Analytical model of the multiple-mode sideways forces in tokamaks

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1. Introduction. The task is related to evaluation of the sideways forces on the vacuum vessel wall in tokamaks [1–7]. Such forces up to 4 MN have been observed in experiments on Joint European Torus (JET) tokamak [8]. They led to significant displacement of the vessel in JET [1] and are expected to be an order of magnitude larger in ITER [8].

In simulations with M3D code [3, 5], the maximal sideways force was found at $\gamma\tau_w \approx 1$, where γ is the kink growth rate and τ_w is the resistive wall time. An analytical model was proposed in [3], but, when properly corrected [6], it yields a different result. Recently, the presence of such maximum at some γ has been theoretically predicted in [7], but in general terms without precise indication of its position on the $\gamma\tau_w$ scale.

Here we develop a model to find the sideways force on the wall as a function of $\gamma\tau_w$ for the helically deformed plasma separated from the conducting wall by a vacuum gap. Mostly the derivations are performed in cylindrical geometry within the thin-wall model that is widely used in the resistive wall mode (RWM) studies, see details and references in [9]. The approach is based on the results of [6], but now we additionally incorporate more harmonics of the kink perturbations than it was done in [2–6]. Also, in contrast to previous studies [1–6], here the condition [7] is explicitly used that the sideways force on plasma is much smaller than that on the wall. The derived expressions explicitly reveal the dependence of the wall force on $\gamma\tau_w$, which is in agreement with numerical results [3, 5] and general analytical predictions [7].

2. Formulation of the problem. The sideways force on the wall is defined by

$$F_X \equiv \int_{wall} (\mathbf{j}\times\mathbf{B})\cdot\mathbf{e}_X \, \mathrm{dV} = \oint \left\{ (\mathbf{B}\cdot\mathbf{e}_X)\mathbf{B} - \frac{\mathbf{B}^2}{2}\mathbf{e}_X \right\} \cdot \mathrm{d}\mathbf{S} \Big|_{in}^{out} . \qquad (1)$$

Here $\mathbf{B}$ is the magnetic field (subject to $\nabla\cdot\mathbf{B} = 0$), $\mathbf{j} = \nabla\times\mathbf{B}$ is the current density, $\mathbf{e}_X$ is the unit vector along a fixed horizontal direction. The first integral is over the wall volume, and the second one is over its outer "*out*" and inner "*in*" sides of the toroidal wall.

With $\mathbf{B}=\mathbf{B}_0+\mathbf{b}$, where $\mathbf{B}_0$ is the axisymmetric equilibrium field ($\partial\mathbf{B}_0/\partial t=0$) and $\mathbf{b}$ is the time varying perturbation $\propto \exp(\gamma t)$, Eq. (1) and natural $F_X(\mathbf{B}_0)=0$ give us

$$F_X = F_X^{out} - F_X^{in}, \tag{2}$$

$$F_X^{\alpha} \equiv \oint_{S_\alpha} \left\{ (\mathbf{b}\cdot\mathbf{e}_X)\mathbf{B}_0 + (\mathbf{B}_0\cdot\mathbf{e}_X)\mathbf{b} + (\mathbf{b}\cdot\mathbf{e}_X)\mathbf{b} - \left(\mathbf{B}_0\cdot\mathbf{b} + \frac{\mathbf{b}^2}{2}\right)\mathbf{e}_X \right\}\cdot \mathrm{d}\mathbf{S} \tag{3}$$

with $\alpha = out$ or $\alpha = in$ denoting the proper wall surface.

3. Calculations of the integrals. The tokamak toroidicity is accounted for by the use of

$$\mathbf{e}_X = \mathbf{e}_R \cos\zeta - \mathbf{e}_\zeta \sin\zeta, \qquad \mathbf{e}_R = \mathbf{e}_r\cos\theta - \mathbf{e}_\theta \sin\theta \tag{4}$$

with ζ counted from the $\mathbf{e}_X$ direction. In the large-aspect ratio approximation $\mathrm{d}\mathbf{S} = \mathbf{e}_r r_w R_0 d\theta d\zeta$, $\mathbf{B}_0\cdot\mathbf{e}_r = 0$ and then with (4) the linear term in (3) is

$$F_X^{\alpha} = -r_w R_0 \oint_{S_\alpha} \left(B_{0\theta} b_r \sin\theta\cos\zeta + B_{0\zeta} b_r \sin\zeta + (\mathbf{B}_0\cdot\mathbf{b})\cos\theta\cos\zeta \right) d\theta d\zeta . \tag{5}$$

The coordinates $(r,\theta,z=R_0\zeta)$ are quasi-cylindrical with $2\pi R_0$ the length of the equivalent torus so that θ and ζ correspond to the poloidal and toroidal angles. The plasma with radius r_{pl} and the wall with radius r_w, thickness d_w and conductivity σ are coaxial when $\mathbf{b}=0$.

