Cherenkov radiation from short laser pulses in a magnetized plasma with variable density

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Cherenkov radiation is electromagnetic radiation emitted when a charged particle passes through a dielectric medium at a speed greater than the phase velocity of light in that medium. Also while propagation of the laser pulse with high velocity in magnetized plasma Cherenkov radiation is observable. Using the external magnetic field perpendicularly to the laser pulse propagation the wake-field becomes electromagnetic and propagated with nonzero group velocity. The magnetization of the plasma gives rise to the appearance of a frequency interval between the plasma ω_p and upper hybrid ω_h frequencies where the phase velocity of the extraordinary XO mode is less than the speed of light. Thus, the laser pulse may propagate in magnetized plasma at a speed greater than the phase velocity of light because of the plasma medium is very dispersive. Then the XO mode from the frequency interval $\omega_p < \omega < \omega_h$ can be emitted [1].

In this study the generation of Cherenkov radiation in plasma is simulated. In this research, using the motion equation, continuity equation and Maxwell equations, the equations governing on the generation of Cherenkov radiation are obtained. Then the equations are solved by Laplace transform. In this study, we consider the density of plasma can change as linear and nonlinear. Results show that the strength of external magnetic field and plasma density distribution leads to change in amplitude of generated Cherenkov radiation and the angle of conical of the radiation.

The starting point is the Maxwell's equations governing on propagation of the laser pulse,

$$\frac{\partial E_z}{\partial x} + \frac{1}{v_0}\frac{\partial E_x}{\partial \xi} = \frac{1}{c}\frac{\partial B_y}{\partial \xi} \qquad (1)$$

$$\frac{1}{v_0}\frac{\partial B_y}{\partial \xi} = \frac{1}{c}\frac{\partial E_x}{\partial \xi} - \frac{4\pi Ne}{c} v_x \qquad (2)$$

$$\frac{\partial B_y}{\partial x} = \frac{1}{c}\frac{\partial E_z}{\partial \xi} - \frac{4\pi Ne}{c} v_z \qquad (3)$$

Here we assume that the laser pulse propagated in z direction with electric field E and magnetic field B. Which v_0 is velocity of laser pulse through plasma, N is plasma density and $\xi = t - \frac{z}{v_0}$. Then using motion equation for x and z components of plasma electrons velocity we have,

$$m\frac{\partial v_x}{\partial \xi} = -eE_x - e\frac{\partial \phi}{\partial x} + \frac{eB_0}{c}v_z \quad (4) \qquad m\frac{\partial v_z}{\partial \xi} = -eE_z + \frac{e}{v_0}\frac{\partial \phi}{\partial \xi} - \frac{eB_0}{c}v_x \quad (5)$$

That $\phi(x,\xi) = \phi_0(\xi)e^{-\frac{x^2}{l^2}}$ is the average of pondermotive potential in Gaussian form, B_0 is the amplitude of external magnetic field in y direction and l is the laser pulse half-width. We can using Laplace transform rewrite above equations as below,

$$c\frac{\partial \tilde{E}_z}{\partial x} + \frac{s}{\beta}\tilde{E}_x = s\tilde{B}_y \quad (6) \qquad \frac{s}{v_0}\tilde{B}_y = \frac{s}{c}\tilde{E}_x - \frac{4\pi Ne}{c}\tilde{v}_x \quad (7)$$

$$c\frac{\partial \tilde{B}_y}{\partial x} = s\tilde{E}_z - 4\pi Ne\tilde{v}_z \quad (8) \qquad msc\tilde{v}_x = -ec\tilde{E}_x + eB_0\tilde{v}_z - ec\frac{\partial \tilde{\phi}}{\partial x} \quad (9)$$

$$mcs\tilde{v}_z = -ec\tilde{E}_z + \frac{es}{\beta}\tilde{\phi} - eB_0\tilde{v}_x \quad (10)$$

