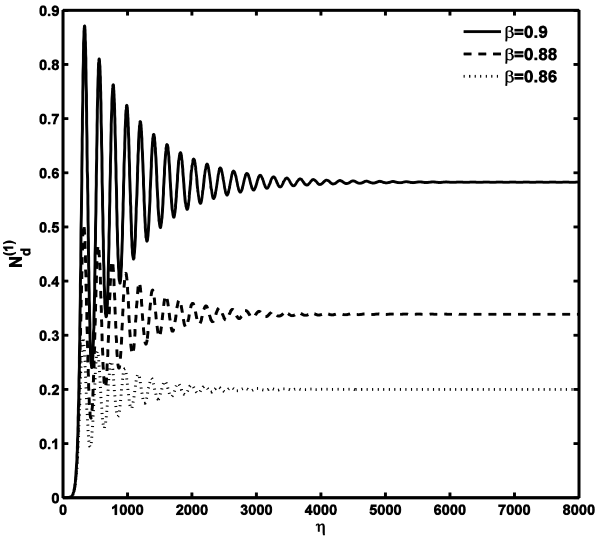


**Figure 1:** Spatial profile of the oscillatory DA shock wave for different values of β=0.86(dotted line), 0.88(dashed line) and 0.9(solid line), with Z=1, $\sigma_+$=1, $\sigma_-$=1,$r_d$=5μm, and θ=2°.

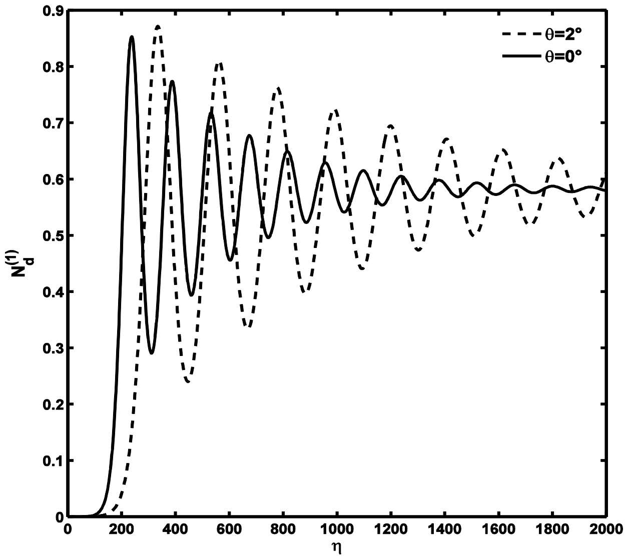


**Figure 2:** Spatial profile of the oscillatory DA shock wave for twi different values of θ=0° (solide line) and 2° (dashed line), with with Z=1, $\sigma_+$=1, $\sigma_-$=1,$r_d$=5μm, and β=0.9.

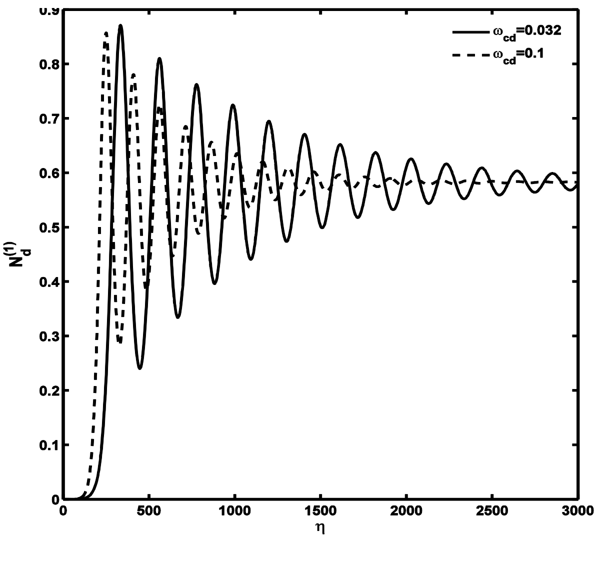


**Figure 3** : Spatial profile of the oscillatory DA shock wave for two different values of $\omega_{cd}$(~$B_0$)=0.03 (solide line) and 0.1 (dashed line), with with Z=1, $\sigma_+$=1, $\sigma_-$=1,$r_d$=5μm, β=0.9, and θ=2°.