

The contribution of spin radiation reaction forces of classical particles in plasmas in inhomogeneous electromagnetic fields

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Abstract

We study a fully classical covariant relativistic plasma theory including the effect of the spin in the radiation reaction effects of charged particles immersed in inhomogeneous electromagnetic fields. We include the effect of the radiation reaction due to dynamical spin. We calculate the contribution of the spin-electromagnetic coupling in the radiation reaction for an accelerated particle, in the same fashion than classical radiation reaction owing to its charge. We also obtain the contribution of the radiation reaction to the evolution of the spin dynamics.

The dynamics of particles (or plasmas) under the influence of very strong electromagnetic fields in extremely high energy requires an appropriated description that includes a variety of relativistic phenomena. Recently, it has been shown [1] that the spin forces overcome the radiation reaction effects in high-energy plasmas. In this work we generalize these results to include the effect of the radiation reaction due to dynamical spin. We use the Tamm-Good equations [2, 3] for classical spinning particles. These equations are a generalization of the Bargmann-Michel-Telegdi equations [4]. As a result, the complete description of the system involves not only the motion of the charged particle but also the evolution of its spin, considering that the radiation reaction forces on the particle are modified by the spin degrees of freedom.

The covariant equations of motion of a particle with mass m , charge q and with a spin magnetic moment $\mu = q\hbar/(mc)$ are [2, 3]

$$a^\mu \equiv \frac{d\nu^\mu}{d\tau} = \frac{q}{mc} F^{\mu\nu} v_\nu + \frac{\mu}{2m} D^\mu F^{\alpha\beta} \Pi_{\alpha\beta}, \quad (1)$$

$$\frac{d\Pi^{\alpha\nu}}{d\tau} = \frac{q}{mc} (F^\alpha{}_\rho \Pi^{\rho\nu} - F^\nu{}_\rho \Pi^{\rho\alpha}) - \frac{\mu}{2mc^2} (\nu^\alpha \Pi^{\nu\beta} - \nu^\beta \Pi^{\alpha\beta}) D_\beta F^{\rho\sigma} \Pi_{\rho\sigma}. \quad (2)$$

where ν^μ is the four-velocity of the particle, $\Pi^{\mu\nu}$ is the antisymmetric spin tensor, $F^{\alpha\beta}$ is the electromagnetic field tensor, and the operator

$$D^\mu = \partial^\mu + \frac{\nu^\mu \nu^\beta}{c^2} \partial_\beta \equiv \zeta^{\mu\beta} \partial_\beta, \quad \zeta^{\mu\beta} = \delta^{\mu\beta} + \frac{\nu^\mu \nu^\beta}{c^2}. \quad (3)$$

These equations do not include the effect of radiation reaction. The degree of freedom of the dynamical variables are constrained $v_\mu v^\mu = -c^2$, $v_\mu \Pi^{\mu\nu} = 0$, and $\Pi_{\mu\nu} \Pi^{\mu\nu} = 2$ (due to $v_\mu \zeta^{\mu\beta} \equiv 0$).

Prescription to incorporate radiation reaction

For a spinless particle, the radiation reaction force $F_R^{\alpha\nu}$ is included in the momentum equation under the prescription $F^{\alpha\nu} \rightarrow F^{\alpha\nu} + F_R^{\alpha\nu}$, where $F_R^{\alpha\nu}$ is calculated a priori [5]. In order to follow the same prescription, we re-write the previous Eqs. (1) and (2) as

$$\frac{dv^\alpha}{d\tau} = \frac{q}{mc} \mathcal{F}^{\alpha\nu} v_\nu, \quad v_\alpha \frac{d\Pi^{\alpha\nu}}{d\tau} = \frac{q}{mc} \mathcal{F}_{\rho\sigma} v^\sigma \Pi^{\nu\rho}, \quad (4)$$

where $\mathcal{F}^{\alpha\nu} = F^{\alpha\nu} + (\mu c/q)(D^\alpha F^{*\beta\nu} s_\beta - D^\nu F^{*\beta\alpha} s_\beta)$ represents the total antisymmetric force tensor, and $F^{*\beta\nu}$ is the dual of $F^{\beta\nu}$. To include now the radiation reaction force we perform $\mathcal{F}^{\alpha\nu} \rightarrow \mathcal{F}^{\alpha\nu} + F_R^{\alpha\nu}$, where $F_R^{\alpha\nu}$ is the *total* radiation reaction force, including both charge-coupling and spin-coupling of the particle with the electromagnetic field. The final equations, including the radiation reaction effects, will read

$$\frac{dv^\alpha}{d\tau} = \frac{q}{mc} \mathcal{F}^{\alpha\nu} v_\nu + \frac{q}{mc} F_R^{\alpha\nu} v_\nu, \quad v_\alpha \frac{d\Pi^{\alpha\nu}}{d\tau} = \frac{q}{mc} \mathcal{F}_{\rho\sigma} v^\sigma \Pi^{\nu\rho} + \frac{q}{mc} F_R_{\rho\sigma} v^\sigma \Pi^{\nu\rho}. \quad (5)$$

