Verifying PB3D: a new code for 3D ideal linear peeling-ballooning stability

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abstract

Magnetic nuclear fusion devices are a promising candidate for the confinement of thermonuclear plasmas but various instabilities set important limits on their operation. Peeling-ballooning perturbations, which can be described appropriately using high-n linear ideal MHD stability theory, are two of them, where high-n indicates that the perturbations are localized along the magnetic field lines [1].

*A new numerical code, called PB3D (**P**eeling **B**allooning in **3D**) was written to investigate the stability of these instabilities in a fast and reliable way, solving the generalized eigensystem presented first in [8]. The important new aspect of this theory is that it describes stability of full 3-D equilibrium configurations that are allowed to perturb the plasma edge, in contrast with previous treatments such as used in the ELITE [9] or MISHKA code [5] that both treat the stability of axisymmetric equilibria.*

3D effects are important for numerous reasons: In tokamaks axisymmetry is often broken, either deliberately, such as when RMP techniques are used to suppress periodic plasma relaxations called ELMs, or due to imperfections in the axisymmetric design, such as the toroidal ripple introduced by discrete toroidal field coils. Stellarators devices, on the other hand, are inherently 3D and cannot be approximated using axisymmetric theory.

In this work, the verification of PB3D with stability results for axisymmetric equilibria is presented, indicating that these are accurately reproduced, and non-intuitive first 3-D results are given.

Introduction

This work employs the ideal linear MHD stability model, to investigate the behavior of so-called *high-n* modes, which are normal modes of the form $\boldsymbol{\xi}(\mathbf{r})\exp i\omega t$ with complex frequency ω describing plasma displacements that are spatially strongly localized around the magnetic field lines, while varying slowly along them, so they are referred to as *fluted* modes. As they are very localized, fluted modes can be excited relatively easy, but at the same time they have the power to directly couple the hot plasma core to the cold vessel walls and these high power fluxes can be detrimental to both the confinement and the devices [3]. For example, axisymmetric[1] simulations of so-called *peeling-ballooning* modes have provided a credible model for most types of Edge Localized Modes (ELMs), which are periodic outbursts that occur in the High-confinement mode (H-mode) observed in most tokamaks [9].

In [8] ideal linear high-*n* theory was derived for full 3-D configurations, without making any axisymmetric assumption on the equilibrium, and valid for modes that are allowed to perturb the plasma edge, including the effect of a surrounding vacuum. Correct treatment of edge effects is important as the stability of peeling modes, which can be thought of as a high-*n* edge version of the general interchange mode, depends on the presence of a low rational surface just outside of the edge [4]. Full 3-D treatment is also of importance, as many configurations that can be approximated well by axisymmetric (i.e. 2-D) treatment, contain important modifications that introduce 3-D effects, such as ferromagnetic Tritium Breeder Modules (TBMs) in ITER, discrete toroidal field coils that introduce a toroidal magnetic ripple,

[1] In this text, this means stability of *axisymmetric equilibria*. The perturbations themselves are 3-D.

and Perturbation Coils that deliberately break axisymmetry in order to control ELMs. Apart from this, many configurations are inherenlty 3-D, such as stellarators. As Eikonal theory, originally often used in the study of high-n stability, is not applicable, the theory here was developed using Fourier Modes.

An important result of the theory developed in [8] is the fact that, due to their high-n nature, perturbation modes pertaining to different surfaces are decoupled. Introducing the field line label $\alpha = \zeta - q\theta$, the decoupling in the α direction is the 3-D extension of the decoupling of modes in the toroidal direction ζ described in [9], and, therefore, the 3-D problem is inherently of the same complexity as the 2-D problem. This motivates the usage of this 3-D theory in a new numerical code **PB3D** (**P**eeling-**B**allooning in **3**-**D**) of which the first verification results are presented here.