Solving equations system involve above equations we will find electric field component of electromagnetic wave behind laser pulse as below,

$$\tilde{E}_x(x,s) = \frac{1}{(s^2+\omega_p^2)^2 + s^2\omega_c^2}\left[\frac{s^2}{\beta}(s^2+\omega_h^2)\tilde{B}_y - c\omega_c\omega_p^2\frac{\partial \tilde{B}_y}{\partial x} + \frac{s^2\omega_c\omega_p^2}{\beta c}\tilde{\phi} - \omega_p^2(s^2+\omega_p^2)\frac{\partial \tilde{\phi}}{\partial x}\right] \quad (11)$$

$$\tilde{E}_z(x,s) = \frac{1}{[(s^2+\omega_p^2)^2 + s^2\omega_c^2]}\left[\frac{s\omega_c\omega_p^2}{\beta}\tilde{B}_y + sc(s^2+\omega_h^2)\frac{\partial \tilde{B}_y}{\partial x} + \frac{s\omega_p^2}{\beta c}(s^2+\omega_p^2)\tilde{\phi} + s\omega_c\omega_p^2\frac{\partial \tilde{\phi}}{\partial x}\right] \quad (12)$$

That $\omega_p^2 = 4\pi e^2 N/m$ is plasma frequency, $\beta = v_0/c$, $\omega_c = eB_0/mc$ is the cyclotron frequency, $\omega_h^2 = \omega_c^2 + \omega_p^2$ is the upper hybrid frequency,

Using taking inverse Laplace transform and expression of $\tilde{B}_y$ in reference [1] we have,

$$E_x(x,\xi) = -\frac{l\omega_c\omega_p^2}{2\sqrt{\pi}c^3}\int_{\omega_p}^{\omega_h}\frac{\omega^2}{g(\omega_h^2-\omega^2)}\left|\tilde{\phi}_0\right|e^{-\frac{g^2l^2}{4}}[\sin(\omega\xi+gx+\varphi)+\sin(\omega\xi-gx+\varphi)]d\omega +$$

$$\left(-\frac{l\omega_c^2\omega_p^4}{2\sqrt{\pi}c^2}\right)\int_{\omega_p}^{\omega_h}\frac{1}{(\omega_h^2-\omega^2)}\left|\tilde{\phi}_0\right|e^{-\frac{g^2l^2}{4}}[\cos(\omega\xi+gx+\varphi)-\cos(\omega\xi-gx+\varphi)]d\omega +$$

$$\frac{\frac{eE_l^2}{2m\omega_l^2}\omega_p}{c\sqrt{4+(\frac{\omega_c}{\omega_p})^2}}\left[-\sin(\frac{\omega_1\tau}{2})\sin\omega_1(\frac{\tau}{2}-\xi)+\sin(\frac{\omega_2\tau}{2})\sin\omega_2(\frac{\tau}{2}-\xi)\right]e^{-\frac{x^2}{l^2}} + \quad (13)$$

$$\frac{\frac{e\tau E_l^2}{4m\omega_l^2}\omega_p\frac{x}{l^2}}{\omega_c\sqrt{4+(\frac{\omega_c}{\omega_p})^2}}\left[\frac{(\omega_p^2-\omega_1^2)}{\omega_1}\frac{\sin(\frac{\omega_1\tau}{2})}{\frac{\omega_1\tau}{2}}\sin\omega_1(\frac{\tau}{2}-\xi) - \frac{(\omega_p^2-\omega_2^2)}{\omega_2}\frac{\sin(\frac{\omega_2\tau}{2})}{\frac{\omega_2\tau}{2}}\sin\omega_2(\frac{\tau}{2}-\xi)\right]e^{-\frac{x^2}{l^2}}$$

$$E_z(x,\xi)=\frac{l\omega_c^2\omega_p^4}{2\sqrt{\pi}c^3}\int_{\omega_p}^{\omega_h}\frac{\omega}{g(\omega_h^2-\omega^2)^2}\left|\tilde{\phi}_0\right|e^{-\frac{g^2l^2}{4}}[\cos(\omega\xi+gx+\varphi)+\cos(\omega\xi-gx+\varphi)]+$$

$$\frac{l\omega_c\omega_p^2}{2\sqrt{\pi}c^2}\int_{\omega_p}^{\omega_h}\frac{\omega}{(\omega_h^2-\omega^2)}\left|\tilde{\phi}_0\right|e^{-\frac{g^2l^2}{4}}[\sin(\omega\xi-gx+\varphi)-\sin(\omega\xi+gx+\varphi)]+$$