Radiation reaction forces

Now we can calculate the radiation reaction forces following standard procedures [5]. The radiation field is $F_R^{\mu\nu} = (1/2)(F_{ret}^{\mu\nu} - F_{adv}^{\mu\nu})$, where the retarded and advanced fields are given by the potentials $F_{ret,adv}^{\mu\nu} = \partial^\mu A_{ret,adv}^\nu - \partial^\nu A_{ret,adv}^\mu$. The spin contribution to the radiation reaction is included by expressing the potential as the contribution of two sources $A_{ret,adv}^\nu = A(e)_{ret,adv}^\nu + A(\mu)_{ret,adv}^\nu$, where $A(e)_{ret,adv}^\nu$ is the usual potential, due to the charge, and $A(\mu)_{ret,adv}^\nu$ is the contribution due to the effective charge of the spin current. Therefore we finally have $F_R^{\mu\nu} = F_R(e)^{\mu\nu} + F_R(\mu)^{\mu\nu}$.

The retarded and advanced potentials [5] due to the current density of the particle is $A(e)_{ret,adv}^\mu = qv^\mu/(c\rho_{ret,adv})$ with $\rho_{ret,adv} = \mp v_\mu R_{ret,adv}^\mu/c$, and $R^\mu = x^\mu - z^\mu$, where ρ represents the distance from the retarded (advanced) position z^μ of the charged particle to the field point x^μ [5]. Thus, $F(e)_{ret,adv}^{\mu\nu} = \pm(2q/\rho_{ret,adv}c^2)d_\tau(v^{[\mu}R_{ret,adv}^{\nu]}/\rho_{ret,adv})$, where $a^{[\mu}b^{\nu]} \equiv (1/2)(a^\mu b^\nu - a^\nu b^\mu)$. Calculating the strength of the field in the neighborhood of the world line of the particle [5], we find the radiation reaction field owing to the electromagnetic radiation emitted by the charge

$$F_R(e)^{\mu\nu} = -\frac{2q}{3c^4} (\dot{a}^\mu v^\nu - \dot{a}^\nu v^\mu). \quad (6)$$

This is the classical radiation reaction that leads to the Lorentz-Abraham-Dirac (LAD) equation.

On the other hand, to calculate $F_R(\mu)$, we invoke the spin contributions to the retarded and advanced potentials [6, 7] $A(\mu)_{ret,adv}^\alpha = (\mu/4\pi)\partial_\mu(\Pi^{\alpha\mu}/\rho_{ret,adv}) = \mp(\mu/4\pi c\rho)Z^\mu$, where $Z^\mu = d_\tau(\Pi^{\mu\alpha}R_\alpha/\rho)$. Thereby we can find

$$F(\mu)_{ret,adv}^{\mu\nu} = \frac{\mu}{2\pi c^2 \rho} \frac{d}{d\tau} \left(\frac{R^{[\mu} Z^{\nu]}}{\rho} \right), \quad (7)$$

where we have used $\partial^\mu Z^\nu = \dot{Z}^\nu \partial_\mu \tau$. Expanding the quantities in the vicinity the world line of the particle, we find the radiation reaction field due to the spin

$$F_R(\mu)^{\mu\nu} = -\frac{\mu}{2\pi c^4} \left(v^{[\mu} \dot{Z}^{\nu]} + a^{[\mu} \dot{Z}^{\nu]} + \frac{1}{3} \dot{a}^{[\mu} Z^{\nu]} - \frac{a^2}{3} v^{[\mu} Z^{\nu]} \right), \quad (8)$$

where $a^2 = a_\mu a^\mu$, and the Z -terms in the right-hand side are evaluated in the limit $\tau, \rho \rightarrow 0$, such that $Z^\nu = -(1/c)v_\alpha \ddot{\Pi}^{\nu\alpha}$, $\dot{Z}^\nu = -(1/c)d_\tau(v_\alpha \ddot{\Pi}^{\nu\alpha})$, and $\ddot{Z}^\nu = -(3/2c)a_\alpha \ddot{\Pi}^{\nu\alpha} - (1/c)\dot{a}_\alpha \ddot{\Pi}^{\nu\alpha} - (a^2/c^3)v_\alpha \ddot{\Pi}^{\nu\alpha}$.