Numerical Problem

A useful result of the theory is the description of the stability of the modes by a coupled set of 2^{nd} order ordinary differential equations (ODE) in the Fourier components m of the perturbation normal to the flux surfaces $X = \boldsymbol{\xi} \cdot \nabla\psi$ where ψ is a flux coordinate

$$\sum_m \left\{ \overline{L}^0_{k,m} X_m - \left(\overline{L}^{1*}_{m,k} X_m \right)' + \overline{L}^1_{k,m} X'_m - \left(\overline{L}^2_{k,m} X'_m \right)' \right\} = 0 \ , \tag{1}$$

primes indicating derivatives in ψ. The components of $\overline{\mathbf{L}}$ are defined as $\overline{\mathbf{L}} = \overline{\mathbf{P}} - \lambda \overline{\mathbf{K}}$, so that this equation corresponds to the minimization of the plasma potential energy (resulting in $\overline{\mathbf{P}}$), with a normalized plasma kinetic energy (resulting in $\overline{\mathbf{K}}$), and $\lambda = \omega^2$ constitutes an eigenvalue of the problem, so that its sign indicates the (in)stability of the perturbation modes.

Subsequently, the system of equations is closed by the boundary conditions, the first being the assumption that the modes should vanish deep in the plasma, and the second one resulting from a minimization of the energy due to the perturbed plasma surface, which has the form

$$\sum_m \left\{ \left(\delta^{\text{vac}}_{k,m} + \overline{L}^{1*}_{m,k} \right) X_m + \overline{L}^2_{k,m} X'_m \right\} = 0 \ , \tag{2}$$

where $\boldsymbol{\delta}^{\text{vac}}$ is a term due to the perturbation of the vacuum potential energy, which is always stabilizing.

Furthermore, the bar notation of the quantities $\overline{\mathbf{L}}$ indicates its elements are defined as

$$\overline{L}_{k,m} = \int_\alpha \mathscr{J} e^{\mathrm{i}(k-m)\theta} L_{k,m} \, \mathrm{d}\theta \ , \tag{3}$$

related to the definition of the $(k-m)^{\text{th}}$ Fourier mode, but with the integration in the magnetic coordinate θ along a field line with label α and the quantity $L_{k,m}$ depending on k and m. $\mathscr{J}$ is the Jacobian. The fact that this integral is 1-D in θ only, is an expression of the fact that only modes pertaining to the same field line α couple.

Finally, in PB3D, equations 1 and 2 are discretized on normal positions ψ_i using central finite differences in such a way that the Hermiticity of ideal MHD is preserved, as described in [7]. The final result is then the generalized eigenvalue problem

$$\mathbf{AX} = \lambda \mathbf{BX} \ , \tag{4}$$

which is solved in PB3D for the eigenvalue λ and eigenvalue $\mathbf{X}$ that bundles the Fourier modes X_m at the different flux surfaces ψ_i.

Verification

The PB3D results are given for the stability of two test cases. The first test case is the axisymmetric CBM18 model, designed to be ballooning unstable through a steep pressure gradient [2], as shown in figure 1. Furthermore, CBM18A is modified to a 3-D case by varying the radius by 10% in the toroidal direction, between $\zeta = 0$ and $\zeta = \pi$, to create an artifical test case for verification.

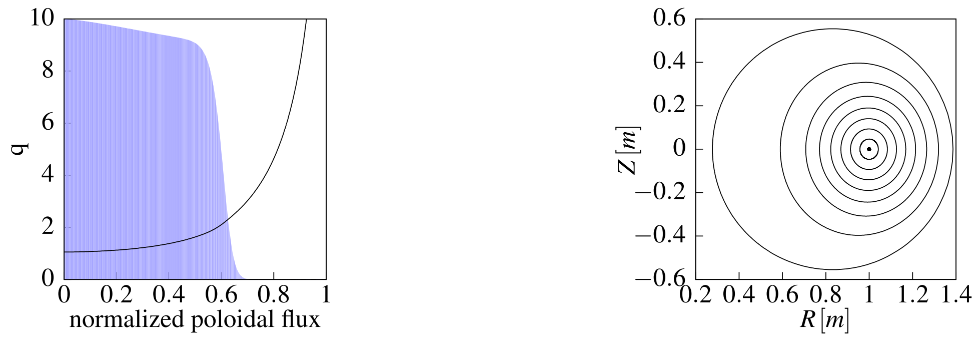

Figure 1: (left) Safety factor and pressure profile (shaded, normalized to value at magnetic axis) and (right) cross-section for CBM18 [7].