$$\frac{\frac{e\tau E_l^2}{4m\omega_l^2}\omega_p}{c\omega_c\sqrt{4+(\frac{\omega_c}{\omega_p})^2}}[(\omega_1^2-\omega_p^2)\frac{\sin(\frac{\omega_1\tau}{2})}{(\frac{\omega_1\tau}{2})}\cos\omega_1(\frac{\tau}{2}-\xi)-(\omega_2^2-\omega_p^2)\frac{\sin(\frac{\omega_2\tau}{2})}{(\frac{\omega_2\tau}{2})}\cos\omega_2(\frac{\tau}{2}-\xi)]e^{-\frac{x^2}{l^2}}+ \tag{14}$$

$$\frac{\frac{e\tau E_l^2}{2m\omega_l^2}\frac{x}{l^2}\omega_p}{\sqrt{4+(\frac{\omega_c}{\omega_p})^2}}[\frac{\sin(\frac{\omega_1\tau}{2})}{(\frac{\omega_1\tau}{2})}\cos\omega_1(\frac{\tau}{2}-\xi)-\frac{\sin(\frac{\omega_2\tau}{2})}{(\frac{\omega_2\tau}{2})}\cos\omega_2(\frac{\tau}{2}-\xi)]e^{-\frac{x^2}{l^2}}$$

That $\omega_1=\frac{1}{2}(\omega_c+\sqrt{\omega_c^2+4\omega_p^2}),\omega_2=\frac{1}{2}(-\omega_c+\sqrt{\omega_c^2+4\omega_p^2})$ and $g=\frac{\omega_p}{c}\sqrt{\frac{\omega^2-\omega_p^2}{\omega_h^2-\omega^2}}$. Here we use integration contour in complex s plane as Fig.1. For example for $\tilde{\phi}_0(i\omega)=\left|\tilde{\phi}_0\right|e^{i\varphi}$ that $\left|\tilde{\phi}_0\right|=\frac{e\tau E_L}{4m\omega_L^2}\frac{\sin(\omega\tau/2)}{(\omega\tau/2)}$ and $\varphi=-\frac{\omega\tau}{2}$ we have investigated the z component of electric field through plasma with variable density. First we assume that plasma density changes in x direction linearly so that the plasma frequency increased as $\omega_p=\omega_{p0}(\left|l\bar{x}/l_0\right|+1)^{1/2}$ or decreased as $\omega_p=\omega_{p0}(-\left|l\bar{x}/l_0\right|+1)^{1/2}$. Figures 2, 3 show the propagation of E_z in mentioned plasma. Here l_0 is wide of plasma medium, $\bar{x}=x/l$, $\bar{\xi}=c\xi/l$, $\omega_{p0}=0.2\omega_L$ and $\omega_c=0.7\omega_{p0}$. It is clear from figures conical Cherenkov radiation in both cases has been generated. In Fig.2 while we close to edges of plasma medium the amplitude of Cherenkov radiation decreased because of by decreasing ω_p the conditions of generation Cherenkov radiation is lost. In fig.3 in centre of plasma medium has been observed any Cherenkov radiation because of the condition of the generation Cherenkov radiation is not satisfied. By closing to edges and

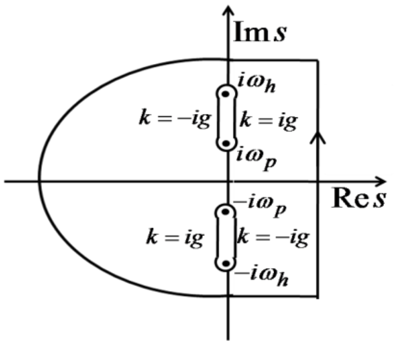

Fig.1: the integration contour in complex s plane.