Total radiation reaction force

A consistent dynamics can be achieved following the prescription of Rohrlich [8, 9, 10], which consists in computing the right-hand sides of Z^ν , \dot{Z}^ν and \ddot{Z}^ν , but omitting the radiation reaction force. The total force (4) on the charged spinning particle is $(\mathcal{F}^{\mu\nu} + F_R^{\mu\nu})v_\nu = \mathcal{F}^{\mu\nu}v_\nu + F_R(e)^{\mu\nu}v_\nu + F_R(\mu)^{\mu\nu}v_\nu$. Recalling that $a_\mu v^\mu = 0$, $Z_\mu v^\mu = 0 = Z_\mu a^\mu$, and $a_\alpha \dot{Z}^\alpha = -\dot{a}_\alpha Z^\alpha$, we find that Eq. (1) becomes

$$a^\mu = \frac{q}{mc} F^{\mu\nu} v_\nu + \frac{\mu}{2m} D^\mu F^{\alpha\beta} \Pi_{\alpha\beta} + \frac{2q^2}{3mc^3} \zeta^\mu_\nu \dot{a}^\nu - \frac{q\mu}{4\pi mc^3} \left(\ddot{Z}^\mu + \frac{1}{2c^2} v^\mu \dot{a}_\alpha Z^\alpha \right). \quad (9)$$

Consistently, the spin tensor evolution equation becomes

$$\begin{aligned} v_\alpha \frac{d\Pi^{\alpha\nu}}{d\tau} &= \frac{q}{mc} F_{\rho\sigma} v^\sigma \Pi^{\nu\rho} + \frac{\mu}{2m} D_\rho F^{\alpha\beta} \Pi_{\alpha\beta} \Pi^{\nu\rho} + \frac{2q^2}{3mc^3} \zeta_{\rho\sigma} \dot{a}^\sigma \Pi^{\nu\rho} \\ &\quad - \frac{q\mu}{4\pi mc^3} \left(\ddot{Z}_\rho + \frac{1}{2c^2} v_\rho \dot{a}_\alpha Z^\alpha \right) \Pi^{\nu\rho}. \end{aligned} \quad (10)$$

The dynamical system composed of Eqs. (9) and (10) generalizes the radiation reaction force presented in Ref. [5] to include the effects of particle spin. Note that Eqs. (9) and (10) are exact.

Electromagnetic wave

Consider a simple electromagnetic wave in a plasma and let us study the effect of radiation reaction of the constituents in its propagation [11]. Let assume that the plasma is cold so the plasma fluid dynamics can be described by the equation

$$\begin{aligned} v^\mu &= \frac{q}{mc} F^{\mu\nu} v_\nu + \frac{\mu}{2m} D^\mu F^{\alpha\beta} \Pi_{\alpha\beta} + \frac{2q^3}{3m^2 c^4} \zeta^\mu_\nu \left(\dot{F}^{\nu\alpha} v_\alpha + \frac{q}{mc} F^{\nu\alpha} F_{\alpha\beta} v^\beta \right) \\ &\quad + \frac{q^3 \mu}{3m^3 c^4} \zeta^\mu_\nu F^{\nu\alpha} D_\alpha F^{\gamma\delta} \Pi_{\gamma\delta} + \frac{q^3 \mu}{3m^3 c^4} \zeta^\mu_\nu D^\nu F^{\alpha\beta} F_{[\alpha\rho} \Pi^{\rho\beta]}. \end{aligned} \quad (11)$$

Here we have approximated the momentum equation to its first order in μ and consider that the radiation reaction effects are small. For sake of simplicity let us assume that the particle spin is constant and aligned with a background magnetic field. So, we can neglect the dynamical spin equation. We study a small amplitude wave propagating in the z -direction, whereas the perturbed electric E and magnetic fields B are in the x and y -directions respectively. The background magnetic field B_0 is also in the y -direction, so the spin becomes relevant in this configuration. Thus, the plasma perturbations oscillates in the x -direction as

$$v^x = \frac{iqE}{\omega m} \left[1 - i \frac{2q^2\omega}{3mc^3} \left(1 + \frac{\mu B_0 k^2}{m\omega^2} \right) \right], \quad (12)$$

where ω and k are the frequency and wave-vector of the electromagnetic wave. The radiation reaction effects (proportional to $q^2/mc^3 = \alpha\hbar/mc^2$, where α is the fine-structure constant) appear correcing the velocities. Using the Maxwell equations, we can find the dispersion relation of the wave in the limit where the radiation reaction effects are small. In this case the dispersion relation is

$$\omega \approx \sqrt{\omega_p^2 + c^2 k^2} - i \frac{2q^2 \omega_p^2}{3mc^3} \left[1 + \left(\frac{\mu B_0}{mc^2} \right) \frac{c^2 k^2}{\omega_p^2 + c^2 k^2} \right], \quad (13)$$

where ω_p is the plasma frequency. Thus, we can see that the radiation reaction introduces a damping to the wave. The spin radiation reaction effects (proportional to μ) enhances this behavior, and the can be dominant in the strong magnetized limit. The new effects become relevant near the Schwinger limit for the electromagnetic field, establishing that this effects could be relevant on extreme astrophysical environments.

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