The first term in (5) does not vanish if $b_r \propto \sin\theta\cos\zeta$. With $\mathbf{b}=\nabla\varphi$ in vacuum this requires

$$\varphi^{lin} \equiv \varphi_{1,1}\sin(\theta-\zeta) + \varphi_{1,-1}\sin(\theta+\zeta). \tag{6}$$

Such and only such choice also makes non-zero the contribution of the term $\propto \mathbf{B}_0\cdot\mathbf{b}$ and nullifies the remaining term $\propto \sin\zeta$. Then Eq. (5) yields

$$F_X^{\alpha} = -\pi^2 R_0 B_{0\theta}(r_w)\left\{ r_w(\varphi_{1,1}+\varphi_{1,-1})' + (1-q)\varphi_{1,1} + (1+q)\varphi_{1,-1} \right\}\Big|_{\alpha}, \tag{7}$$

where we disregard a small difference in $B_{0\theta}$ at radial positions r_w and r_w+d_w, the prime means the derivative with respect to r, $q \equiv rB_{0\zeta}/(R_0 B_{0\theta})$ so that $q \propto r^2$ outside the plasma.

For the perturbations prescribed by (6), at $nr \ll mR_0$ equation $\nabla^2\varphi = 0$ gives us

$$\varphi_{1,n} = -r_w\left[x^{-1} + 0.5\Gamma_1(x^{-1}+x)\right] b_r^{1,n}(r_w) \tag{8}$$

with $x \equiv r/r_w$, $b_r^{1,n}$ is taken at the wall, $\Gamma_1 = 0$ behind the wall and

$$\Gamma_1 = \gamma\tau_w \quad \text{with} \quad \tau_w \equiv \sigma r_w d_w \tag{9}$$

in the plasma-wall gap [9]. For the magnetically thin wall, the amplitudes $b_r^{1,n}$ must be the same for the both regions. Here we assume real γ (locked modes) as in simulations [3, 5].

By using Eq. (8) we obtain for the expression in the brackets in (7):

$$\{...\} = r_w(b_r^{1,1} - b_r^{1,-1})[q + \Gamma_1(1+q)] - 2r_w\Gamma_1 b_r^{1,1}. \tag{10}$$

4. The sideways force on the plasma. The asymmetric force on the plasma is

$$\mathbf{F}_p \cdot \mathbf{e}_X \equiv \int_{plasma} (\mathbf{j}\times\mathbf{B})\cdot\mathbf{e}_X \, \mathrm{dV} = \oint_{S_{in}} \left\{ (\mathbf{B}\cdot\mathbf{e}_X)\mathbf{B} - \frac{\mathbf{B}^2}{2}\mathbf{e}_X \right\} \cdot \mathrm{d}\mathbf{S} = F_X^{in}. \tag{11}$$

Here we extend the integration volume because the gap with $\mathbf{j}=0$ does not affect the result.

Estimates [7] show that the disruption-induced $\mathbf{F}_p$ is negligible compared to the wall force. With

$$F_X^{in} = 0 \tag{12}$$

from (7) and (10) we obtain at the wall ($r = r_w$)

$$b_r^{1,1} - b_r^{1,-1} = \frac{2\Gamma_1}{q_w + (1+q_w)\Gamma_1} b_r^{1,1}. \tag{13}$$

This requires, in particular, that $b_r^{1,-1} \neq 0$. Incorporation of $b_r^{1,-1}$ coupled with $b_r^{1,1}$ is the main difference of our analytical model from those in [2–4, 6] initially assuming $b_r^{1,-1} = 0$. In the latter case, Eq. (13) is satisfied by either $\Gamma_1 = -q_w/(q_w - 1) < 0$ or $b_r^{1,1} = 0$ and, consequently, $F_X = 0$. Then we have to consider quadratic effects in Eq. (3) and additional harmonics of the perturbation, but it can hardly produce a large force on the wall.