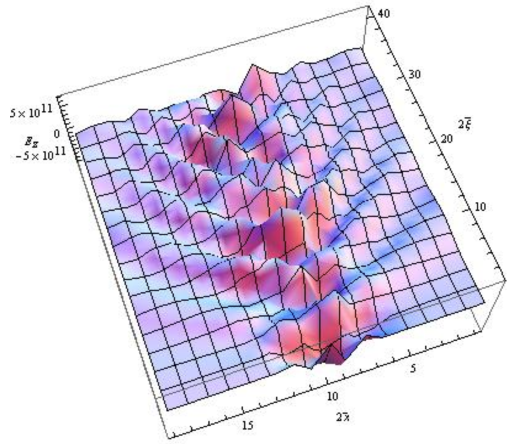

Fig.2. 3D plot of E_z versus transverse distance from propagation axis and along propagation axis for $\omega_p = \omega_{p0}(-|l\bar{x}/l_0|+1)^{1/2}$.

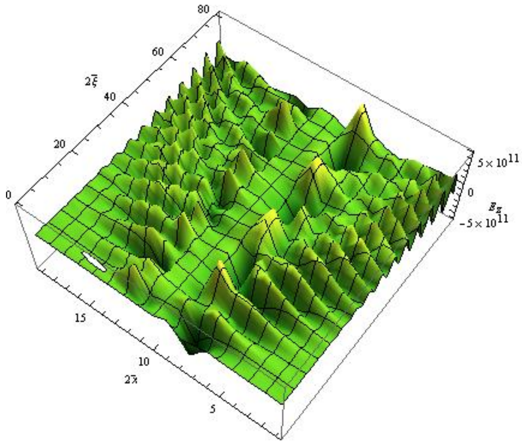

Fig.3. 3D plot of E_z versus transverse distance from propagation axis and along propagation axis for $\omega_p = \omega_{p0}(|l\bar{x}/l_0|+1)^{1/2}$.

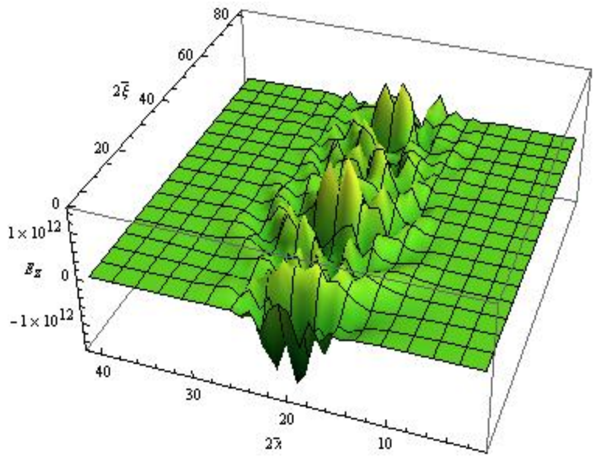

Fig.4. 3D plot of E_z versus transverse distance from propagation axis and along propagation axis for $\omega_p = \omega_{p_0} e^{-4l^2\bar{x}^2/l_0}$.

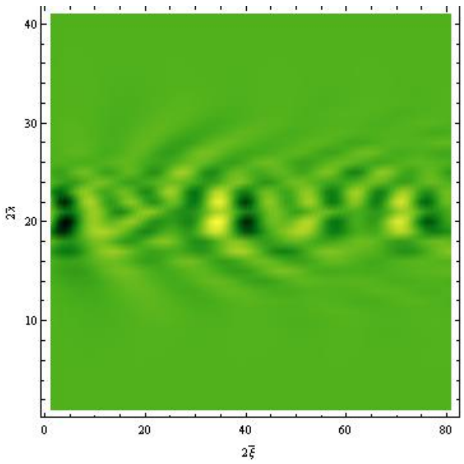

Fig.5. Density plot of E_z versus transverse distance from propagation axis and along propagation axis for $\omega_p = \omega_{p_0} e^{-4l^2\bar{x}^2/l_0}$.

increasing plasma density the Cherenkov radiation has been observed.

In figures 4 and 5 amplitude E_z has been plotted for Gaussian distribution of plasma density. In this case the conical Cherenkov radiation has been generated too. But by closing to edge the condition of generation of the Cherenkov radiation is lost fast.

Results show that plasma density distribution leads to change in amplitude of generated Cherenkov radiation and angle of cone of radiation. So that by selecting suitable distribution of plasma density we can tune the amplitude of radiation and exit point of radiation from plasma medium.

Reference:

[1] M. I. Bakunov, S. B. Bodrov, A. V. Maslov, and A. M. Sergee, PHYSICAL REVIEW E 70, 016401 (2004).