For the axisymmetric test case CBM18, figure 2 shows a comparison of the most unstable eigenvalue, for various values of the primary mode number n, between PB3D, MISHKA [5] and COBRA [6]. MISHKA is a general-n axisymmetric code that has been used extensively to benchmark other codes, such as ELITE, and COBRA is an infinite-n 3-D code that makes use of an Eikonal formulation and yields the limit case for $n \to \infty$.

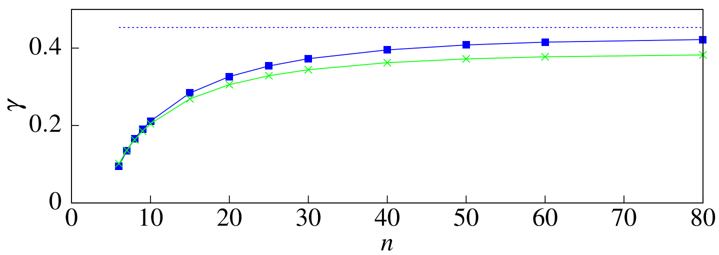

Figure 2: Comparison between results from PB3D (squares), MISHKA (crosses) and COBRA (dashed line) [7].

It can be seen that the PB3D results have the same dependence on n as the MISHKA results, but generally a bit more unstable. This could be due to the fact that the problem solved in PB3D is fundamentally different, through the high-n assumptions that are taylored to unstable modes such as peeling-ballooning modes. As a result PB3D may tend to sligthly overestimate the instability for ballooning cases. This is a characteristic also seen for ELITE, for example in [9, fig. 4].

Furthermore, figure 3 shows the familiar mode structure, each Fourier mode resonating around its resonant surface $q = \frac{m}{n}$ (not shown).

For the 3-D modified version of CBM18, where the radius of the poloidal cross section is decreased by 10% for $\zeta = \pi$, compared to $\zeta = 0$, the results are given in figure 4, and this time comparison is only

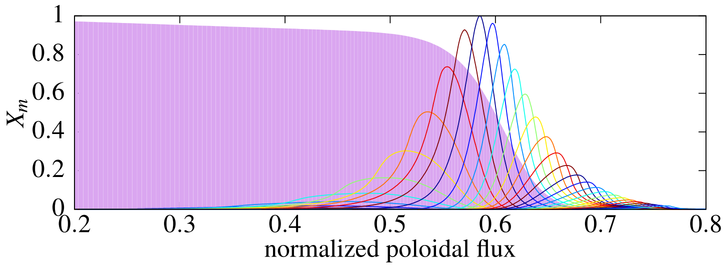

Figure 3: Modes X_m at midplane and pressure p (shaded, normalized to value at magnetic axis [7].

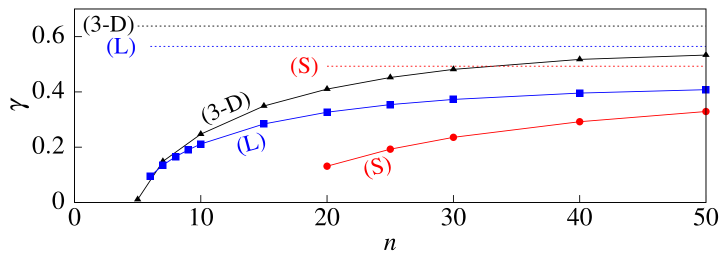

Figure 4: Comparison between results from PB3D (squares) and COBRA (dashed line) for original axisymmetric CBM18 (blue), small version (red) and 3-D modified version (black) [7].

with the 3-D code COBRA. It can be seen that the result is *more* unstable than for the axisymmetric case, even though a small version of the axisymmetric case is *less* unstable! This is an example of a non-intuitive result for the study of 3D modification of stability and this is what PB3D was written for. Detailed analysis of these effects in more realistic geometries is planned as future work.

Conclusions

The new numerical code PB3D is capable of fast analysis of the ideal ideal high-n stability of 3-D edge configurations. This makes it an appropriate new tool for the study of the 3-D effects on the important peeling-ballooning modes, such as observed in toroidal devices. The aim is to employ PB3D to study the influence of the breaking of axisymmetry, such as happens when perturbation coils are used to control ELMs, as well as the stability of general 3-D configurations. To this end, in this work the first verification results of axisymmetric as well 3-D configurations configurations is provided.

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