5. The sideways force on the wall. Behind the wall, $\Gamma_1 = 0$. Then Eq. (7) with (10) gives

$$F_X^{out} = -\pi^2 R_0 q_w r_w B_{0\theta}(b_r^{1,1} - b_r^{1,-1}) \tag{14}$$

with $B_{0\theta}$, $b_r^{1,1}$ and $b_r^{1,-1}$ taken at the wall. Under the condition (13) this turns into

$$F_X^{out} = -\pi^2 R_0 q_w r_w B_{0\theta}(r_w) \frac{2\Gamma_1}{q_w + (1+q_w)\Gamma_1} b_r^{1,1}(r_w). \tag{15}$$

According to (8), at any intermediate point in the plasma-wall gap, $r_{pl} \le r \le r_w$, we have

$$\frac{b_r^{1,n}(r)}{b_r^{1,n}(r_w)} = \left[1 + 0.5\Gamma_1(1-x^2)\right]\cdot x^{-2}. \tag{16}$$

Therefore, Eq. (15) with (2) and (12) at $\Gamma_1 = \gamma\tau_w$ is equivalent to

$$F_X = -4\pi^2 R_0 r_{pl} \kappa^2 b_r^{1,1}(r_{pl}) B_J f(\gamma\tau_w) \tag{17}$$

with $\kappa \equiv r_{pl}/r_w$, b_r given at the plasma surface, $B_J \equiv B_{0\theta}(r_{pl})$ and

$$f(\gamma\tau_w) \equiv \frac{q_{pl}}{q_{pl} + (\kappa^2 + q_{pl})\gamma\tau_w} \frac{\gamma\tau_w}{2 + \gamma\tau_w(1-\kappa^2)}. \tag{18}$$

Equations (17)–(18) describe the sideways force on the wall as non-monotonically varying with γ from $F_X = 0$ at $\gamma = 0$ to $F_X \to 0$ at $\gamma\tau_w \to \infty$ with a maximum in-between at

$$\gamma\tau_w = \left(\frac{2q_{pl}}{(1-\kappa^2)(\kappa^2 + q_{pl})} \right)^{1/2} . \qquad (19)$$

6. Discussion. The position of this maximum weakly depends on q_{pl}. At $r_w / r_{pl} = 1.1$ it falls into the range $0 < \gamma\tau_w < 4$ at any q_{pl}. Such low values of $\gamma\tau_w$ are in agreement with numerical results in [3, 5]. The function (18) is plotted in Fig. 1 using $r_w / r_{pl} = 1.3$ and $q_{pl} = 3$. Then Eq. (19) gives $\gamma\tau_w \approx 2$.

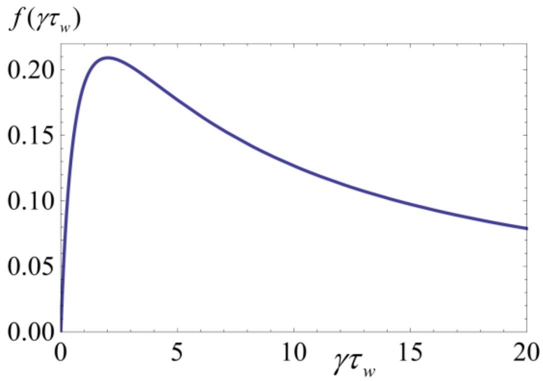

Fig. 1. Dependence of F_X on $\gamma\tau_w$ described by function $f(\gamma\tau_w)$ given by (18). The calculation parameters are $r_w / r_{pl} = 1.3$ and $q_{pl} = 3$.

In contrast to the single-mode analytical modelings [4, 6], the linear analysis presented here proves that, from the point of view of the sideways forces, the most dangerous must be slow RWMs. However, even the largest force must be tolerable. Because of the mutual cancellation of the contributions from two coupled modes, the resulting force (14) is about one order of magnitude smaller than that predicted by the single-mode models [2–4, 6].

Here we have proved that the sideways force associated with a kink mode must be maximal at $\gamma\tau_w = O(1)$. The demonstrated force dependence on $\gamma\tau_w$, not accounted for in [1, 2, 4], is similar to the earlier findings in numerical calculations with M3D code [3, 5]. However, the force amplitude in our model cannot reach the level comparable to that in [3]. Maybe, this can be attributed to the fact that, in our model, the wall is separated from the plasma by a vacuum gap (no halo currents). We conclude that a search of a large sideways force should be done either at the next stages of disruptions or with realistic 3D wall models.

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